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Bernard Bolzano's Contributions to Logic and Ontology. First Part: A - B

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1. Morscher, Edgar. 2008. *Bernard Bolzano's Life and Work*. Sank Augustin: Academia Verlag.
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"Despite the enormous increase of interest in Bolzano's philosophy during the last decades, an up-to-date monograph on Bolzano's philosophy is still a desideratum. The last book that might be called a monograph on Bolzano's philosophy dates from almost 100 years ago; it is Shmuel Hugo Bergmann's *Das philosophische Werk Bernard Bolzanos* (Halle/S. 1909), written in the spirit of the Brentano school, in particular of Bergmann's teacher Anton Marty.
When I was invited by the Editors of the *Stanford Encyclopedia of Philosophy* to contribute the entry on Bernard Bolzano, I took it as a challenge for starting my long-standing plan to write a monograph on Bolzano's philosophy. The present book is, to be clear, merely the first step toward this end. In this respect I can benefit from the generous copyright regulations of the Stanford Encyclopedia of Philosophy which allow the entries to appear also in print. The author welcomes any kind of comments and criticism to the present printed version of the Internet article in order to take them into consideration in his projected monograph on Bolzano's philosophy.
(...)
I dedicate this book to the greatest and most meritorious Bolzano scholar ever, Jan Berg, without whom Bernard Bolzano would not be seen as the outstanding philosopher as we now know him to be." (From the Preface)

2. Rusnock, Paul, and Šebestik, Jan. 2019. *Bernard Bolzano: His Life and Work*. New York: Oxford University Press.
Contents: A Note on Citations XIII; Acknowledgements XV; Preface XVII; Chronology XX; Extracts XXXI-XXXII; Introduction 1; 1. Bolzano's Life 5; 2. Ethics 83; 3. Political Philosophy 105; 4. Philosophy of Religion 139; 5. Catholicism and the Catholic Church 167; 6. Logic 187; 7. Theory of Knowledge 337; 8. Ontology and Metaphysics 405; 9. 9 Mathematics 502; 10. Aesthetics, the Science of Beauty 544; Afterword 595; Bibliography 599; Index of Persons 647; Index of Subjects 653-667.
"In the English-speaking world, Bolzano is best known for his work in logic and mathematics. There are certainly things of great importance and beauty in these parts of his work. We have already written, each of us, on these matters, and will have more to say about them in this book. But a faithful portrait of Bolzano cannot limit itself to this, for until he was 40 years old, he was only able to pursue these subjects in his spare time. With his considerable gifts in these non-controversial areas, he certainly might have led a distinguished life of speculation as a mathematician or philosopher. Instead he chose quite deliberately to plunge into the

turbulent political life of his homeland, applying his formidable intelligence, energy, and determination to the reform of his society and its institutions. It is here that we shall begin." (p. 3)

Bibliographies

1. Lapointe, Sandra. 2019. Bernard Bolzano. *Oxford Bibliographies Online*: 1-13.
"Introduction
Bernard Bolzano's (b. 1781-d. 1848) originality and numerous anticipatory insights have deserved him a unique position in the history of philosophy. While scholarship trudged for more than a hundred years after his death, in the second half of the 20th century, Bolzano emerged at once as the most significant logician between Leibniz and Frege, one of Kant's most scrupulous and formidable critics, and what may have been one of the greatest single influences on Brentano's students, in particular Twardowski and Husserl, beside Brentano himself. For a variety of reasons—e.g. methodological and thematic proximity—analytic philosophers have found in Bolzano a congenial interlocutor. As a result, most commentaries and discussions tend to focus on aspects of Bolzano's views on logic and its philosophy, in particular his treatment of questions relating to analyticity, deducibility, and grounding in his *opus magnum*, the *Theory of Science* (1837). But the wealth of ideas we find throughout his work is far from exhausted. Because Bolzano research is young, there still subsist substantial gaps in the literature. More importantly, perhaps, there is ample space for reassessments of standard interpretations. The present bibliography is designed so as to both provide interested researchers and prospective scholars with a sense of those issues that constitute the poles of current discussions and leave room for ulterior updates."

Studies on Bolzano's Logic and Ontology

Abbreviations: WL = Bolzano's Wissenschafslehre (1837)

1. *Bernard Bolzano, 1781-1848 Bicentenary. Impact of Bolzano's Epoch on the Development of Science*. 1982. *Acta Historiae Rerum Naturalium Necnon Technicarum*. Prague: Institute of Czechoslovak and General History CSAS
Proceedings of the Bernard Bolzano's Bicentenary Conference, Prague, September 7-12, 1981.
Contents: (only the contributions on Bolzano are cited): Zdenèk Ceska: B. Bolzano and the Charles University 3-8; Lubos Novy: Bolzano's contribution to science and society 9-23; Günter Kröber: Bernard Bolzano und das Problem des Wissenschfatlich-technischen und sozialen Fortschrittes 103-128; Jan Janko: Veränderungen der Naturgeschichte und ihre Differenziation in der I. Hälfte des 19. Jahrunderts 129-154; Joseph W. Dauben: Progress of mathematics in the early 19th century: Context, contents and consequences 223-260; Jan Berg: 415-425; Karel Berka: B. Bolzano's philosophy of science 427-442; Jonathan Cohen: Bolzano's theory of induction 443-457; Helmut Metlzer: Bernard Bolzano Beitrag zum Gestaltwandel der Logik 479-489.
2. "Bolzano Studien." 1987. *Philosophia Naturalis* no. 24:351-499
Content (Essays in English):
II. Basic Questions of Logic and Semantics.
Jan Berg: Bolzano and Situation Semantics: Variations on a Theme of Variation 373-377; Peter Simons: Bolzano, Tarski, and the Limits of Logic 378-405;

III. On the Problem of Paradoxis.
Jan Berg: Is Russell's Antinomy Derivable in Bolzano's Logic? 406-413;
IV: Probability, Induction and Syllogistic.
Jan Berg: Bolzano on Induction 442-446;
V. Contributions on Bolzano's Metaphysics.
Rolf George: Bolzano on Time 452-468.

3. *Bolzano's Wissenschaftslehre 1837-1987. International Workshop (Firenze, 16-19 September 1987)*. 1992. Firenze: Olschki
Content: Premessa V; Rolf George: Concepts of Consequence 3; Detlef D. Spalt: Bolzano's Zahlbegriffe. Bislang Übersehene Marksteine Feudal-absolutistischer Mathematik 27; Ettore Casari: An Interpretation of Some Ontological and Semantical Notions in Bolzano's Logic 55; Jan Berg: The Connection Between Bolzano's Logic of Variation and His Theory of Probability 107; Benson Mates: Bolzano and Ancient Pyrrhonism 121; Karel Berka: Bolzanos Lehre vom naturlichen Schliessen 141; Jan Sebestik: The construction of Bolzano's Logical System 163; Carlo Cellucci: Bolzano and Multiple-Conclusion Logic 179; Rudolf Haller: Bolzano and Austrian Philosophy 191; Massimo Mugnai: Leibniz and Bolzano on the "Realm of Truths" 207; Bob van Rootselaar: Axiomatics in Bolzano's Logico-Mathematical Research 221-230.

4. "Bolzano and Analytic Philosophy." 1997. *Grazer Philosophische Studien* no. 53:1-266
Edited by Wolfgang Künne, Mark Siebel, Mark Textor.
Proceedings of the International Symposium held in Hamburg 3rd-t5th January 1997.
Contents: Preface; VI; Dagfin Føllesdal: Bolzano's Legacy 1; Jan Berg: Bolzano, the Prescient Encyclopedist 13; Jan Šebestik: Bolzano, Exner and the Origins of Analytical Philosophy 33; Paul Rusnock: Bolzano and the Traditions of Analysis 61; Peter Simons: Bolzano on Collections 87; Ali Behboud: Remarks on Bolzano's Collections 109: Mark Siebel: Variation, Derivability and Necessity 117; Edgar Morscher: Bolzano's Method of Variation: Three Puzzles 139; Rolf George: Bolzano's Programme and Abstract Objects 167; Mark Textor: Bolzano's Sententialism 181; Wolfgang Künne: Propositions in Bolzano and Frege 203: Michael Dummett: Comments on Wolfgang Künne's Paper 241; Carsten Uwe Gieske: Bolzano's Notion of Testifying 249-266.
"From January 3rd to January 5th 1997 the international symposium *Bolzano and Analytical Philosophy* took place in Hamburg.
(---)
Michael Dummett once called Bernard Bolzano the "great-grandfather of analytical philosophy".[*] The aim of the symposium was to explore whether Bolzano's analytical great-grandchildren can still learn from their Bohemian ancestor. We hope the symposium will stimulate further systematic and exegetical research in thisarea." (from the *Preface*)
[*] Michael Dummett, *Origins of Analytic Philosophy*, Cambridge (MA): Harvard University Press 1993, p. 171.

5. "Bernard Bolzano." 1999. *Revue d'Histoire des Sciences* no. 52:339-506
The following contributions are in English:
Charles Chihara: Frege's and Bolzano's rationalist conceptions of arithmetic 343; Joëlle Proust: Bolzano's theory of representation 363; Johannes Hafner: Bolzanos' criticism of indirect proofs 385; Paul Rusnock: Philosophy of mathematics: Bolzano's responses to Kant and Lagrange 399.

6. "Bolzano and Kant." 2012. *Grazer Philosophische Studien* no. 85
Guest Editor: Sandra Lapointe.
Table of Contents: Sandra Lapointe: Introduction 1; Sandra Lapointe: Is Logic Formal? Bolzano, Kant and the Kantian Logicians 11; Nicholas F. Stang: A Kantian Reply to Bolzano's Critique of Kant's Analytic-Synthetic Distinction 33; Clinton Tolley: Bolzano and Kant on the Place of Subjectivity in a *Wissenschaftslehre* 63;

Timothy Rosenkoetter: Kant and Bolzano on the Singularity of Intuitions 89; Waldemar Rohloff: From Ordinary Language to Definition in Kant and Bolzano 131-149.

7. Adair-Toteff, Christopher. 2002. "Bolzano's *Gesamtausgabe*." *British Journal for the History of Philosophy* no. 10:127-133
"Shortly after Bolzano's death there was an attempt to collect his works as a *Gesamtausgabe*, but there was little interest. Another attempt was made to honour the sixtieth anniversary of his death in 1908 but that also failed.
The *Wissenschaftslehre* was republished in four volumes in the early 1930s, but it was not until the late 1960s that a number of international Bolzano scholars succeeded in planning the Bernard Bolzano *Gesamtausgabe*. The first of a projected 100+ volumes appeared in 1969 – an introductory book that was a biography. The *Gesamtausgabe* is composed of four series:
I Writings (*Schriften*);
II Posthumous writings (*Nachlaß*);
III Correspondence (*Briefwechsel*);
IV Documents (*Dokumente*)." (p. 128)
(...)
"Bolzano's *Wissenschaftslehre* was published in four volumes in Germany in 1837. It consists of five Books: Theory of Fundamentals, Theory of Elements, Theory of Knowledge, Heuretic, and Theory of Science Proper.
In the first book Bolzano defines *Wissenschaftslehre* as the attempt to provide an account of science in general. Its function is not to discuss any individual sciences but rather to determine the rules by which all truths can be determined to belong to the individual sciences. He acknowledges that he is using science and doctrine in slightly unusual terms but insists that his account is superior to either Fichte's or Hegel's. Bolzano maintains that Fichte's *Wissenschaftslehre* offers a mistaken attempt to provide a doctrine of knowledge in general. Bolzano does admit that he does not really understand Fichte's philosophy. Bolzano also admits to a similar lack of comprehension of Schelling and Hegel.
Bolzano uses a number of technical terms. One is a 'proposition in itself' (*Satz an sich*) which he takes to be an assertion that something is or is not the case. A proposition in itself need not be uttered or even thought. In a similar vein Bolzano speaks of 'truth in itself' (*Wahrheit an sich*) or 'objective truth' that does not have real existence in contrast to recognized truths that do exist. And, there are 'representations in themselves' (*Vorstellungen an sich*). Examples of ideas in themselves would be 'Caius' and 'wisdom' in the proposition 'Caius has wisdom'. Representations in themselves have neither truth nor existence. Much in the first three books appears to be an attack on Kantian philosophy – he has no use for Kant's psychology and his *Ding an sich*. But, these books are not simply negative. He claims to have shown that there are truths and that we can recognize them and he sets out the conditions under which we can recognize them." (p. 129)

8. Bar-Hillel, Yehoshua. 1950. "Bolzano's Definition of Analytic Propositions." *Methodos*:32-55
Published also in *Theoria* 16, 1950, pp. 91-117.
Reprinted in: Y. Bar-Hillel, *Aspects of language. Essays and lectures on philosophy of language, linguistic philosophy and methodology of linguistics*, Jerusalem: The Magnes Press - The Hebrew University, 1970, pp. 3-28.
"In view of recent discussions on the nature of analytic truth, it should be rather interesting to inquire into the treatment which this subject received by the most outstanding logician of the first half of the 19th century, the Austrian philosopher, theologician, and: physicist Bernard Bolzano.
Our investigation will turn upon section 148 of Bolzano's four volumed masterwork *Wissenschaftslehre* (1837). Only occasionally shall we need to refer to other parts of this work. This section, headed "Analytic or Synthetic Propositions", comprises pages 83-89 of the second volume and is divided into three subsections of less than two pages altogether, followed by four annotations, filling the next five pages. I

dwell so long upon these bibliographical particulars only to bring into full light the wealth of systematic and historic material contained in these few pages.
1. Pre-History.
Bolzano's aim, in § 148, was to define a concept which could serve as an adequate explication for what is now commonly termed 'logical truth'. Though this aim is nowhere explicitly stated, there can be no doubt about it, just as Kant before him and many logicians after him doubtless aimed at the same target when they proposed their respective definitions.
Bolzano devotes the greater part of his fourth annotation the discussion of many such attempts made by his predecessors and contemporaries. He mentions *Aristotle, Locke, Crusius* (the German logician of the first half of the 18th century who was probably the first to use the terms 'analytic' and 'synthetic' in their Kantian senses), Kant and many other minor philosophers. He easily succeeds in proving the inadequateness of Kant's two definitions for analytic', the one given in his *Logik* and equating, in effect, Analytic (1) with Identical, the other much better known in the introduction to the Critique of Pure Reason, where he proposes to call propositions 'analytic', whose predicate-concept is contained (perhaps in a hidden manner) in the subject-notion. Bolzano points out (p. 87) the vagueness of the term 'contained' and argues that, according to a quite natural interpretation of this term, the proposition « The father of Alexander, King of Macedonia, was King of Macedonia » ought to be analytic, a consequence which Kant certainly did not intend to be drawn.
But to even more refined versions of Kant's definition, given by some of his followers, replacing the vague 'contained' by more concise terms, such as those making use of 'essential characteristics', Bolzano objects that only one type of proposition conforms to them, namely 'A (which is B) is B'. But should not, continues Bolzano, also propositions of the type 'Every object is either B or non-B' be counted among the analytic propositions?
Having thus convinced himself of the inadequateness of all prior approaches, he started to attack the subject along a new and highly original line." (pp. 3-4 of the reprint).

9. ———. 1952. "Bolzano's Propositional Logic." *Archiv für Mathematische Logik und Grundlagenforschung* no. 1:65-98
Reprinted in: Y. Bar-Hillel, *Aspects of language. Essays and lectures on philosophy of language, linguistic philosophy and methodology of linguistics*, Jerusalem: The Magnes Press - The Hebrew University, 1970, pp. 33-68.
"1848 is a remarkable year not only in general history; in the history of human culture and thought it will be remembered also as the birth year of *G. Frege*, "the greatest logician of the 19th century",(2) and should be remembered as the year in which the death of the greatest logician between Leibniz and Frege, the Czech Bernard Bolzano, occurred. So far, little has been done to evaluate his important contributions to logical theory,(3) and I hope that the present article will help to undo this undeserved wrong.
The purpose of this article is very restricted: only a small part of Bolzano's investigations will be dealt with, i.e. his propositional logic, and even this in a limited degree. This theory is in my opinion not only a master-work of outstanding historical interest, I also believe that it contains many features neglected even by modern symbolic logic and nevertheless worthy of close study. I am convinced that such a study will considerably enrich our logical technique and terminology.
Since our principal aim is to emphasize the impact which Bolzano's ideas should have on contemporary logic, I shall allow myself to depart, sometimes considerably, from his original account and even to disregard parts of his theory unacceptable to us which do not play any decisive role in its construction, all this, of course, after due warning shall have been given.
I shall summarize the contents of §§147, 154-160 of Bolzano's *Wissenschaftslehre* (1837), with which alone this study is concerned, in 28 definitions and 95 theorems. Most of these theorems will not be proved, for the sake of brevity, but the reader will, in general, be able to supplement the proofs by himself. Many definitions and a

few theorems will be illustrated by simple examples. Major departures from Bolzano's original account will be specially mentioned and justified.
In the second part of the study I shall outline the place of Bolzano's contribution within the framework of modern semantics, by its detailed comparison with the corresponding parts of R. Carnap's two volumes of Studies in Semantics. This comparison will give us a certain perspective on the bearing of Bolzano's highly original innovations for modern research, and on the other hand enable us to see clearly the precise nature of some of his shortcomings." (pp. 33-34)
(1) This article has been written as an outcome of conversations with Professor Hugo Bergman of the Hebrew University, Jerusalem, and a joint reading of the relevant passages of Bolzano's *Wissenschaftslehre*. It is to Professor Bergman that I owe the general ideas on which this paper is based.
(2) According to A. Tarski, *Introduction to Logic*, 1941, p. 19.
(3) The following is a list of the most important articles dealing mainly with Bolzano's contributions to logic which have appeared in the last two decades:
W. Dubislav Bolzano as Vorlaufer der mathematischen Logik", Philosophisches Jahrbuch der Görres-Gesellschaft, vol. 44 (1931), pp. 448-456.
H. Scholz, "Die Wissenschaftslehre Bolzanos", Semesterberichte, 9. Semester, 1936/37, pp. 1-53.
H. Scholz, "Die Wissenschaftslehre Bolzanos", Abhandlungen der Fries'schen Schule, n. s. vol. 6 (1937), pp. 399-472.
H. R. Smart, "Bolzano's Logic", The Philosophical Review, vol. 53 (1944), pp. 513-533.
I have not been able to get hold of Scholz's second article, but since it is, according to the *Journal of Symbolic Logic*, only a somewhat broader version of his first article, the loss is probably not too great. My quotations from Scholz will therefore refer always to his first article.

10. ———. 2006. "Bolzano, Bernard." In *Encyclopedia of Philosophy: Second edition. Vol. 1*, edited by Borchert, Donald M., 646-648. New York: Thomson Gale
First edition 1967.
"Bernard Bolzano, a philosopher, theologian, logician, and mathematician, was born in Prague, where his father, an Italian art dealer, had settled; his mother was a German merchant’s daughter. Bolzano studied mathematics, philosophy, and theology in Prague and defended his doctor’s thesis in mathematics in 1804; he was ordained a Roman Catholic priest the following year. Shortly thereafter he was appointed to a temporary professorship in the science of religion at Karlova University in Prague and two years later was given a newly established chair in this field. Some time later he was accused of religious and political heresy and was removed from his teaching position in December 1819. Bolzano spent much of his time thereafter with the family of his friend and benefactor, A. Hoffmann, at their estate in southern Bohemia. He had difficulty getting his later publications through the Metternich censorship. Some of his books were put on the Index, and many appeared only posthumously. Some manuscripts are yet to be published; the most important of these are in the National Museum and the University Library in Prague, others are in the Österreichische Nationalbibliothek in Vienna. In December 1848, Bolzano died of a respiratory disease from which he had suffered for most of his life." (p. 646)

11. Behboud, Ali. 1997. "Remarks on Bolzano's Collections." *Grazer Philosophische Studien* no. 53:109-115
"With his "zoology of general kinds of collective entities"[*], Peter Simons has sketched - in a very helpful way - some main alternatives for possible interpretations of Bolzano's collections. As he pointed out, we may not have more thanjust a best fit - and, in fact, he proposes that Bolzano' s account is "a distinct and distinctive theory of collections". I do agree with Simons that there are many difficulties we have to face when we try to fit Bolzano' s account into one of our theories. Also, ignoring the considerable historical interest for the moment, the price the technical inconveniences as well as the conceptual complexities which such a fit might

require in the end could be too high to be of any practical interest. Nevertheless, I would like to try my luck for a best fit. (1)" (p. 105)
(1) Simons is clearly right that Bolzano does not develop a systematic theory of collections. However, collections play a fundamental role for Bolzano (even beyond his mathematical theories), since anything whatsoever is either a collection or an "atom" (*einfach*). So it is no surprise thatthe notion of a collection is almost ubiquitous in Bolzano's works.
[*] P. Simons, *Bolzano on Collections*, (1997), p. 87.

12. Bellomo, Anna, and Massas, Guillaume. 2021. "Bolzanos' Mathematical Infinite." *The Review of Symbolic Logic*:1-55
Abstract: "Bernard Bolzano (1781–1848) is commonly thought to have attempted to develop a theory of size for infinite collections that follows the so-called part–whole principle, according to which the whole is always greater than any of its proper parts. In this paper, we develop a novel interpretation of Bolzano's mature theory of the infinite and show that, contrary to mainstream interpretations, it is best understood as a theory of infinite sums. Our formal results show that Bolzano's infinite sums can be equipped with the rich and original structure of a non-commutative ordered ring, and that Bolzano's views on the mathematical infinite are, after all, consistent."

13. Benoist, Jocelyn. 2002. "Husserl and Bolzano." In *Phenomenology World-Wide: Foundations, Expanding Dynamisms, Life-Engagements. A Guide for Research and Study*, edited by Tymieniecka, Anna-Teresa, 98-100. Dordecht: Kluwer
"Bolzano's influence on Husserl has recently come to be appreciated at its true worth. It is actually an extremely important one.
Husserl recalls that he attended a lecture given by Brentano on *The Paradoxes of the Infinite*. But he mayalso have heard of Bolzano from his mathematics professor Karl Weierstrass. Papers written by the Brentanist Benno Kerry (*Ueber Anschauung und ihre psychische Verarbeitung* 1885-1891) also had a certain bearing on Husserl's knowledge of Bolzano. The insightful discussion of the Bolzanian thesis of "representations without object" to be found in Twardowski's book *On Content and Object of Presentations* succeeded in interesting Brentano's pupil in that author definitively.
The psychological point of view adopted by Husserl in the *Philosophy of Arithmetic* does seem to be very far removed from that of Bolzano. However, on the other hand, Husserl's break with psychologism, which took place during the years 1894-1896, appears to have had a direct bearing on Husserl's better acquaintance with Bolzano's *Wissenschaftslehre* during that period. We now know that in 1896 Husserl gave a course which was not, as is commonly believed, a draft of the *Prolegomena*, but was rather a survey of Bolzano's *Wissenschaftslehre*. [*]
What really matters is that the break with Brentanian psychologism was indeed a Bolzanian move. Such a move allows us to speak of a "Bolzanian tum" in Husserl's thought, taking place around 1896. From that point of view, Husserl's thought, "phenomenology", may and must be understood as a (quite strange) kind of compromise between Brentanian descriptive psychology and Bolzanian propositionalism." (p. 98)
[*] E, Husserl, *Logik. Vorlesung 1896*, edited by Elisabeth Schuhmann, Dordrecht: Springer 2001.

14. Benthem, Johan van. 1984. *Lessons from Bolzano*. Stanford: Center for the Study of Language and Information, Leland Stanford Junior University
"Bernard Bolzano's contributions to logic, largely unnoticed in the 19th century, have been receiving ever more attention from modern logicians (cf. Scholz, 1937; Berg, 1962; Corcoran, 1975). As a result, it has already become something of a commonplace to credit Bolzano with the discovery of the notion of logical consequence in the semantic sense. Now, this particular attribution, whether justified or not, would at best establish a historical link between modern logical concerns and Bolzano's work. The purpose of the present note, however, is to bring

out three important aspects of that work that are still of contemporary systematic interest. No detailed textual study of Bolzano is needed to substantiate our suggestions. We shall refer to well-documented 'public' aspects of the 'Wissenschaftslehre' (Bolzano, 1837), pointing out their more general logical significance." (p. 1).
References
Jan Berg, *Bolzanos's Logic*, Stockholm: Almqvist & Wiksell 1962.
John Corcoran, "Meanings of Implication." *Diálogos*, 9, pp. 59-76, 1975.
Heinrich Scholz, "Die Wissenschaftslehre Bolzano's: Eine Jahrhundert-Betrachtung." *Abhandlungen der Fries'schen Schule*, 6, pp- 399-472, 1937.

15. ———. 1985. "The Variety of Consequence, According to Bolzano." *Studia Logica* no. 44:389-403
Abstract: "Contemporary historians of logic tend to credit Bernard Bolzano with the invention of the semantic notion of consequence, a full century before Tarski. Nevertheless, Bolzano's work played no significant role in the genesis of modern logical semantics. The purpose of this paper is to point out three highly original, and still quite revelant themes in Bolzano's work, being a systematic study of possible types of inference, of consistency, as well as their meta-theory. There are certain analogies with Tarski's concerns here, although the main thrust seems to be different, both philosophically and technically. thus, if only obliquely, we also provide some additional historical perspective on Tarski's achievement."

16. ———. 2003. "Is There Still Logic in Bolzano's Key?" In *Bernard Bolzanos Leistungen in Logik, Mathematik und Physik*, edited by Morscher, Edgar, 11-34. Sankt Augustin: Academia Verlag
"Bolzano is widely seen as the philosopher of abstract propositions, far removed from psychological blemishes. Nevertheless, many themes in this paper suggest links with the actual reasoning performed by non-Platonic humans like us. We saw this with attention to diverse styles of task-dependent reasoning, with degrees of logicality for the expressions of natural language that we actually use, with inferences transferring information across discourse situations, with global architecture of reasoning styles, or with mixtures of such neatly compartmentalized logical activities as semantic evaluation and proof. When we take all this seriously, it becomes hard not to go one step further, and do something which Frege has forbidden - but probably also Bolzano: take the psychological facts seriously. All the above topics border on cognitive science and the experimental study of human reasoning, and the eventual agenda of modem logic will also have to come to better terms with that than the by now pretty stale slogan of 'anti-psychologism'.
Conclusions.
We have surveyed some aspects of Bolzano's logic from a modern standpoint, stressing in particular his different styles of consequence, the essential ternary nature of consequence when language is taken into account, and the mixed notion of consequence in a model. In all three cases we included some new technical observations to show that the issues are still alive. But the more general thrust is this.
Bolzano's work remains interesting for logic today, both in its general sweep, and in some of its details. Partly, it is attractive precisely because it is so non-mainstream, and hence valuable for modern agenda discussions. Its themes crossing logic and philosophy of science reflect current rapprochements, while its thrust also seems to fit with some themes from AI. Classical mathematical logic has had an Austrian icon in Kurt Gödel: modem logic might consider at least having a Czech-Austrian patron saint." (pp. 30-31).

17. ———. 2013. "Bernard Bolzano's *Wissenschaftslehre*." *Topoi* no. 32:301-303
"In this review, I will focus on Mr. Bolzano's thoughts about logic, even though he offers much more than that to readers interested in theory of science and general philosophy. Modern logic has become more and more technical, cutting itself loose from its broader origins as the study of reasoning, and philosophers of logic

slavishly play up to this trend by devising ever more arcane criteria of 'logicality' that apply only to a small elite of 'logical constants', making it harder and harder for new themes to enter the field. Refreshingly, Mr. Bolzano does none of this. He resolutely ignores received wisdom in logic textbooks, and deftly avoids entanglement in the scholasticism of our modern age. Instead, he just goes back to what logic is about, and rethinks it afresh." (p. 301)

18. Berg, Jan. 1962. *Bolzano's Logic*. Stockholm: Almqvist & Wiksell
Contents: Preface 5; Introduction 7; Abbreviations 12; I. Bolzano's life and work 13; II. A logical frame 33; III. Bolzano's fundamental notions 41; IV. Bolzano's logic of variation 92; V. Other logical theories 146; VI. Bolzano's logic of entailment 151; VII. Bolzano's philosophy of mathematics 165; Bibliography 179; Index of proper names 213-214.
"Bernard Bolzano made essential contributions to, *inter alia*, theology, logic, and mathematics. For political reasons, however, he was prevented from influencing to a full extent the age in which he lived. As a mathematician his name has survived, although many of his most remarkable results were not published until a century after their conception. As a logician he has begun to appear again in scattered articles and comments. In this study I have tried to give an exposition and evaluation of the main ideas of his logic from a modern viewpoint.
Important parts of Bolzano's theories of logic and semantics were new with him, and when these ideas reappear later they were independent of him. This position, in part outside of the historical development, makes it highly pertinent to compare Bolzano's theories directly with modern logic. When tracing the lineage of some of Bolzano's ideas I have even projected the earlier theories onto a modern scheme of reference.
In the first chapter I shall briefly mention the genesis of Bolzano's main works. But I am not primarily interested in the genetic aspect of Bolzano's theories. Therefore, as often as possible I shall consider Bolzano's various formulations of his ideas as if they were parts of a simultaneous whole.
My analysis of Bolzano's achievements in logic, semantics, and mathematical philosophy is based on his mature production after 1820, when he started writing his *magnum opus*, the *Wissenschaftslehre*. The works chiefly consulted are:
(1) *Wissenschaftslehre*, I-IV (Bolzano (15) mostly abbreviated "WL"). [Names followed by parenthesized numerals refer to the bibliography; see the last paragraph of this introduction.]
(2) *Der Briefwechsel B. Bolzano's mit F. Exner* (Bolzano (54), "BE").
(3) *Erste Begriffe der allgemeinen Grössenlehre* (Bolzano (78), "AG").
(4) *Reine Zahlenlehre. Erster Abschnitt. Von dem Begriffe, den allgemeinsten Beschaffenheiten und der Bezeichnungsart der Zahlen* (Bolzano (79), "RZ").
(5) *Unendliche Grössenbegriffe* (Bolzano (81), "UG").
(6) *Bolzano's Wissenschaftslehre und Religionswissenschaft in einer beurtheilenden Uebersicht* (Bolzano (z5), "WU").
(7) *Einleitung zur Grössenlehre* (Bolzano (83), "EG").
(8) *Paradoxien des Unendlichen* (Bolzano (45), "PU").
For (3)-(5) and (7) I have utilized Bolzano's unpublished manuscripts. In general I have presupposed that some editions of (1), (2), and (8) are accessible to the reader.
Chapter I presents in a concentrated form some biographical and bibliographical facts about Bolzano. The manuscripts used are described and dated. Some topics lying outside of logic proper and dealt with in published or unpublished works of Bolzano are also touched upon. The most representative portraits of Bolzano - one of which has not been published before - are reproduced.
Chapter II describes the logical machinery to be used in the formalization and the comparative analysis of Bolzano's logic. This chapter may be read cursorily and used as future occasion may require. The system expounds the so-called elementary logic, i.e., classical predicate logic of first order with identity. In view of its simplicity and non-controversial character, elementary logic seems at present to be an expedient object for comparison in research in the history of logic. To be sure,

we know nothing for certain about the future of logic; maybe the logics of tomorrow will differ greatly from those of today. However, it is likely that our elementary logic will be translatable into or representable within these conceivable new systems. But there is always the possibility that new systems of logic will promote a deeper understanding of certain features of the objects analysed.
Chapters III-VII expound those aspects of Bolzano's theories of logic, semantics, and mathematical philosophy which seem to me fundamental. Certain sections of these chapters are subdivided into two parts; part A describes Bolzano's ideas and part B offers commentaries on A. In order to shorten the exposition, definitions and arguments are sometimes formalized even in A, and sometimes auxiliary notions not found in Bolzano are introduced with explicit caution. In doing so I have always attempted to stay within or very close to Bolzano's sphere of ideas. This does not mean, of course, that I have always followed the order in which Bolzano presents his definitions and theories in the *Wissenschaftslehre* or elsewhere. Nor have I stinted myself at times in giving very free paraphrases of Bolzano's mode of expression. In proving theorems I try to reproduce Bolzano's line of thought without copying his manner of speaking. In part B, Bolzano's logic is compared with modern theories. I attempt to show how certain gaps could be filled in and how Bolzano's theories could be elaborated and made more precise. Moreover, under B some forerunners of Bolzano and selected parts of the modern literature concerning him are discussed.
In the annotated bibliography, part A embraces the literature on Bolzano and his own works. References to part A are given by names followed by numerals within parenthesis. References to part B of the bibliography are effected by placing a "B" after the parenthesized numerals. In references to manuscripts, folio numbers are qualified by "r" and "v", meaning, as usual, recto and verso respectively." (*Introduction*).

19. ———. 1966. "Bolzano's Notion of Proposition." In *Ost und West in der Geschichte des Denkens und der kulturellen Beziehungen. Festschrift für Eduard Winter zum 70. Geburtstag. Mit einem Geleitwort von A.P. Juskevic*, edited by Steinitz, Wolfgang, 519-526. Berlin: Akademie-Verlag.

20. ———. 1967. "What Is a Proposition?" *Logique et Analyse* no. 39/40:293-306
"Certain theories of modern logic have the purpose of defining interesting classes of linguistic expressions, such as the set of sentences of a language, or relations between expressions, such as derivability among formulas. Other theories aim at describing semantic relations between linguistic expressions and nonlinguistic objects, such as the relation of being the meaning of an expression. Yet a third kind of theories may give a direct analysis of non-linguistic objects which could stand in semantic relations to linguistic expressions.
This paper first propounds and discusses certain constructions of the second kind and then attempts an explication of the third kind of the notion of non-linguistic proposition. However, only a limited class of propositions (called "elementary propositions") will be explained, viz., propositions corresponding to the sentences of a language of elementary logic. Admittedly, this explication will have merely a remote connection with the problems of ordinary language. On the other hand, a tradition of logical semantics has accumulated since the 19th century dealing with technical and more or less formalized languages, and it may be worth while to attempt a solution of some problems encountered in such studies.
In writing this paper I have profited from comments and criticism of Professor A. Wedberg, University of Stockholm." (p. 293)

21. ———. 1972. "Bolzano's Theory of an Ideal Language." In *Contemporary Philosophy in Scandinavia*, edited by Paul, Anthony, Olson, Raymond and Wright, Georg Henrik von, 405-415. Baltimore: John Hopkins Press
"In his logical inquiries Bolzano employed a partly formalized language embracing an ordinary language extended by constants, variables, and certain technical expressions. In the second volume of the Wissenschaftslehre he investigated the

relations of this semiformalized philosophical language to colloquial language (WL, sections 127-46, 169 84). He believed that all sentences of colloquial language were 'reducible' to sentences of certain canonical forms expressed in the philosophical language. These canonical sentences were said to mirror their corresponding propositions in the sharpest way.
Had Bolzano's theory of reduction been completely developed it might have resulted in the construction of an ideal language for philosophical analysis. In this ideal language, however, sentences of canonical form would not play quite the same role as the atomic sentence forms on the basis of which more complex forms are built up in modern quantification theory. It seems, on the contrary, that Bolzano intended even the most complicated sentences to have canonical forms or to be reducible to sentences having such form.
This paper attempts a reconstruction of an extensional Bolzanian ideal language on the level of elementary logic. After some preliminary explanations of fundamental notions in Bolzano's logic, the main points of his theory of reduction of sentences are described. Two principles that determine the construction of an elementary Bolzanian ideal language emerge from the exposition. We then move toward building such a language and begin by modifying the standard representation of elementary logic, replacing the universal and existential quantifiers by Hilbert's e-operator. By further modifications of both the syntax and the underlying semantics, a logical language satisfying the two principles is obtained." (pp. 405-406)

22. ———. 1977. "Bolzano's Contribution to Logic and Philosophy of Mathematics." In *Logic Colloquium '76*, edited by Gandy, Robin O. and Hyland, John Martin, 147-171. Amsterdam: North-Holland
"The *Wissenschaftslehre* was intended merely as a prelude to Bolzano's work on mathematics. His main ambition was to recreate the whole body of contemporary mathematics in accordance with the vision of an abstract hierarchy of true propositions. For Bolzano this task implied the creation of entirely new foundations for certain branches of mathematics, as may be seen from his highly interesting efforts directed toward basing geometry on topological concepts.
In carrying out this program, most of the means of expression of modern quantification theory were in essence available to Bolzano.
He came very close to modern notions of satisfaction, logical truth, consistency and logical consequence. On the other hand, the formal deductive machinery of quantification theory is practically non-existent in Bolzano's works. This syntactic machinery appears only in Frege, who created the first strictly logistical system at the end of the 19th century. Bolzano's lack of interest in developing particular logical calculi most probably stems from his aspects of logic and mathematics and of science in general.
The notion of calculus in the modern logistical sense was first clearly considered by Leibniz. His basic dream was of an effectively decidable, interpreted calculus embracing all "eternal" truth. Bolzano was justifiably critical of this overambitious program and presented instead his own theory of the *Abfolge* structure of nonlinguistic propositions, thereby taking his stand away from that line of development in logic which leads to modern syntactic concept formation. A reason for Bolzano's general lack of interest in questions of logical syntax was no doubt his profoundly intensional, non-linguistic approach to logic.
Bolzano's central thesis, that there are abstract objects which differ from both mental occurrences and all kinds of linguistic expressions, has been advocated by later philosophers of the German-speaking countries, inter alia by Lotze, Brentano in his earlier period, Meinong and Frege. Lotze and Frege never refer to Bolzano's work, though, and the others protested their independence of Bolzano. Husserl admits that he received vital influences from Bolzano, but his notions of "ideal" objects derive from Lotze's and not from Bolzano's logic.
Among the great Western philosophers Bolzano is perhaps the least influential. In epistemology, logic and mathematics his most fervent disciples were not able to propagate his ideas with sufficient vigor.

His keen criticism of German idealistic philosophy and his important discoveries in logic: semantics and mathematical philosophy silently died away.
A contributing cause of Bolzano's lack of influence on the development of the philosophical disciplines was, of course, the fact that most of his works were, for political reasons, published anonymously in editions not easily accessible. Furthermore, an immense number of unpublished manuscripts in a partly almost indecipherable handwriting is to be found in archives in Prague and Vienna. Several unfruitful attempts have been made in the last 150 years to bring out more or less complete editions of Bolzano's works. It is to be hoped that the latest venture launched in Stuttgart, West Germany, will prove more successful." (pp. 170-171)

23. ———. 1982. "A Requirement for the Logical Basis of Scientific Theories Implied by Bolzano's Logic of Variation." *Acta Historiae Rerum Naturalium Necnon Technicarum* no. 12:415-425
Bernardo Bolzano (1781-1848) - Bicentenary. Impact of Bolzano's epoch on development of Science - Conference papers, Prague 7-13 September 1981.

24. ———. 1986. "A Logic of Terms with an Existence Operator." In *Logic and Abstraction. Essays Dedicated to Per Lindström on his Fifthieth Birthday*, edited by Furberg, Mats, Wetterström, Thomas and Aberg, Claes, 71-94. Göteborg: Acta Universitatis Gothoburgensis
"1. Introduction

In this paper a language L^* of elementary logic satisfying the following two conditions will be constructed:

(C 1) each expression in L^* consists of an n-ary function symbol **f** applied to n arguments ($n > 0$);

(C 2) if in L^* **f** is a predicative function symbol, $\mathbf{t_i}>$ a term, and J an interpretation for a particular i ($1 < i < n$) under which $\mathbf{t_i}>$ is empty, then $\mathbf{f}(t_1 \cdot \cdot \cdot t_n$ is false under J.

Here **f** is a predicative function symbol of L^* if **f** under a suitable mapping, corresponds to a predicate of one of the standard versions of elementary logic.

The language L^* differs from standard representations of elementary logic in that it replaces the universal and existential quantifiers with Hilbert's ε-operator and modifies both the syntax and the underlying semantics accordingly. An elementary logic with a non-trivial existence predicate is in itself interesting, and when combined with a logic of terms can be utilized for research in the history of logic. (Cf. Berg [1972]."
References
J. Berg, Bolzano's theory of an ideal language. R. E. Olson & A. M. Paul (Eds.): *Contemporary philosophy in Scandinavia* pp. 405-415 (Baltimore).

25. ———. 1987. "Bolzano and Situation Semantics: Variations on a Theme of Variation." *Philosophia Naturalis* no. 24:373-377
"The distinction between logical and non-logical notions plays a significant role in Bolzano's theory and he is fully aware of its importance even though he has to admit that various scholars may differ in their opinion on what a *logical idea* is (WL § 148.3). Tarski has tried to clarify the distinction between logical and non-logical notions (Adam Tarski, *What are logical notions?* History and Philosophy of Logic, 7, 1986, pp. 143-154). A notion of Euclidean geometry, e.g., is invariant under all similarity transformations, and a topological notion is invariant under all continuous transformations. Analogously, a *logical notion* may be conceived of as a concept which is invariant under all bijective mappings of the domain of individuals onto itself. (In this sense even the classical reduction problem of critical realism can be solved: A physical notion is a concept invariant under a Galilei or a Lorentz transformation.)
It is possible to vary not only the non-logical ideas-as-such contained in propositions but even some or all logical ideas. Actually, such a variation is implied by the algebraic approach to logic. The propositions of Bolzano correspond to the values of the propositional variables of modern logic. (p. 374)

(...)
Bolzano seems to have intended, however, variation exclusively over non-logical ideas-as-such. That he did not allow a variation of the copula is abundantly clear from his way of introducing the notion of variation in his *Einleitung zur Grossenlehre* (Bolzano (2A7), p. 62). Here he presupposes that only the subject and predicate ideas of a proposition or parts thereof be varied. Furthermore, his proofs of certain variation-logical theorems show that he would not allow a variation of the logical constants of negation, truth, and the copula in the form of an inclusion between ideas-as-such (WL §§ 154.19, 155.21)." (p. 375)

26. ———. 1987. "Is Russell's Antinomy Derivable in Bolzano's Logic?" *Philosophia Naturalis*:406-413
"In his encyclopedic work *Wissenschaftslehre* Bernard Bolzano expounded a theory of logical truth which constitutes an outstanding achievement in the history of Western thought. This informal theory is essentially based on a substitutional truth-value semantics without certain existence presuppositions and contains a general proof theory. In his substitutional semantics Bolzano introduced notions such as universal validity, consistency, consequence, analyticity, and probability by means of the technique of variation of concepts. In his proof theory he treated the notion of entailment, which is a generalization of a special case of the relation of logical consequence, and studied proof trees generated by the relation of entailment which exhibit the objective connection between all true propositions. In view of this wealth of important notions it seems worthwhile to investigate the possibility of a consistent reconstruction of Bolzano's logic. In particular, it must be examined how his theory fares with a fundamental set-theoretic antinomy such as that of Russell." (p. 406)
(...)
"There is, therefore, no such thing as an idea of all ideas which are not objects of themselves, and Bolzano could scarcely be blamed for having no idea of something which does not exist in any sense at all. Under a reasonable interpretation of Bolzano's theory of ideas-as-such there is no trouble-maker around who could generate an antinomy analogous to that of Russell." (p. 411)

27. ———. 1987. "Bolzano on Induction." *Philosophia Naturalis* no. 24:442-446
"Bolzano combined the fundamental notions of his theory of probability and his proof theory to achieve a logical analysis of the principles of induction. The relation between the conclusion and the premisses of an inference of incomplete induction or analogy is an interior probability relation in Bolzano's sense. The principles of induction endow the relation between the premisses and the conclusion with the character of a relation between ground and consequence. From Bolzano's subsumption of the rule of incomplete induction under the syllogistic rule of Barbara, it follows that this relation is a special case of Bolzanian derivability." (p. 442)

28. ———. 1992. *Ontology Without Ultrafilters and Possible Worlds: An Examination of Bolzano's Ontology*. Sankt Augustin: Academia Verlag
Contents: Vorwort der Herausgeber [Edgar Morscher] 7; Einleitung von Edgar Morscher 13;
Jan Berg: Ontology Without Ultrafilters and Possible Worlds 29
Introduction 31; § 1. Collections, sets, and sums 34; § 2. Numbers, infinite sets, and infinitesimals 39; § 3. Ontology without ultrafilters 48; § 4. Ideas, properties, and intuitions 52; § 5. Propositions, sentences, and judgements 64; § 6. Validity, derivability, and entailment 79; § 7. Substances, adherences, and causes 88; § 8. Ontology without possible worlds 91;
List of special symbols 95; References 97-100.
"The first basic notion of Bolzano’s ontological system is the part relation. Its domain, i.e., the set of all objects bearing it to something, embraces concrete substances, abstract objects, and collections. The converse domain of the part relation, i.e., the set of all objects to which it is borne, contains collections only.

Some collections are concrete entities existing in space and time, the rest are abstract sums or other sets. Concrete sums are composed of substances and adherences, i.e., forces. Forces applied to certain substances give rise to subjective ideas or judgements. Further results of such applications are the concrete sentence occurrences. A subjective idea is a part of a judgement which is not itself a judgement. The set of judgements is ordered by a special causal relation.
Bolzano's abstract world is constituted of sets, abstract sums, certain attributes (i.e., properties or relations), ideas-as-such, and objects constructed on the basis of these entities. Thus, sentence shapes are a kind of properties, and certain complexes of ideas-as-such constitute propositions. The notion of an idea-as-such can be constructed from expressions of a language by means of axioms for the relation of being an object of something. Analogously, properties can be generated by axioms for the relation of something being applied to an object. The converse of this relation, i.e., the relation of an entity having a property, and the relation of being an object of an idea-as-such are fundamental ontological constants of Bolzano's." (p. 31)
(...)
"The question whether a rational reconstruction of Bolzano's ontology is possible will be sustained like a pedal point throughout the present study. In many respects, indeed, his ontological system is a model of thrift, comprehensiveness, and deductive cogency. He shows us how to grasp a self-contained, abstract "third" world (in Popper's sense) embracing the realms of classical logical truth and additive probability spaces without indulging in possible worlds, states of affairs, facts, and all that. Admittedly, from a modem point of view certain aspects of his ontology may look like Dr. Johnson's dog walking on its hind legs: it is not always done quite well, but you are surprised to find it done at all. To rational bipeds of our time it should be more instructive, though, to watch this performance rather than amazing at metaphysical cephalopods wallowing in clouds of ontological splendors, or gazing at recondite cogitators crawling on all fours through a self-induced verbal fog." (p. 33)

29. ———. 1992. "The Connection Between Bolzano's Logic of Variation and His Theory of Pprobability." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 107-120. Firenze: Leo S. Olschki
"In his monumental four-volume work *Wissenschaftslehre* (1837) - in the sequel denoted by 'WL' - Bolzano introduced several new concepts for the analysis of the structure of scientific theories. In particular, he tried to lay down a logically satisfactory foundation of mathematics and the theory of probability. During the search for such a foundation he became aware of the distinction between the actual thoughts of human beings and their linguistic expressions on the one hand, and abstract propositions (*Sätze an sich*) and their components which exist independent of these thoughts and expressions on the other hand.
Bolzano described the relations of propositions to other relevant notions such as those of sentence, truth, existence, and analyticity. Furthermore, he studied relations among propositions and defined highly interesting notions of validity, consistency, derivability, and probability, based on the method of «replacing» certain components in proposition. A proposition in Bolzano's sense is a structure of ideas-as-such (*Vorstellungen an sich*). According to Bolzano, each complex idea-as-such can be analyzed into a sequence of simple ideas which include certain logical constants such as those expressed by the words 'not', 'and', 'some', 'all', 'to have', or 'ought' (WL §§ 61, 78.1, 116.3). The manner in which a complex idea-as-such is built up from simple ones may be expressed in a language by a chain of definitions." (p. 1907)

30. ———. 1994. "The Ontological Foundations of Bolzano's Philosophy of Mathematics." In *Logic and Philosophy of Science in Uppsala*, edited by Prawitz, Dag and Westerstähl, Dag, 265-271. Dordrecht: Kluwer
"The basic notion of Bolzano's ontological system is the part relation.

Its domain embraces concrete substances, abstract objects, and collections; the converse domain contains collections only.
Some collections are concrete entities existing in space and time, the rest are abstract sets.
Bolzano's notion of a set implies that a set cannot be a member of itself. Hence, there is no danger of an antinomy similar to that of Russell arising in Bolzano's ontological system of sets.
Bolzano's abstract world is constituted of sets, certain attributes (i.e., properties and relations), ideas-as-such, and objects constructed on the basis of these entities. Thus, certain complexes of ideas-as-such constitute propositions. The notion of an idea-as-such can be constructed from expressions of a language by means of axioms for the relation of being an object of something. Analogously, properties can be generated by axioms for the relation of something being applied to an object. The converse of this relation and the relation of being an object of an idea-as-such are fundamental ontological constants of Bolzano's." (p. 265)

31. ———. 1997. "Bolzano, the Prescient Encyclopedist." *Grazer Philosophische Studien* no. 53:13-32
Abstract: "In his *Wissenschaftslehre* Bernard Bolzano tried to lay down a logically satisfactory foundation of mathematics and theory of probability. Thereby he became aware of the distinction between the actual thoughts and judgments of human beings, their linguistic expressions and the abstract propositions *(Sätze an sich)* and their components (*Vorstellungen an sich).* This ontological distinction is fundamental in Bolzano's thinking paired with a universal world view in the sense that philosophy, mathematics, physics and metaphysics should be build upon the same logical foundations. Bolzano's enterprise is sketched in the light of examples from his logical semantics, proof theory, number theory, theory of truth and his variation logic."

32. ———. 2000. "From Bolzano's Point of View." *The Monist.An International Quarterly Journal of General Philosophical Inquiry* no. 83 (1):47-67
"I am going to present logic, logical semantics, ontology, proof theory, the foundations of mathematics, and certain aspects of the philosophy of nature from Bolzano's point of view.
In his monumental four-volume work *Wissenschaftslehre* (1837) Bolzano introduced several new concepts for the logical analysis of the structure of scientific theories. In particular, he tried to lay down a logically satisfactory foundation of mathematics and the theory of probability.
During the search for such a foundation he became aware of the distinction between the actual thoughts and judgements of human beings, their linguistic expressions, and the abstract propositions (*Sätze an sich*) and their components which exist beyond space and time. This ontological distinction is fundamental in Bolzano's philosophy. In his terminology, real things have actuality whereas abstract objects have logical existence bare of actuality.
Bolzano worked extensively with the relation of being an object of an idea-as-such (a *Vorstellung an sich*). The object of an idea-as-such can be either an abstract object or a concrete object existing in space and time.
The relation of being an object of an idea-as-such corresponds in modem semantics to the relation of being an element of the extension of a concept." (47)
(...)
"Bolzano is indubitably one of the greatest philosophers of the German language. His world view was a universal one in the sense that philosophy, mathematics, physics, and metaphysics should build upon the same logical foundations. In fact, he already recognized many of the essential things to come in logic and the foundations of mathematics." (p. 67)

33. ———. 2003. "Bolzano's Heuristics." In *Bernard Bolzanos Leistungen in Logik, Mathematik und Physik*, edited by Morscher, Edgar, 35-56. Sankt Augustin: Academia Verlag

"In the fourth part of the *Wissenschaftslehre* [WL], contained in the third volume of the original 1837 edition, Bolzano treats *heuristics* or the "art of discovery", i.e., the "rules to be observed in the search for new truths" (§ 9. Note 3; cf. also § 15.2). The first main section of Bolzano's heuristics embraces the *general rules* of this discipline (§§ 325 -348).
Logic in Bolzano's sense is a theory of science the objects of which are the different sciences and their linguistic representations (§ 15). According to Bolzano a science is a set of true propositions (*Sätze an sich*) worthy of representation in a textbook. Logic or the *theory of science* is a set of rules which are necessary and sufficient for a representation to satisfy certain criteria concerning scientific textbooks (§ 1). In view of this very broad conception of logic it is fairly obvious that heuristics is an integrant part thereof." (p. 35)
This paper was already presented in 1991 at the International Bolzano Symposium in Salzburg, but has never been published since.

34. ———. 2003. "The Importance of Being Bolzano." In *Bernard Bolzanos Leistungen in Logik, Mathematik und Physik*, edited by Morscher, Edgar, 153-166. Sankt Augustin: Academia Verlag
"1. Logical consequence
Ever since Aristotle philosophers have occupied themselves with the question whether a given statement follows from another statement. The first published precision of this notion in modern times was undertaken by the Polish logician Alfred Tarski in 1936. Accordingly, a closed formula F is a logical consequence of a set of formulas F if and only if F is true under every interpretation of the nonlogical constants under which all elements of F are true. Logical constants are inter alia connectives of sentential logic (expressed by words like "not", "and", "or", "if - then") and quantifiers of predicate logic (such as "for all" and "there is"); hence, the interpretation of these constants is determined.
But who conceived this notion of logical consequence (mutatis mutandis) already a hundred years earlier?'
Right: The Bohemian philosopher, ontologist, logician, mathematician and theologian Bernard Bolzano!
Upon substitution of abstract nonlinguistic propositions for closed formulas and variants of propositions for interpretations, we get precisely a special case of Tarski's notion of logical consequence. (A variant of a proposition P is a proposition identical with P up to at least one nonlogical component.) Incidentally, at the university of Warsaw Tarski was a student of Lukasiewicz's who lectured inter alia on Bolzano's logic.
Just like Bolzano Tarski admitted being unable to exactly distinguish between logical and nonlogical constants. Not until thirty years later did he formulate a necessary condition for the property of being a logical constant. Furthermore, if all constants of the formal language in question were regarded as logical, the notion of material implication, would emerge. Even this weakest of all notions of consequence was introduced by Bolzano and is playing an important role in some of his deduction rules.
Tarski presupposed a fixed domain as a realm of reference for the interpretations. Even Bolzano did not conceive of a combined quantification over domains and components of propositions. (By introducing a predicate for domains and letting the quantifiers refer to this predicate, however, one can represent all theorems of the model theory developed later on.)
Nowadays we know, of course, that Tarski's notion of logical consequence is unsuitable if the set-theoretic language is enlarged by a generalized existence quantifier expressing that there is an absolutely infinite class C (in the sense that C does not include exactly K elements for any cardinal number K). This esoteric fact of modern set theory cannot, however, diminish our appreciation of Bolzano's achievement.
2. Analytic propositions

A fundamental distinction in Kant's Critique of Pure Reason is that between analytic and synthetic judgements. In modern logical semantics analyticity is often considered a relation between a sentence S, a set of definitions, and a language L. For instance, one can say that S in L is analytic with respect to D if S is a logical consequence of D in L which embraces S and the elements of D.
But who formulated an analogous explication of analyticity within the system of abstract propositions already in the 1830s?
Right: Bernard Bolzano!" (pp. 153-154)
(...)
"6. Situation semantics
In modern so-called situation semantics, established at the beginning of the 1980s by the American logician and linguist Jon Barwise, a notion of consequence is introduced which is stronger than that of Tarski. In situation semantics certain set-theoretic structures are considered models and a situation is a partial submodel thereof. The primitive notion is the confirmation of a sentence in a model by a situation. For example, a sentence of the form of "A or not A" is a logical consequence of any sentence in the sense of Tarski but not a strong consequence of it.
But who discovered this notion of strong consequence even a hundred years earlier?
Right: Bernard Bolzano!
In his logic Bolzano considered not only the variants with respect to the sequence of all nonlogical components of propositions but also the variants with respect to all subsequences. By that counterparts of main laws of situation semantics turn into theorems of Bolzano's logic." (p. 156)
(...)
17. Estimation
Thus some outstanding achievements of Bolzano's on the fields of logic, semantics, and mathematics have been delineated. The fact that the connection of most of these achievements with modern research remained unknown until the 1960s is due to the circumstance that the study of Bolzano's work took a new turn then and that eventually editions of the often hardly legible manuscripts of the literary remains could be published in the Collected Works of Bernard Bolzano.
Moreover, particularly in Bolzano's logical semantics there are many original ideas which have no precise affinity with modern theories. In addition to that he accomplished extensive investigations into concepts of epistemology, philosophy of nature, physics, metaphysics, ethics, and theology." (p. 165)

35. Berka, Karel. 1982. "Bolzano’s Philosophy of Science." In *Bernard Bolzano, 1781-1848 Bicentenary. Impact of Bolzano's Epoch on the Development of Science*, 427-442. Prague: Institute of Czechoslovak and General History CSAS.

36. ———. 1983. "The Ideal of Mathematization in B. Bolzano." In *Nature Mathematized. Historical and Philosophical Case Studies in Classical Modern Natural Philosophy. Vol. 1*, edited by Shea, William R., 291-298. Reidel: Kluwer
"In my contribution I would like to draw attention to the views on the ideal of mathematization held by B. BoJzano, a later follower of Leibnizian rationalism. This analysis will show the evolution of conceptions elaborated in the epoch of *mathesis universalis* on this topic in a period basically influenced by the philosophy of Kant and other representatives of German classical philosophy." (p. 291)
(...)
"The discussions concerning the acceptability of the fifth postulate of Euclid's *Elements* and the various attempts to prove it, seem to him to be clear evidence that the problem in question does not lie in the demonstration of the certainty of this postulate, but in finding the objective ground of its validity. In his work *Die drey Probleme der Rectification, der Complanation und der Cubirung* (1811), he claims that we cannot accept as a basic truth any proposition which admits a further ground of its truth. BoJzano does not doubt that this postulate is true, requiring only to have its validity grounded in an objective way, independently of our subjective feeling of certainty." (p. 291)

(...)
"Bolzano's conception, which extends and modifies the Leibnizian project of mathematization is explicitly proclaimed in part II, "On the mathematical method", of his *Beyträge zu einer begründeteren Darstellung der Mathematik* (1810) and further elaborated in other mathematical works, especially in his *Einleitung zur Grossenlehre*, and in the *Wissenschaftslehre*, where the logical aspects of mathematics and its methodology are taken into consideration." (p. 292)

37. ———. 1988. "Natural Deduction in Bolzano's *Wissenschaftslehre*." In *Intensional Logic, History of Philosophy and Methodology. To Imre Ruzsa on the Occasion of his 65th Birthday*, edited by Bodnàr, István M., Maté, András and László, Pólos, 203-212. Budapest: Department of Symbolic Logic, Eotvos University.

38. ———. 1998. "Bernard Bolzano. A Historian of Logic." *History of Science and Technology* no. 31:121-130
Abstract: "Bolzano´s *Theory of Science* (*Wissenschaftslehre*) contains a great amount of very valuable information concerning the development of logic from its beginnings in Aristotle till the post-Kantian period. In a critical exposition Bolzano presents views of his predecessors and compares them with his own standpoint. The paper presents a selective survey of various conceptions developed by Aristotle, G. W. Leibniz and his followers G. Ploucquet, J. H. Lambert and S. Maimon together with their Bolzanian interpretation. The historical analyses in his principal logical work are, thus, at the same time a witness of his own opinions toward different topics in logic."

39. Betti, Arianna. 1998. "*De Veritate*: Another Chapter. The Bolzano-Lesniewski Connection." In *The Lvov-Warsaw School and Contemporary Philosophy*, edited by Kijania-Placek, Katarzyna and Wolenski, Jan, 115-137. Dordrecht: Kluwer
"In 'De Veritate: Austro-Polish contributions to the theory of truth from Brentano to Tarski' Jan Wolenski and Peter M. Simons related an intriguing story of the "Austro-Polish obsession with truth". Wolenski and Simons mention the Bohemian philosopher Bernard Bolzano several times, with particular reference to absoluteness and sempiternity of truth in Twardowski and Lesniewski.
(...)
In the following I wish to point out three issues. First, in the so-called prelogistic writings the early Lesniewski defines truth of sentences in such a way that truth conditions are the same - *mutatis mutandis* - as Bolzano's.
Secondly, from this point of view the links between the early and the late Lesniewski, in this case between some parts of his early writings and some aspects of Ontology, are closer than they are commonly believed to be. Thirdly, in this perspective it can be shown that some of Bolzano's views come near to Lesniewski's Ontology. In discussing Bolzano's views I shall mostly follow Casari's reading of Bolzano's *Wissenschaftslehre*." (p. 115)

40. ———. 2006. "Sempiternal Truth. The Bolzano-Twardowski-Lesniewski Axis." In *The Lvov-Warsaw School: The New Generation*, edited by Jadacki, Jacek Jusliuz and Pasniczek, Jacek, 371-399. Amsterdam: Rodopi
"Twardowski [*] had revived Bernard Bolzano's ideas on the subject [eternity and sempiternity of truth], and, mainly thanks to him, these became known in the Lvov-Warsaw School (see, for instance, Jadacki 1993, p. 191). There is no doubt that Lesniewski knew Twardowski's ideas and it seems evident that the latter influenced him: Lesniewski's results are mostly compatible with the "absolutistic" content of Twardowski's 1900 article. And, similarly, no doubts can be raised about the Bolzanian origin of the aspects of eternity and sempiternity of truth defended by Twardowski in *Relative Truths* (see, for instance, Wolenski and Simons 1988, p. 430, n. 24; and Simons 1992, Ch. 2, p. 15, n. 11; see also Smith 1988, p. 325): though his name is not quoted, traces of Bolzano's legacy can be found even in the examples given by Twardowski, some of which are the same as used by Bolzano in his *Wissenschaftslehre*. Yet, since Bolzano, Twardowski and Lesniewski supported different theories of meaning with different ontological presuppositions,

"sempiternity of truth" actually stands for three different conceptions. This paper is a survey of these three conceptions. I suggested elsewhere a comparison between Bolzano and the early Lesniewski as to their theories of meaning and truth, claiming the possibility of a (direct or indirect) influence of Bolzano upon Lesniewski. The analysis presented here is also meant as a contribution to the picture sketched there." (p. 372, notes omitted)
[*] "On the So-Called Relative Truths" (1900) in J. Brandl and J. Wole?ski (eds.), *Kazimierz. Twardowski - Actions, Products and other Topics in Philosophy,* Amsterdam: Eodopi 1999, pp. 147-168. J. Brandl and J. Wole?ski (eds.),
References
Jadacki, J.J. (1993). Kazimierz Twardowski's Descriptive Semiotics. In: Coniglione et *al.,* eds. (1993), pp. 191-206.
Coniglione, F., R. Poli and J. Wolenski, eds. (1993). Po*lish Scientific Philosophy: The Lvov-Warsaw School.* Poznan Studies in the Philosophy of the Sciences and the Humanities, vol. 28. Amsterdam: Rodopi.
Simons, P.M. (1992). Ph*ilosophy and Logic in Central Europe from Bolzano to Tarski. D*ordrecht: Kluwer.
Smith, B. (1988). Ka*simir Twardowski: An Essay on the Borderlines of Ontology, Psychology and Logic.* In: Szaniawski, ed. (1988), pp. 313-375.
Szaniawski, K., ed. (1988). The *Vienna Circle and the Lvov-Warsaw School. T*he Hague: Nijhoff.
Wolenski, J. and P.M. Simons (1988). D*e veritate: Austro-Polish Contributions to the Theory of Truth from Brentano to Tarski. I*n: Szaniawski, ed. (1988), pp. 391-443.

41. ———. 2006. "The Strange Case of Savonarola and the Painted Fish. On the Bolzanization of Polish Thought." In *Actions, Products, and Things. Brentano and Polish Philosophy*, edited by Chrudzimski, Arkadiusz, 55-81. Frankfurt: Ontos Verlag
"I have previously discussed in several papers specific Bolzanian elements present in the Polish tradition. This paper will not, for the most part, add anything in particular to that. The new - and rather blunt hypothesis to be put forward here is that, despite appearances, Twardowski also contributed *de facto* to slowing down the reception of Bolzano's most modem logical discoveries. For in Poland Bolzano was to remain one logician among many for rather long. It was chiefly thanks to two factors that Bolzano's star could, slowly, begin to rise in Poland, or, at least, that the fundamental achievements of his logic could be known. One factor is antipsychologistic (more precisely Platonistic) influence coming from Husserl and from Twardowski's student Lukasiewicz. The other factor is the change in the conception of logic which took Polish logic from, say, Sigwart, to Tarski through Lesniewski and Lukasiewicz," (p. 55)

42. ———. 2010. "Explanation in Metaphysics and Bolzano's Theory of Ground and Consequence." *Logique et Analyse* no. 56:281-316
"In "Troubles with Truth-making: Necessitation and Projection." Erkenntnis 64: 61-74 (2006a, and in "Truth-Making without Truth-Makers." Synthese 152: 21-46 (2006b), Benjamin Schnieder criticizes truthmaking as a relation between entities in the world and the truths those entities 'make true'. In (2006b), his criticism exploits a notion of conceptual explanation that is very similar to Bolzano's grounding. In the first part of this paper, I offer an analysis of Bolzano's grounding. I discuss some open problems and argue that Bolzano's grounding is not a systematization of the ordinary notion of 'because' as others have maintained, but of the technical notion of explanatory proof in the context of an axiomatic conception of (proper) science. On the basis of this analysis, in the second part, I offer a critical discussion of Schnieder 2006b's arguments against truthmaking. I conclude that the latter are not very effective from a methodological point of view and that Bolzano's original position fares better in this respect; still, truthmaker theorists will be able to defend truthmaking only at a high price."

43. ———. 2012. "Bolzano's Universe: Metaphysics, Logic, and Truth." In *Categories of Being. Essays on Metaphysics and Logic*, edited by Haaparanta, Leila and Koskinen, Heikki J., 167-208. New York: Oxford University Press
"Thanks to a handful of publications from the last decade, however, Bolzanian metaphysics has begun to receive more attention than ever before.(1)
It is not difficult to show why Bolzanian metaphysics matters. Bolzano's logic builds on firm ontological and mereological foundations. Logic as a science has a realm of its own, that of the *an sich*, in the strong sense that logic is the science of a special kind of object, namely, propositions-in-themselves and ideas, and their qualities. Furthermore, the edifice of logic rests on a mereological conjecture regarding the basic form of propositions and is constructed by exploiting mereological relations between propositions and ideas, plus a device of semantic ascent, involving very special ideas with very special qualities, called symbolic ideas.
Th e first and main aim of this essay is to present an overview of Bolzano's universe from the point of view of his metaphysics and its relationship to logic, relying fundamentally on his major work, the *Wissenschaftslehre.* This I shall do in sections II–VI. Although these sections are chiefly intended as an exposition of the state of the art on the matter, I shall make no secret of preferring a reading of Bolzano as a "Platonistic nominalist," as Textor puts it—as a Platonist about propositions and a nominalist about properties. (2) My second aim, in sections VII–IX, shall be to answer the open question of whether in Bolzano there is any "ontology of truth," as one may call it, though with some hesitation." (pp. 167-168)
(1) Among others, Künne 1998; Schnieder 2002; Textor 2004.
(2) Textor 2004 , 10. That Bolzano is a Platonist about propositions is the predominant view, which I follow here. Among those who disagree, cf. Cantù 2006 , 10.
References
Cantù, Paola. 2006. Bolzano et les propositions en soi: une théorie objective des vérités. In *Propositions et états de choses* , ed. J. Benoist. Paris: Vrin.
Künne, Wolfgang. 1998. Substanzen und Adhärenzen—Zur Ontologie in Bolzanos Athanasia. *Philosophiegeschichte und logische Analyse* 1: 233–50.
Schnieder, Benjamin. 2002. *Substanz und Adhärenz: Bolzanos Ontologie des Wirklichen*. Sankt Augustin: Academia.
Textor, Mark. 2004. Bolzanos Ontologie. In *Die Bedeutung Bernard Bolzanos für die Gegenwart*, ed. K. Strasser. Prague: Filosofia.

44. Beyer, Christian. 2004. "Bolzano and Husserl on Singular Existential Statements." In *Phenomenology & Analysis: Essays on Central European Philosophy*, edited by Chrudzimski, Arkadiusz and Huemer, Wolfgang, 69-88. Frankfurt: Ontos Verlag
"Which form does the propositional content take that is judged when a given speaker sincerely utters a sentence in order to assert a singular existential statement? Two thought-provoking answers to this question have been proposed by Bernard Bolzano and, when commenting upon Bolzano's proposal, by Edmund Husserl. In Section 1 of this paper the author clarifies what he means by "singular existential statements". In Section 2 Bolzano's proposed analysis is sketched. In Section 3 the author exposes the earlier Husserl's conception of "logical reflection" and draws upon it to explain why Husserl, around 1900, subscribed to Bolzano's proposal. Following this, he reconstructs and considers in detail the later Husserl's discussion of that proposal and Husserl's own mature theory of singular existential statements as manifested in a 1917/18 lecture series, both of which shed light upon a conception that is of central importance for Husserlian phenomenology: the conception of "noematic sense" (Section 4)." (p. 71)

45. Bodnar, Joanne. 1976. *Bolzano and Husserl: Logic and Phenomenology*
Unpublished Ph.D. thesis, State University of New York at Buffalo, available at ProQuest Dissertation Express.
Contents: Introduction 1; I. Bolzano's Anti-Psychologism 5; II. Bolzano's Theory of Meaning 41; III. Bolzano's Basic Logical Relations 51; IV. Truth to Bolzano 63; V.

Husserl's Anti-Psychologism 76; VI. Husserl's Theory of Meaning 99; VII. Basic Logical Relations in Husserl 113; VIII. Truth to Husserl 128; IX. Recapitulation 144; X. Conclusion 151; Bibliography 164-168.

"Bernard Bolzano and Edmund Husserl both present some form of ontological framework for logic rather than a linguistic framework. Their works predate the pragmatic and semantic theories of Tarski and Carnap. Bolzano's *Wissenschaftslehre* appeared in 1837. (2)And Husserl's major logical thinking was formulated before 1935. (3) But neither of them seem receptive to a semantic foundation for logic, because of their rationalist-platonist leanings. Both strongly oppose the view that logic is taken from psychological experience by generalization. They have a viewpoint which is perhaps closer to the classical outlook than to either of the others, since they consider the foundation of logic to be the acceptance of meanings as entities — entities which are in some important ways related to actual and possible being and its structure.

Bolzano and Husserl each make crucial modifications on traditional platonism as a philosophy of logic. A basic thesis which they both do accept is that the logical entities such as the proposition with its elements and its relations are ideal unities, which are independent of their being thought. But the logical entities are not platonic forms in which spatio-temporal existences "participate" — nor are they determined by spatio-temporal existence in any way. They are independent of the subject or knower as well as of the facts of material existence.(4)

Thus the logical entities are what they are whether they ever come to expression or not. They have a character similar to that of numbers or other "abstract" mathematical objects, but it cannot be said that they arise in experience as abstractions from the empirical world. Although there is disagreement about the question of abstraction in Bolzano — with some Bolzano commentators such as Rolf George seeing little difference between Bolzano and Carnap — this tendency to read Bolzano's work as if he were a pragmatist obscures the originality of Bolzano.(5)

His differences from semantic and empiricist thinking are well worth investigating. Husserl's approach too deserves consideration for its uniqueness. Crediting Bolzano with giving a starting point in philosophy of logic, Husserl "discovered" Bolzano and brought his work out of obscurity. He makes use of the work of Bolzano however, only to transform it thoroughly. If certain common themes are selected for exposition, the positions of Bolzano and Husserl are both seen to be modifications of the classical platonism. This provides a basis for a comparison of Bolzano and Husserl." (pp. 3-5).

(2) Bolzano's work under consideration is *Theory of Science*, ed. and trans. by R. George (Berkeley, 1972), hereafter cited simply as Bolzano. English paginations are used, but section numbers apply to all German editions as well. The Theory of Science is a condensation of Wissenschaftslehre, vols. 1-4 (Sulzbach, 1837). R. George follows in large measure the F. Kambartel edition of Bolzano's *Wissenschaftslehre* vols. 1-2, entitled *Grundlegung der Loqik* (Hamburg, 1963) in which Bolzano's original has been condensed, with the omitted passages summarized by the editor.

(3) Works by Husserl principally under consideration are *Logische Untersuchungen* (1900 and 1913), trans. by J. N. Findlay (New York, 1976); and *Formale und Transzendentale Loqik* (1929), trans. by D. Cairns (Hague, 1969).

(4) See: U. Neemann, *Bernard Bolzanos Lehre von Anschauung und Begriff in ihrer Bedeutung fŭr erkenntnistheoretische und pädaqogische Probleme* (Paderborn, 1972) pp. 81 and 144. for discussions of how Bolzano's logical entities differ from Plato's forms and from Kant's subjective categories.

(5) See: R. George, "Editor's Introduction" in Bolzano's *Theory of Science*, (Berkeley, 1972) p. xxx. Also note J. Berg, *Bolzano's Logic* (Stockholm, 1962) pp. 49-50, where he expresses the view that Bolzano takes logical entities as abstractions.