Some Remarks on Stoic Logic

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For Class 1 C
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Stoicism is an ancient school of philosophy. It was founded by Zeno of Citium in Cyprus around 300 B.C. Except for fragments, most of the writings of the stoics are lost. What little we know about the stoics is that they had a theory of ethics and a theory of logic. In stoic ethics virtue is the only good and the virtuous person is a person who has attained happiness through knowledge. The virtuous person has mastered his, or her, passions and emotions, and finds happiness in him/herself. Perhaps Professor Addona will say more to you on this subject. I will say some things about stoic logic here, although I am sure that Professor Addona has already told you much about the subject.

In ancient times, stoic logic was viewed by many philosophers as an alternative to, and perhaps even opposed to, Aristotle's logic, and that led to some disagreements between the two schools. Today we know that these two logical theories are compatible and today each is a different part of modern mathematical logic. Stoic logic was an early form of propositional logic, and most, but not all, of what the stoics had to say on this subject is now developed as truth-functional propositional logic. Aristotle's logic is the logic of syllogisms, which deals with the logic of such quantifier words as 'all' and 'some', which now can be developed as part of monadic predicate logic, that is, that part quantificational predicate logic that is restricted to one-place predicates.

1 Syllogistic Logic

Of course, one can also study syllogistic as a separate logic the way it had been studied in past centuries. The kind of propositions that are studied in that case are called **categorical propositions**. They are of four types, called **A**, **E**, **I**, and **O**. The **A** and **I** are from the Latin **A**ff**I**rmo, and are of the universal affirmative form 'All A are B' and the particular affirmative form 'Some A are B'. The **E** and **O** are from the Latin n**E**g**O**, and are of the universal negative form 'No A are B' and the particular negative form 'Some A are not B'. Aristotle gave rules for immediate inferences between categorical propositions as well rules about syllogisms, which are arguments having two categorical propositions as premises and a third categorical proposition as conclusion. There are rules for deciding which syllogistic forms are valid and which are invalid. Professor Addona probably has covered this subject with you already.

2 Truth-Functional Logic

One can also study truth-functional propositional logic in a separate way as well, that is without being a part of mathematical logic. In that case we begin grammatically with simple "atomic" sentences of the subject predicate form that do not contain propositional forms as parts, and from these we construct complex, molecular sentences by means of negation and the propositional connectives 'and', 'or', 'if..., then...', and others as well. These are truth-functional connectives, which means that the truth values (truth or falsehood) of molecular sentences constructed by means of these connectives are determined in terms of the sentential parts connected.

Can you think of some other truth-functional connectives in addition to those listed above? There are other connectives, but all of the truth-functional ones can be defined in terms of just negation and conjunction. The stoics also considered causal propositions, such as 'Socrates died because he drank hemlock.' The word 'because' is not a truth-functional connective. Can you think of an example that shows why it isn't truth-functional?

If we use the letters $p, q, r, ...$ to represent atomic sentences, then we can symbolize negation, conjunction, disjunction and conditional sentences as follows:

'not-p' is symbolized as '$\neg p$'.
'p and q' is symbolized as '$p \wedge q$'.
'p or q' is symbolized as '$p \vee q$'.
'If p, then q' is symbolized as '$p \rightarrow q$'.

The stoics distinguished between inclusive and exclusive disjunction, and they preferred the exclusive to the inclusive. Apparently they thought of the inclusive 'or' as somehow deficient. The distinction is explicit in Latin, which uses '*vel*' for the inclusive 'or' and '*aut*' for the exclusive 'or'. Today logicians prefer to use the inclusive 'or', symbolized by $\vee$ for the Latin 'vel'. The exclusive 'or', the symbol for which we can use $\bar{\vee}$, can be defined in terms of the inclusive 'or' and conjunction and negation as follows:

$$p \bar{\vee} q =_{def} (p \vee q) \wedge \neg(p \wedge q).$$

You can represent truth-functions very clearly today by means of truth tables. It is a good exercise to write the truth tables for each of the above connectives.

3 Modality

Stoic logic was developed by Chrysippus (280–207 B.C.) from the teachings of the Megarians. Megaris was an ancient city in east central Greece. It was the capital of Megaris, a mountainous region of ancient Greece on the isthmus of Corinth. One of the most important of the Megarian logicians was Diodorus Cronus. He is famous for his view of modality, which I briefly mentioned in my lecture at the liceo last November. Diordorus claimed that a proposition is possible if it either is or will be true, and hence that a proposition is necessary if it is true and will henceforth always be true. We can represent Diodorus's notion of possibility and necessity by means of the symbols $\Diamond^f$ and $\Box^f$, with the superscript 'f' to indicate that these modalities are with respect to the future. If we also use $\mathcal{F}$ as the future-tense operator, read as 'it will be the case that ...', then Diodorus's definition can then given as follows:

$$\Diamond^f p =_{def} p \vee \mathcal{F}p$$

$$\Box^f p =_{def} p \wedge \neg\mathcal{F}\neg p.$$

Note that we are representing the phrase 'it always will be the case that p' by '$\neg\mathcal{F}\neg p$'. Can you see why that is correct?

Diodorus had an argument, a trilemma, which the ancients called "the Master Argument", that he used to defend his interpretation of possibility and necessity. It is a very interesting argument that can best be formulated in tense logic. Perhaps Professor Addona will tell you a little about it.

Chyrsippus and Diodorus's pupil, Philo, both disagreed with Diodorus on this interpretation of possibility and necessity. Both seemed to think that a proposition is possible if, as Philo said, "in its internal nature it is susceptible of truth," by which he apparently meant something like "self-consistent".

Diodorus and Philo were the first logicians in history to debate the nature of conditionals, i.e., how the truth conditions of propositions of the form 'If P, then Q', are to be understood. Philo gave the now standard truth-functional definition:

'If P, then Q' is true if, and only if, the conjunction , 'P and not-Q' is not true,

which can be expressed in symbols as:

$$(p \rightarrow q) =_{def} \neg(p \wedge \neg q).$$

Diodorus, on the other hand, thought of the conditional as a kind of implication. In particular, Diodorus's interpretation was connected to his temporal view of necessity and possibility, and it amounted in effect to the claim that 'If P, then Q' is true if, and only if, P is not now true and Q false nor will P ever be true and Q false in the future. That is, in terms of the future-tense operator $\mathcal{F}$, Diodorus's notion of the conditional was defined as follows:

$$(If\ p,\ then\ q) =_{def} \neg(p \wedge \neg q) \wedge \neg\mathcal{F}(p \wedge \neg q).$$

This analysis is equivalent to the following:

$$(If\ p,\ then\ q) \leftrightarrow \Box^f(p \rightarrow q),$$

which explains the conditional in terms of the weak truth-functional connective $\rightarrow$ and Diodorus's temporal notion of necessity. Can you see why this is equivalent to Diodorus's definition?

Some philosophers speak of the truth-functional conditional as a "material implication", whereas Diodorus's conditional is a necessary implication, as the above equivalence makes clear. The truth-functional conditional does not represent implication, however, and we should avoid speaking of it that way. Note that the truth-functional conditional $(p \rightarrow q)$ is false in only one circumstance, namely when p is true and q is false. That means that the conditional $(p \rightarrow q)$ is true when the antecedent p is false regardless of whether or not q is true or false; that is,

$$\neg p \rightarrow (p \rightarrow q)$$

is a **tautology**, by which we mean a **logical truth** on the level of truth-functional propositional logic. It also means that the conditional $(p \rightarrow q)$ is true when q is true regardless of whether or not p is true or false; that is,

$$q \rightarrow (p \rightarrow q)$$

is also a tautology. Write a truth table for these formulas to see that they cannot be false.

Some philosophers speak of the above tautologies as "paradoxes", but they are not really paradoxes; and they seem odd only if we think of the truth-functional conditional as an "implication". This seems to be one of the confusions that caused so many of the debates between the stoics and other philosophers, as well as between the stoics themselves. And it is a debate that still goes on today.

Diodorus's interpretation of the conditional as an implication was also criticized, because as an implication it seems too weak. If instead of Diodorus's temporal notion of necessity we use $\Box$ for logical necessity, then we can define implication, the symbol for which we can use $\Rightarrow$, as follows:

$$(p \Rightarrow q) =_{def} \Box(p \rightarrow q).$$

In other words, 'p **implies** q' means that it is logically necessary that if p then q.

Some philosophers still object to this notion of implication, because on this interpretation a logically impossible proposition implies every proposition, i.e., then

$$\neg\Diamond p \rightarrow (p \Rightarrow q)$$

is valid; and a logically necessary proposition is implied by every proposition; that is, then

$$\Box q \rightarrow (p \Rightarrow q)$$

is valid. These are sometimes called the "paradoxes" of implication, but just as with the truth-functional conditional, these are not really paradoxes, because no contradiction follows from them. Nevertheless, today some logicians have developed different kinds of "relevant logics" to represent their interpretation of the conditional as an implication, and in particular one in which the antecedent p is "relevant" to the consequent q of the conditional 'if p, then q'.

Finally, note that modal propositional logic, i.e., truth-functional propositional logic extended to include formulas of the form $\Box p$ and $\Diamond q$ is not truth-functional in its modal part. For example, if p is true, then although $\Diamond p$ then must also be true, we do not automatically know if $\Box p$ is true or false; and if p is false, $\Box p$ must also be false, but we do not automatically know if $\Diamond p$ is true or false. In other words, you will not be able to completely fill in a truth table for $\Diamond p$ or $\Box p$.

4 Propositions as Lekta

What we have been calling propositions the stoics called **lekta** ($\lambda\epsilon k\tau\acute{\alpha}$) that are either true or false. The word 'lekton' means 'what is meant'. There was some debate among Stoics as to whether a proposition, i.e., a lekton, is just a thought, a sentence spoken or written, or something incorporeal, i.e., an abstract entity. Most stoics seem to think propositions were incorporeal, abstract entities. But that is strange because the stoics were materialists and did not believe in abstract entities. In fact, they held that our minds and thoughts, as well as particular spoken or written sentences, are all material objects. So how could they say that propositions (lekta) are incorporeal, abstract entities?

The question of the ontological status of propositions is still an important issue today, and there is much disagreement on this among philosophers. Some philosophers, called nominalists, think that propositions can only be (declarative) sentences. Others, called conceptualists, think that propositions can only be judgments, i.e., certain kinds of mental acts; and yet others, called realists, think that propositions are real, abstract entities that exist independently of the mind and society in a kind of Platonic realm of "meanings". My own position is that of conceptual intensional realism. For me propositions, and intensional objects in general, are abstract entities, but they do not pre-exist the evolution of consciousness and culture, and although they are not independent of mind and society they nevertheless have a certain amount of autonomy. Could the stoics perhaps have meant something like this when they said that propositions are incorporeal, abstract entities?

In addition to propositions, i.e., the lekta that are true or false, the stoics also spoke of questions, commands, and even prayers as lekta that are not either true or false. In other words, some lekta are neither true nor false. What about **future contingent propositions**, i.e., propositions about the future that are not necessarily true or necessarily false? Does the principle of excluded middle fail in those cases? That is, are they neither true nor false?

According to some philosophers, Aristotle, in his famous sea battle example, held that future contingent propositions are neither true nor false. In this example, the Athenians were planning a sea battle that was to occur the next day, and the position was taken by some that if the proposition that the Athenians will win the battle the next day is already true, then it's already settled, i.e., determined, and hence they need not even bother to prepare for the battle (which is a really dumb thing to say or do).

Notice that the principle of excluded middle has the form:

$$(p \vee \neg p).$$

Construct a truth table for this formula and note that it always comes out true, i.e., that it is a tautology. That seems to be the stoics view, i.e., because propositions are lekta that are either true or false, the principle of excluded middle must be a truth of logic, and hence even contingent future propositions must be either true or false.

But what about Aristotle's sea battle argument? How do we answer the problem there? Note that because $(p \vee \neg p)$ is logically true, then so is $\Box(p \vee \neg p)$. But from this it does not follow that $(\Box p \vee \Box\neg p)$ is also logically true. In other words, the distribution of $\Box$ over a necessary disjunction is not valid. Thus, where p is the proposition expressed by the Athenians that they will win the sea battle the next day, it is indeed necessary that they will either win or not win the battle, but it does not follow that it is necessary that they will win or that it is necessary that they will not win the battle. Determinism does not follow in other words. Do you think maybe that was really Aristotle's point about this example?

There are more things that can be said about the stoic's theory of logic, but I think we have covered enough ground here.