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The Philosophy of Bernard Bolzano: Logic and Ontology

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Some appreciations of Bolzano's philosophy

"Bernhard Bolzano's *Wissenschaftslehre*, published in 1837, a work which in its treatment of the logical 'theory of elements' far surpasses anything that world-literature has to offer in the way of a systematic sketch of logic. Bolzano did not, of course, expressly discuss or support any independent demarcation of pure logic in our sense, but he provided one de facto in the first two volumes of his work, in his discussions of what underlay a *Wissenschaftslehre* or theory of science in the sense of his conception; he did so with such purity and scientific strictness, and with such a rich store of original, scientifically confirmed and fruitful thoughts, that we must count him as one of the greatest logicians of all time.
He must be placed historically in fairly close proximity to Leibniz, with whom he shares important thoughts and fundamental conceptions, and to whom he is also philosophically akin in other respects." (Chapter Ten, Appendix: References to F. A. Lange and B. Bolzano, § 61, p. 142)

From: Edmund Husserl, *Logical Investigations,* vol. I, *Prolegomena to a Pure Logic* [1900], London and New York: Routledge 1970.

"While the idealists were removing every trace of objectivity from Kant's semantics, there was in a corner of the Austro-Hungarian empire, ignored by the leaders of German philosophy, a Czech priest by the name of Bernard Bolzano, who was engaged in the most far-reaching and successful effort to date to take semantics out of the swamp into which it had been sinking since the days of Descartes. Bolzano was the first to recognize that transcendental philosophy and its idealistic sequel were a *reductio ad absurdum* of the semantics of modem philosophy. He was also the first to see that the proper prolegomena to any future metaphysics was a study not of transcendental considerations but of what we say and its laws and that consequently the *prima philosophia* was not metaphysics or ontology but semantics. The development of these ideas in his monumental *Wissenschaftslehre* and in a variety of other writings established Bolzano as the founder of the semantic tradition. Bolzano's philosophy was the kind that takes from and then gives life to science. His approach to semantics was developed in dialectical interplay with the decision to solve certain problems concerning the nature of mathematical knowledge. Kant had not even seen these problems; Bolzano solved them. And his solutions were made possible by, and were the source of, a new approach to the content and character of a priori knowledge." (p. 23)

From: J. Alberto Coffa, *The Semantic Tradition from Kant to Carnap. To the Vienna Station*, Cambridge: Cambridge University Press 1991.

"Bernard Bolzano was a lone forerunner both of analytical philosophy and phenomenology.
Born in Prague in the year when Kant's first Critique appeared, he became one of the most acute critics both of Kant and of German Idealism. He died in Prague in the same year in which Frege was born; Frege is philosophically closer to him than any other thinker of the nineteenth or twentieth century.
Bolzano was the only outstanding proponent of utilitarianism among German-speaking philosophers, and was a creative mathematician whose name is duly remembered in the annals of this discipline. His *Wissenschaftslehre* (Theory of Science) of 1837 makes him the greatest logician in the period between Leibniz and Frege.
The book was sadly neglected by Bolzano's contemporaries, but rediscovered by Brentano pupils: its ontology of propositions and ideas provided Husserl with much of his ammunition in his fight against psychologism and in support of phenomenology, and through Twardowski it also had an impact on the development of logical semantics in the Lwow-Warsaw School." (p. 823)

From: Wolfgang Künne, "Bolzano, Bernard" in: Edward Craig (ed.), *Routledge Encyclopedia of Philosophy*, vol. II p. 823-827, New York: Routledge 1998.

An overview of Bolzano's philosophy

"It is as logician, methodologist, and epistemologist that Bolzano, after a long period of neglect, regained philosophical attention in the twentieth century. Mainly in order to combat radical skepticism, he found it necessary to base his teachings in these fields on certain ontological conceptions. He was convinced that there exist truths-inthemselves *(Wahrheiten an sich*) prior to and independent of language and man. These truths he carefully distinguished from truths expressed in words and conceived truths. The set of truths-in-themselves is a subset of the set of propositions (in-themselves) (*Sätze an sich*), again to be distinguished from propositions expressed in words and conceived propositions. Propositions consist of terms (ideas-in-themselves, *Vorstellungen an sich*). These are likewise to be distinguished, on the one hand, from the words or word sequences by which they are denoted and, on the other, from subjective ideas that occur in our mind. Although linguistic entities and conceived entities exist concretely, terms, propositions, and truths do not. Terms were equally carefully distinguished from their objects, whether or not these objects themselves existed concretely. Though Bolzano was a Platonist (in the modern sense), his ontology was rather remote from that of Plato or, for that matter, from that of Immanuel Kant, in spite of the common *an sich* terminology.

Beyond these negative determinations, Bolzano had little positive to say on the ontological status of terms and propositions except that they are the matter (*Stoff*) or sense (*Sinn*) of their correlates in language and thought.

Terms can be either simple or complex and either empty (*gegenstandslos*) or nonempty (*gegenständlich*); if nonempty, they are either singular or general. Examples of empty terms are –1, 0, Nothing, Round Square, Green Virtue, and Golden Mountain; absolutely simple terms are Not, Some, Have, Be, and Ought, but Bolzano was uncertain about others. Simple, singular terms he called intuitions (*Anschauungen*).

Propositions are composed of terms and are perhaps best regarded as ordered sequences of terms, while the content (*Inhalt*) of a proposition is the (unordered) set of the simple terms out of which the terms constituting the proposition are composed. The content of a complex term is similarly defined. The terms 3^5 and 5^3 are different, though they have the same content. The terms 2^4 and 4^2 are different, though they have not only the same content but even the same object. With this conception of content, the traditional doctrine of the reciprocity between the extension of a term (the set of objects falling under it) and the content of a term can easily be seen to be invalid.

Among Bolzano's many idiosyncratic convictions, perhaps the most interesting, but also the most strange to the modern mind, was his belief that each branch of science has a unique, strictly scientific presentation, which for him meant not only a unique finite axiom system (a belief he shared with many) but also an essentially unique entailment (*Abfolge*) of each theorem of this science by the axioms, a belief which might well be unique to Bolzano.

This relationship of entailment, as presented by Bolzano, is very peculiar and obscure. Bolzano was never quite sure that he understood it himself, though he was convinced that there objectively must exist some such relationship, that each science must have its basic truths (*Grundwahrheiten*) to which all other truths of that science stand in the peculiar relation of consequence (*Folge*) to ground (*Grund*). Bolzano was constantly struggling to differentiate this relation of entailment from the relation of derivability (*Ableitbarkeit*), which was the basic relation of his logic. Though he did not succeed in putting his theory of entailment into consistent and fruitful shape,- and could not possibly have done so, in view of the chimerical character of his goal,- his acumen, mastery of the contemporary logical and methodological literature, intellectual honesty, and lifelong self-criticism more than made up for his numerous shortcomings. Bolzano remains a towering figure in the epistemology, logic, and methodology of the first half of the nineteenth century." (p. 647)

From: Yeoshua Bar-Hillel, *Bolzano, Bernard*, in: Paul Edwards (ed.), *The Encyclopedia of Philosophy*, New York: Macmillan 1967, vol. 2, pp. 337-338; Second edition: Donald M. Borchert (ed.), New York: Thomson Gale 2006, vol. 1, pp. 646-648.

"Bolzano's philosophy is notable for its clarity and for his reliance on logical argument. This, his monadological metaphysics and his many-sidedness helped to earn him his sobriquet of 'the Bohemian Leibniz'. Bolzano's stalking horse was Kant, whom he respected as an important philosopher but with whom he disagreed on many fundamental matters. A follower, Franz Prihonsky, collected his critical discussions of Kant into a volume entitled *Neue Anti-Kant*. So Neurath's epithet about Austrian philosophy being spared Kant is wrong: Bolzano took Kant very seriously, but disagreed with him.

The most characteristic doctrine of Bolzano's philosophy is his semantic Platonism, which anticipates that of Frege. Bolzano distinguished mental judgements and linguistic sentences (*Sätze*) from what he called *Sätze an sich*, which I shall call 'propositions'. Likewise he distinguished mental ideas (*Vorstellungen)* and linguistic names from *Vorstellungen an sich*, which I shall call 'concepts'. The *an sich* entities, propositions and concepts, are abstract and timeless: they are the meanings of linguistic expressions and the contents of significative mental acts. Bolzano had an argument against scepticism which he thought proved the existence of true propositions. Suppose there were no truths. Then the proposition that there are no truths would be a truth, so by *reductio* there is at least one truth. Since any proposition *p* is distinct from (though equivalent to) the proposition that it is true that *p*, it follows for Bolzano that there are infinitely many truths, and these are all abstract propositions (in themselves). Some years later Dedekind produced a similar (and similarly flawed) argument to try and show the existence of an infinite set. It is important that for Bolzano false propositions have the same ontological status as true ones, and objectless concepts have the same status as concepts under which objects fall.

This Third Realm of the in-itself is brilliantly wielded by Bolzano to define and explain truth and falsity, logical truth and logical falsity, logical consequence, compatibility, derivability, analyticity, logical analyticity, probability, degrees of derivability and probabilistic inference. His definition of logical consequence differs little from that of Tarski, which it anticipated by about a century, and his theory of logical truth anticipates that of Quine. In logic it seems to have been Bolzano's fate to have invented wheels that others more famously reinvented after him. Had his views been widely known and available in readable texts in or shortly after his lifetime, I estimate that the advance of logic would have been accelerated by at least thirty, perhaps even fifty years. Where he falls short of Frege is that he does not have the concept of a formal system, where axioms are laid down and theorems follow by precisely defined syntactic rules of inference. Bolzano on the other hand prefers to work throughout with semantic concepts. The most important of these is the idea of *variation*. If we take a proposition and consider some logical part of it, whether a concept or another proposition, then we can consider what happens when we allow this part to vary and consider the range of its possible variants. For example if we take the proposition *John loves Mary* then we could replace *John* by *Fred, Harry, Elisabeth* etc., usually providing only that the name replacing John always denotes, and consider various properties of the class of variants so obtained. It is amazing how many different logico-semantic concepts Bolzano can define using this one idea. In one respect though he remains old-fashioned and Leibnizian, namely in his affection for the subject,- predicate form of propositions. The basic form of proposition for Bolzano is A has b, where A is the subject-concept and b is an abstract name for a predicate-concept, e.g., instead of *This is red* he would say *This has redness*. He even thought that every proposition could be tortured into this form. Our recent relational example would be *John has love for Mary*. Two philosophically interesting concepts are truth and existence. For *It is true that it rains in Spain* Bolzano has *The proposition that it rains in Spain has truth* and for *Tigers exist* he has *The concept of tiger has objectuality*, meaning that at least one thing falls under it. The latter analysis will evoke memories of Kant and Frege: like them Bolzano considers existence a second-level concept. Even non-existence has subject,- predicate form: *There are no unicorns* becomes *The concept of unicorn has objectlessness*.

Metaphysically Bolzano was an atomist and monadist, his monads, unlike those of Leibniz, having a physical location. Taking the idea of atoms as physical points seriously led him into an odd theory of contact. At a point on its surface a physical body may have an atom (and so be closed there) or lack an atom (or be open there). Consider now two non-overlapping bodies in contact at a certain point. If they were both open there they would fail to be in contact there, since there would be a spatial point between them that neither occupies. If they were both closed there they could not be in contact without sharing a point, in which case they would overlap. Hence contact can only take place where one body is open and the other is closed. Bolzano's chief metaphysical work was *Athanasia, or Reasons for the Immortality of the Soul*. Here he took the standard view that the soul is a monad and

hence indestructible. The book contains an ontology of substance and accidents, which he calls *adherences*." (pp. 112-114)

From: Peter Simons, "Bolzano, Brentano and Meinong: Three Austrian Realists", in: Anthony O'Hear (ed.), *German Philosophy Since Kant*, Cambridge: Cambridge University Press 1999, pp. 109-136.

"[Bolzano] composed his two main works from 1823 though 1841: the *Wissenschaftslehre* (4 vols., 1837) and the posthumous *Grössenlehre*.
(...)
Bolzano recognized a profound distinction between the actual thoughts and judgments (*Urteile*) of human beings, their linguistic expressions, and the abstract propositions (*Sätze an sich*) and their parts which exist independently of those thoughts, judgments, and expressions. A propostion in Bolzano's sense is a preexistent sequence of ideas-as-such (*Vorstellungen an sich*).
Only propositions containing finite ideas-as-such are accessible to the mind. Real thins existing concretely in space and time have subsistence (*Dasein*) whereas abstract objects such as propositions have only logical existence. Adherences, i.e., forces, applied to certain concrete substances give rise to subjective ideas, thoughts, or judgments. A subjective idea is a part of a judgment that is not itself a judgment. The set of judgments is ordered by a causal relation.
Bolzano's abstract world is constituted of sets, ideas-as-such, certain properties (*Beschaffenheiten*), and objects constructed from these. Thus, sentence shapes are a kind of ideas-as-such, and certain complexes of ideas-as-such constitute propositions. Ideas-as-such can be generated from expressions of a language by postulates for the relation of being an object of something. Analogously, properties can be generated by postulates for the relation of something being applied to an object.
(...)
In the *Grössenlehre* Bolzano intended to give a detailed, well-founded exposition of contemporary mathematics and also to inaugurate new domains of research. Natural numbers are defined, half a century before Frege, as properties of "bijective" sets (the members of which can be put in one-to-one correspondence), and real numbers are conceived as properties of sets of certain infinite sequences of rational numbers. The analysis of infinite sets brought him to reject the Eudidean doctrine that the whole is always greater than any of its parts and, hence, to the insight that a set is infinite if and only if it is bijective to a proper subset of itself. This anticipates Peirce and Dedekind.
Bolzano's extension of the linear continuum of finite numbers by infinitesimals implies a relatively constructive approach to nonstandard analysis. In the development of standard analysis the most remarkable result of the *Grössenlehre* is the anticipation of Weirstrass's discovery that there exist nowhere differentiable continuous functions.
The *Wissenschaftslehre* was intended to lay the logical and epistemological foundations of Bolzano's mathematics. A theory of science in Bolzano's sense is a collection of rules for delimiting the set of scientific textbooks. Whether a class of true propositions is a worthwhile object of representation in a scientific textbook is an ethical question decidable on utilitarian principles.
Bolzano proceeded from an expanded and standardized ordinary language through which he could describe propositions and their parts. He defined the semantic notion of truth and introduced the function corresponding to a "replacement" operation on propositions. One of his major achievements was his definition of logical derivability (*logische Ableitbarkeit*) between sets of propositions: B is logically derivable from A if and only if all elements of the sum of A and B are simultaneously true for some replacement of their non-logical ideas-as-such and if all elements of B are true for any such replacement that makes all elements of A true. In addition to this notion, which is similar to Tarski's concept of consequence of 1936, Bolzano introduced a notion corresponding to Gentzen's concept of consequence. A proposition is universally valid (*allgemeingültig*) if it is derivable from the null class. In his proof theory Bolzano formulated counterparts to Gentzen's cut rule.
Bolzano introduced a notion of inductive probability as a generalization of derivability in a limited domain This notion has the formal properties of conditional probability. These features and Bolzano's characterization of probability density by the technique of variation are reminiscent of Wittgenstein's inductive logic and Carnap's theory of regular confirmation functions.
The replacement of conceptual complexes in propositions would, if applied to a formalized language, correspond dosely to a substitution-semantic conception of quantification. His own philosophical language was based on a kind of free logic. In essence, Bolzano characterized a

substitution-semantic notion of consequence with a finite number of antecedents. His quantification over individual and general concepts amounts to the introduction of a non-elementary logic of lowest order containing a quantification theory of predicate variables but no set-theoretical principles such as choice axioms. His conception of universal validity and of the semantic superstructure of logic leads to a semantically adequate extension of the predicate-logical version of Lewis's system S5 of modal logic without paradoxes. It is also possible to simulate Bolzano's theory of probability in a substitution-semantically constructed theory of probability functions. Hence, by means of an ontologically parsimonious superstructure without possible-worlds metaphysics, Bolzano was able to delimit essentially the realms of classical logical truth and additive probability space." (pp. 93-94)

From: Jan Berg, "Bolzano, Bernard", in: Robert Audi (ed.), *The Cambridge Dictionary of Philosophy. Second Edition*, Cambridge: Cambridge University Press 1999.

The importance of Bolzano's logic

"Why look back now? Let me start by stating my non-historian's view of the modem history of logic. Like many scientific disciplines, flourishes while being ill-defined. Despite textbook orthodoxy, the issue what logic should be about is a legitimate topic of discussion, and one to which answers have varied historically. One key topic is reasoning: its valid laws for competent users, and perhaps also its sins: mistakes and fallacies. But the modern core also includes independent concerns such as formal languages, their semantic meaning and expressive power. Moreover, the modem research literature, much of it still in a pre-textbook stage, reveals a wide range of topics beyond reasoning and meaning, dealing with general structures in information, and many-agent activities other than reasoning, such as belief revision or communication. Thus, the agenda of logic keeps evolving, as it should. In this light, going back to the pioneers is not just a matter of piety, but also of self-interest.
One striking feature of older literature is its combination of issues in logic with general methodology of science. One sees this with Bolzano, Mill, or Peirce, but also with major modem authors, such as Tarski, Carnap, or Hintikka. The border line between logic and philosophy of science seems arbitrary. Why have 'confirmation', 'verisimilitude', or 'theory structure' become preserves for philosophers of science, and not for logicians? This separation seems an accidental feature of a historical move, viz. Frege's 'contraction of concerns', which tied up logic closely with the foundations of mathematics, and narrowed the agenda of the field to a point where fundamentalists would say that logic is the mathematics of formal systems. Admittedly, narrowing an agenda and focusing a field may be hugely beneficial. Frege's move prepared the ground for the golden age of logic in the interbellum, which produced the core logic curriculum we teach today. At the same time, broader interests from traditional logic migrated, and took refuge in other disciplines. But as its scientific environment evolved in the 20th century, logic became subject to other influences than mathematics and philosophy, such as linguistics, computer science, AI, and to a lesser degree, cognitive psychology and other experimental disciplines.
Compared with Frege, Bolzano's intellectual range is broad, encompassing general philosophy, mathematics, and logic. This intellectual span fits the above picture. Even so, I am not going to make Bolzano a spokesman for any particular modern agenda. The current professional discussion speaks for itself. But I do want to review some of his themes as to contemporary relevance. Incidentally, the main sources for the analysis in my 1985 paper, besides reading Bolzano himself, have been Kneale & Kneale 1962, and Berg 1962. After the Vienna meeting this autumn of 2002, I learnt about Rusnock 2000, whose logic chapters turned out sophisticated and congenial.

A short summary of Bolzanian themes:

- We quickly enumerate those points in Bolzano's logical system that are the most unusual and intriguing to logicians. These will return at lower speed in later sections.

- The systematic idea of decomposing propositions into general constituents is linguistically attractive, and reminiscent of abstract analyses of constituent structure in categorial grammars (Buszkowski 1997, Moortgat 1997, van Benthem 1991).
- In doing so, looking at different ways of setting the boundary between *fixed* and *variable* vocabulary in judging the validity of an inference is another innovation, which ties up with the recurrent issue of the boundaries of 'logicality'.
- Moving to logical core business, acknowledging different styles of reasoning: 'deducibility', 'strict deducibility', or statistical inference, each with their own merits, is a noteworthy enterprise quite superior to unreflected assumptions of uniformity.
- As to detailed proposals, consider Bolzano's central notion of deducibility. It says that an inference from premises φ to a conclusion Ψ is valid, given a variable vocabulary *A* (written henceforth as $\varphi \Rightarrow A\ \Psi$) if (a) every substitution instance of the *A*'s which makes all premises true also makes the conclusion true, and (b) the premises must be consistent. Clause (a) is like modem validity, modulo the different semantic machinery, but with a proviso (b) turning this into a non-monotonic logic, the hot topic of the 1980s. Moreover, the role of the vocabulary argument A making inference into a ternary relation really, will also turn out significant later.
- But also other notions of inference are reminiscent of modem proposals trying to get more diversity into how people deal with large sets of data, such as 'strict deducibility': using just the minimal set of premises to get a given conclusion.
- Bolzano's statistical varieties of inference involve counting numbers of substitutions that make a given statement true. Such connections between qualitative logic and quantitative probability were still alive in Carnap's inductive logic, a fringe topic at the time, but they are coming back in force in modem logic, too.
- Very striking to logicians at the interface with AI is Bolzano's formulation of systematic properties of his notions of inference, such as versions of transitivity or the deduction theorem, some depending on the fixed/variable constituent distinction. No truth tables, model-theoretic semantics, and their ilk, but instead, some of the more sophisticated structural theory of inference that carne in fashion in the 1980s.

All these themes do, or should, occur in modern logic! Let's take them up now one by one." (pp. 12-14)

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From: Johann van Benthem, "Is There Still Logic in Bolzano's Key?", in: Edgar Morscher (ed.), *Bernard Bolzanos Leistungen in Logik, Mathematik und Physik*, Sankt Augustin: Academia Verlag, 1999.

"In an introductory chapter, Bolzano defines a science as an "aggregate of truths whose known portion is important enough to be set forth in a special book" and logic as the science which deals

with the division of the domain of all truths into suitable parts, and supplies the rules for the composition of the respective treatises. These rules and the division of the domain of all truths are discussed in the final, fourth volume of the German edition. But before the domain of truths can be divided into sections, and treatises written, a sufficient number of truths must first be discovered. Accordingly, the theory of science proper is preceded by a book entitled *Erfindungskunst* (Heuretic), which is concerned with the discovery of truths. This section, in turn, presupposes a discussion of the conditions of human knowledge in general. But epistemology can be transacted only if it is preceded by a theory concerning the entities which are known, namely propositions in themselves and their terms (Theory of Elements). Finally, the first section of the work is the Theory of Fundamentals, in which Bolzano undertakes to prove that there are truths in themselves and that some of them can be known." (pp. XXVIII-XXIX)

From: George Rolf, *Editor's Introduction*, to: Bernard Bolzano, *Theory of Science*, Berkeley: University of California Press 1972.

The main thesis contained in the *Theory of Science* consists in a clear distiction between psychology and logic. This work, as well as Bolzano's other works on logic, was given little consideration by his contemporaries. Husserl was the first to point out the exceptional importance of Bolzano's conception, considering him as "one of the greatest logicians of all times".
In Bolzano's view, logic is "a theory of science" -- Wissenschaftslehre. which explains the title of the above cited treatise on logic.
The work is divided into five parts:
1) *Fundamentallehre* -- fundamental theory. In this part Bolzano points out that truths must be considered in themselves -- *Wahrheiten an sich*, separating the logical content from the corresponding logical process.
2) *Elementarlehre* -- elementary theory. In this part he treats of the theory of representations of sentences and deductions. Here also Bolzano admits, as he did for truth, that there are "representations in themselves -- *Vorstellungen an sich* and "sentences in themselves" -- *Sätzen an Sich.*
3) *Erkenntnislehre* -- the theory of knowledge. That is the theory of the conditions that truth must conform to in relation to human intelligence.
4) *Erfindungskunst* -- the art of discovering truth.
5) *Eigentliche Wissenschaftslehre* -- the theory of science proper. This part is concerned with "truth" in the field of special sciences.
The three fundamental concepts on which Bolzano's theory is based are: "sentence in itself", "representation in itself", and "truth in itself".
By "sentence in itself" he understands that which can be thought in a sentence, irrespective of the fact whether this sentence has been thought or not, expressed or not. In this way, he marks a fundamental distinction between thinking a sentence and the sentence itself. The "sentence in itself" is neither representation, nor judgement; Bolzano does not specify what such a sentence is, but he says what it is not. A "sentence in itself" has no existence whatsoever, since only thought sentences or asserted sentences exist in the mind of the one who thinks; the sentence is the content of thought, which content has no real existence. So, for instance, the sentence "life is not the greatest good of all" is a "sentence in itself", when we consider only its significant content -- its sense --, irrespective of the fact whether it is true or false.
As to the "representation in itself", this does not exist in us, it exists independently of the subject's consciousness; therefore, although several subjects may have the same representation, it is not multiplied but unique, and this is, in fact, Bolzano's argument in favour of the objectivity of representation. Let us take the above quoted sentence, "life is not the greatest good of all"; "life" and "the greatest good of all" are representations in themselves and are elements of the given sentence. The sum of representations in a sentence forms its content. This "objective representation" does not need, like the "proposition in itself" or the objective proposition, a subject who should think of, or express it but, like the latter, "it is not anything existing and yet it is a certain something" -- *Zwar nicht als etwas Seiendes, aber doch als ein gewisses Etwas* (*Wissenschaftslehre*, vol. 1, p. 217). More precisely "representation in itself" consists of something but not of something existing. Therefore "representations in themselves" are neither true nor false.
The third of Bolzano's concepts is "truth in itself", which expresses something as it is, irrespective of whether it was or not thought or expressed by some one. The object of truth needs nothing of what

exists. So, for instance, the truth that "truth is nothing existing", does not require any real object (*op. cit*., vol. I, p. 112).
After this analysis of significations Bolzano proceeds to the examination of other logical concepts, of logical value, logic relation and deduction, and he comes to the conclusion that logic is a science of meaning. This is pure logic -- *Die reine Logik* -- independent of psychology, with an a priori value, but not in the Kantian sense.
Husserl will be influenced by these basic ideas of Bolzano's philosophy and in this way will attempt to definitely eliminate psychologism in logic." (vol. III, pp. 354-355)

From: Anton Dumitriu, *History of Logic*, Tubridge Wells: Abacus Press 1977 (4 volumes).

Bolzano's contribution to logic

"The *Wissenschaftslehre* [*WL*] (1837) by Bernard Bolzano (1781-1848) is one of the masterpieces in the history of logic. In this encyclopedic work Bolzano intended to construct a new and philosophically satisfactory foundation of mathematics. The search for such a foundation brought forth valuable by-products in logical semantics and axiomatics. For example, Bolzano introduced the notion of abstract, non-linguistic proposition and described its relations to other relevant notions such as sentence, truth, existence and analyticity. Furthermore, he studied relations among propositions and defined highly interesting notions of validity, consistency, derivability and probability, based on the idea of "replacing" certain components in propositions. In set theory, he stated the equivalence of reflexivity and infiniteness of sets and considered isomorphism as a sufficient condition for the identity of powers of infinite sets. He conceived of a natural number as a property characterizing sets of objects, even though he did not base his development of arithmetic on this notion, and analyzed sentences about specific numbers in a way reminiscent of Frege and Russell. In a posthumous manuscript from the 1830's (recently published) he developed a theory of real numbers, which differs from those of Dedekind, Weierstrass, Méray and Cantor. Bolzano's real numbers may be identified with certain sequences of rational numbers.
Logic in Bolzano's sense is a theory of science, a kind of metatheory, the objects of which are the several sciences and their linguistic representations. This theory is set forth in Bolzano's monumental four-volumes work *Wissenschaftslehre* (hereafter referred to as *WL*). Bolzano's very broad conception of logic with its strong emphasis on methodological aspects no doubt accounts for the type of logical results which he arrived at. The details of his theory of science proper are given in the fourth volume of the *WL* and belong to the least interesting aspects of his logic. On the other hand, Bolzano's search for a solid foundation for his theory of science left very worthwhile by-products in logical semantics and axiomatics. His theory of propositions in the starting-point of these results.
Bolzano became more and more aware of the profound distinction between the actual thoughts of human beings and their linguistic expressions on the one hand, and the abstract propositions and their components which exist independently of these thoughts and expressions on the other hand. Furthermore, he imagined a certain fixed deductive order among all true propositions. This idea was intimately associated with his vision of a realm of abstract components of propositions constituting their logically simple parts.
For the following presentation of Bolzano's theory of propositions I have to define some terms. A concrete sentence occurrence is a sequence of particles existing in space and time, arranged according to the syntactic rules of a grammar, and contrasting with its surroundings. A simple sentence shape, on the other hand, is a class of similar concrete occurrences of simple sentences. A compound sentence shape is built up recursively from simple sentence shapes by means of syntactic operations. Not every compound sentence shape has a corresponding concrete sentence occurrence. Two compound sentence shapes may be considered identical if they are built up from identical simple sentence shapes in the same way. Two simple sentence shapes are identical if they contain the same sentence occurrences.
Now consider the compound sentence containing the following concrete sentence occurrence: 'a simple sentence shape is a class of similar sentence occurrences or it is not the case that a simple sentence shape is a class of similar sentence occurrences'. In another sense one could say that this sentence shape, which is an abstract logical object outside of space and time, contains two sentence

occurrences, i.e., two abstract "occurrences" of the simple sentence shape containing the following concrete inscription: 'a simple sentence shape is a class of similar sentence occurrences'. In the following, I will use the expression 'sentence occurrence' exclusively in the first, concrete sense.
Bolzano's notion of abstract non-linguistic proposition (*Sätz an sich*) is a keystone in his philosophy and can be traced in his writings back to the beginnings of the second decade of the 19th century. I shall try to characterize Bolzano's conception of propositions by means of certain explicit assumptions. These assumptions also give information about the relation between propositions and other logically interesting objects.
In his logic Bolzano utilizes a concept which is an exact counterpart of the modern logical notion of existential quantification. Therefore, he could have stated that (1) There exist entities, called 'propositions', which fulfill the following necessary conditions (2) through (15). (Cf. *WL* 30 ff.)
Thus, propositions possess the kind of logical existence developed in modern quantification theory. However, (2) A proposition does not exist concretely in space and time *(WL* 19).
According to Bolzano, both linguistic and mental entities such as thoughts and judgments are concrete (*WL*, 34, 291). Hence, propositions could not be identified as concrete linguistic or mental occurrences. Furthermore,(3) Propositions exist independently of all kinds of mental entities (*(WL* 19).
Therefore the identification between propositions and mental dispositions sometimes made in medieval nominalism cannot be applied to propositions in Bolzano's sense.
A proposition in Bolzano's sense is a structure of ideas-as-such. Hence, an idea-as-such (*Vorstellung an sich*) is a part of a proposition which is not itself a proposition (*WL* 48). But to he able to generate propositions we have to characterize ideas-as-such independently of propositions. This is in fact implicit in Bolzano. He worked extensively with the relation of being an object of an idea as-such, which corresponds in modern logic to the relation of being an element of the extension of a concept. In terms of this relation, taken as a primitive by Bolzano, certain postulates may be extracted from his writings which concern the existence and general properties of ideas-as-such.
Independently of human minds and of linguistic expressions there exists a collection of absolutely simple ideas-as-such. As examples Bolzano mentions the logical constants expressed by the words 'not', 'and', 'some', 'to have', 'to be', 'ought' (*WL*, 78); but he admits being unable to offer a more comprehensive list. He seems to mean that each complex idea A can be analyzed into a sequence *S*(A) of simple ideas which would probably include certain logical constants.
I shall call this sequence *S*(A) the 'primitive form' of A. The manner in which a complex idea is built up from simple ones may be expressed by a chain of definitions. So it appears that some complex ideas behave somewhat like the open formulas of a logical calculus. Bolzano assumes that two ideas are strictly identical if and only if they have the same primitive form (*(WL* 92, 119, 557)." (pp. 147-150)

From: Jan Berg, *Bolzano's Contribution to Logic and Philosophy of Mathematics*, in: R. O. Gandy, J. M. Hyland (eds.), *Logic Colloquium '76*, Amsterdam: North Holland 1977, pp. 147-171.

Bolzano's contribution to semiotics

"The Prague philosopher, Bernard Bolzano, in his major work the *Theory of Science* (1837), mainly in the last two of the four volumes, reserves much space for semiotics. The author frequently cites Locke's Essay and the *Neues Organon*, and discovers in Lambert's writings "an semiotics many very estimable remarks", though these are of little use "for the development of the most general rules of scientific discourse", one of the aims Bolzano sets himself (par. 698).
The same chapter of *The Theory of Science* bears two titles, one of which, -- *Semiotik* -- appears in the table of contents (vol. IV, p. XVI), the other of which -- *Zeichenlehre* -- heads the beginning of the text (p. 500); paragraph 637, which follows, identifies both designations -- the theory of signs or semiotics (*Zeichenlehre Oder Semiotik*). If, in this chapter and in several other parts of the work, the author's attention is held above all by the testing of the relative perfection of signs (*Vollkomenheit oder Zweckmässigkeit*) and particularly of signs serving logical thought, then it is in the beginning of the third volume that Bolzan? tries to introduce the reader to the fundamental notions of the theory of signs throughout par. 285 (pp. 67-84) which overflows with ideas and is titled "the designation of our representations" (*Bezeichnung unserer Vorstellungen*).

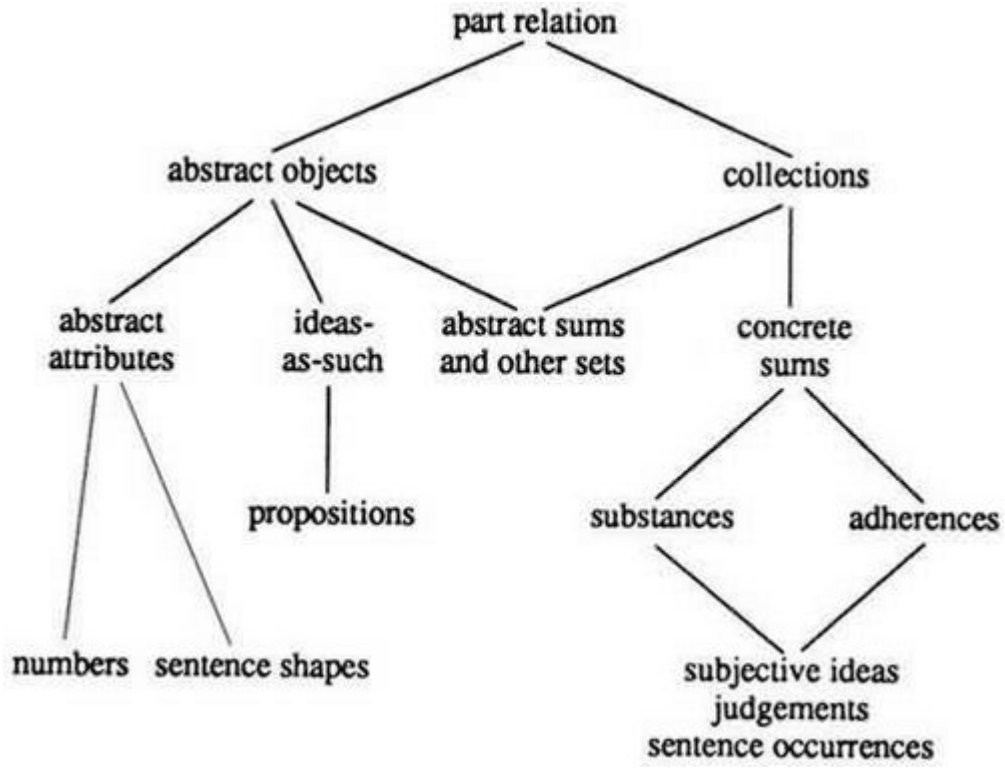
This paragraph begins with a bilateral definition of the sign, "An object through whose conception we wish to know in a renewed fashion another conception connected therewith in a thinking being, is known to us as a *sign*". A whole chain of geminate concepts follows, some of which are very new, while others, referring back to their anterior sources, are newly specified and enlarged. Thus Bolzano's semiotic thoughts bring to the surface the difference between the meaning (*Bedeutung*) of a sign as such and the significance (*Sinn*) that this sign acquires in the context of the present circumstance, then the difference between the sign (1) produced by the addresser (*Urheher*) and (2) perceived by the addressee who, himself, oscillates between understanding and misunderstanding (*Verstehen und Missverstehen*). The author makes a distinction between the thought and expressed interpretation of the sign (*gedachte und sprachliche Auslegung*), between universal and particular signs, between natural and accidental signs (*natürlich und züfallig*), arbitrary and spontaneous *(willkürlich und unwillkürlich*), auditory and visual (*hörbar und sichtbar*), simple (*einzeln*) and composite (*zusamengestzt*, which means "a whole whose parts are themselves signs"), between unisemic and polysemic, proper and figurative, metonymical and metaphorical, mediate and immediate signs; to this classification he adds lucid footnotes on the important distinction to be made between signs (*Zeichen*) and indices (*Kennzeichen*) which are devoid of an addresser, and finally on another pressing theme, the question of the relationship between interpersonal (*an Andere*) and internal (*Sprechen mit selbst*) communication." (pp. 202-203 of the reprint)

From: Roman Jakobson, *A Glance at the Development of Semiotics*, in *The Framework of Language*. Translated from the French by Patricia Baudoin, Ann Arbor: Michigan Studies in the Humanities, Horace R. Rackham School of Graduate Studies 1980 and reprinted in: R. Jakobson, *Selected Writings. Contributions to Comparative Mythology. Studies in Linguistics and Philology,* Berlin: Walter de Gruyter 1985, pp. 199-218.

Bolzano's ontology

"The first basic notion of Bolzano's ontological system is the part relation. Its domain, i.e., the set of all objects bearing it to something, embraces concrete substances, abstract objects, and collections. The converse domain of the part relation, i.e., the set of all objects to which it is borne, contains collections only.
Some collections are concrete entities existing in space and time, the rest are abstract sums or other sets. Concrete sums are composed of substances and adherences, i.e., forces. Forces applied to certain substances give rise to subjective ideas or judgements. Further results of such applications are the concrete sentence occurrences. A subjective idea is a part of a judgement which is not itself a judgement. The set of judgements is ordered by a special causal relation.
Bolzano's abstract world is constituted of sets, abstract sums, certain attributes (i.e., properties or relations), ideas-as-such, and objects constructed on the basis of these entities. Thus, sentence shapes are a kind of properties, and certain complexes of ideas-assuch constitute propositions. The notion of an idea-as-such can be constructed from expressions of a language by means of axioms for the relation of being an object of something. Analogously, properties can be generated by axioms for the relation of something being applied to an object. The converse of this relation, i.e., the relation of an entity having a property, and the relation of being an object of an idea-as-such are fundamental ontological constants of Bolzano's.
Natural numbers are defined as properties of bijective sets, and real numbers are essentially conceived of as properties of sets of certain infinite sequences of rational numbers. The analysis of infinite sets leads to a generalization of the part relation by scrapping the doctrine that the whole is always greater than any of its parts. The extension of the linear continuum of finite numbers by infinitesimals within the coarsest free algebraic filter settles definite limits to Bolzano's approach to analysis.
A part relation in a narrower sense, viz., the relation of being a subsequence of a sequence of abstract objects, holds among ideasas-such and propositions. Furthermore, the relation of derivability holds among propositions, and true propositions are ordered by the relation of entailment.
Among the relations holding between the constituents of the concrete world and the abstract world there are the relations of a substance having a property or being an object of an idea-as-such.

Moreover, the relations of an idea-as-such or proposition being the subject matter of a subjective idea or a judgement, respectively, establish ontologically important connections between the abstract world and the concrete world.
The main features of Bolzano's ontology may be schematized as follows:



(...)
The question whether a rational reconstruction of Bolzano's ontology is possible will be sustained like a pedal point throughout the present study. In many respects, indeed, his ontological system is a model of thrift, comprehensiveness, and deductive cogency. He shows us how to grasp a self-contained, abstract "third" world (in Popper's sense) embracing the realms of classical logical truth and additive probability spaces without indulging in possible worlds, states of affairs, facts, and all that. Admittedly, from a modern point of view certain aspects of his ontology may look like Dr. Johnson's dog walking on its hind legs: it is not always done quite well, but you are surprised to find it done at all. To rational bipeds of our time it should be more instructive, though, to watch this performance rather than amazing at metaphysical cephalopods wallowing in clouds of ontological splendors, or gazing at recondite cogitators crawling on all fours through a self-induced verbal fog." (pp. 31-32)
(...)
"Ontology without possible worlds.
A minimal requirement for pursuing philosophy of science and mathematics is the access to sentence (or formula) shapes, an adequate truth definition, substitution, and some set-theoretic principles. The first three notions allow a semantic demarcation of the
realms of classical logical truth and additive probability spates. Apart from syntactic identity, the strongest semantic principle of individuation for sentence shapes is logical equivalence. If one should insist on abstract objects with stronger semantic identity conditions, as Bolzano did for reasons of philosophical foundations, then non-linguistic propositions may be tendered.
Bolzano proceeded from an expanded and standardized ordinary language by means of which he could describe the universe of propositions and their parts. We have seen that this exposition can be organized into explicit postulate systems. The existence of propositions and their parts being thus guaranteed, Bolzano defined the semantic notion of truth and introduced the function corresponding to a "replacement" operation on propositions. He could also easily have rendered an exact definition of the notion of a sentence shape. The replacement of conceptual complexes in propositions enabled him to develop the essential parts of classical logic and probability theory without resorting to ontologically lavish constructions.
Bolzano's notion of proposition offers an interesting alternative to the corresponding concepts developed in modern possible-world semantics. (For a lucid survey, see Edgar Morscher, *Propositions and all that: Ontological and epistemological reflections*, in: L. M. de Rijk (ed.), *Logos*

and Pragma. Essays on the Philosophy of Language in Honour of Professor Gabriel Nuchelmans, Nijmegen, 1987, pp. 241-257) According to a representative theory of this kind, a proposition is a function sending possible worlds onto truth-values. A possible world is a maximally consistent set of states of affairs. A state of affairs is somehow conceived of as being built up from members of the domain of individuals and their attributes. Moreover, a fact is a real state of affairs. Thus, a concrete object and its attributes can be parts of a state of affairs. For example, the concrete individual Kurt Waldheim and the property of being the 42nd president of the United States of America would, according to this view, be parts of the state of affairs that Kurt Waldheim is the 42nd president of the U.S.

The main flaws of this approach to the ontology of propositions are, first, that propositions expressed by logically equivalent sentences conflate and, second, that a concrete object can never be part 01 a state of Alai's which is nut a fact. lot example, the real Kurt Waldheim can never be part of the state of affairs of someone being the 42nd president of the U.S.

The latter obstacle can be removed by representing concrete things by bundles of world-lines, i.e., by sets of sets of world-points. The real Kurt Waldheim, e.g., is thereby represented by a bundle of world-lines which will never enter into a state of affairs containing the property of being the 42nd president of the U.S. The fictitious Kurt Waldheim figuring in such a state of affairs branches off from the bundle representing the real Kurt Waldheim at a certain space-time point in the world of 1993. In view of the highly abstract character of this approach, an alternative remedy might be to leave states of affairs unanalyzed and take them as primitive entities. From the ontological point of view, however, we could then as well get on directly with propositions.

An attempt to evade the former difficulty of propositions conflating under logical equivalence of the corresponding sentences by proffering new categories of intensional objects will be a great expense to unyielding ontologians. One device may be to take the functions sending the possible worlds onto truth-values in intension. Hence, a practicable theory of propositions based on a possible-world semantics would have to postulate the existence of sets of sets of world-points, and moreover of properties, relations, and function concepts. An attempted entity-saving measure of introducing the attributes by functions in extension from possible worlds onto sets of individuals or sets of n-tuples of individuals would be redundant, however, since attributes are parts of the constituents of possible worlds.

An ontology based on Bolzano's system of propositions would only have to postulate the existence of one category of intensional objects, namely ideas-as-such, and could otherwise employ purely extensional set-theoretic and algebraic methods. A possible objection to Bolzano's ontology might be raised on account of the fact that it cannot yield the semantics of epistemic and other non-classical logics. In these regions outside the analysis of the foundations of science and mathematics, it may be argued, real philosophy begins with the search for new semantic superstructures while the metaphysical dusk of possible worlds approaches."

From: Jan Berg, *Ontology without Ultrafilters and Possible Worlds. An Examination of Bolzano's Ontology,* Sankt Augustin: Academia Verlag 1992.

Bolzano on the limits of knowledge

Bolzano, whose concept ‘truths in themselves’ was employed by Husserl (...), set himself the task of clarifying the concept of unknowability and of finding out whether it has any instances. The § 314 of his *Wissenschaftslehre* bears the title: “Whether our ability to know has definite limits” (“Ob es bestimmte Grenzen für unser Erkenntnisvermögen gebe?”). The target of Bolzano’s criticism is Kant, as the next paragraph (315) makes clear: “The doctrine of the critical philosophy on this” (“Die Lehre der kritischen Philosophie hierüber”).

One would expect that Hartmann carefully studied Bolzano’s reasoning before presenting his own theory. At least, he should have commented on Bolzano’s arguments, and tried to refute them. One’s expectations grow higher due to his citation of Husserl, who obviously received the concept of truths in themselves from Bolzano (Husserl praised Bolzano in the first volume of his *Logische Untersuchungen*). However, nothing seems to indicate that Hartmann was familiar with Bolzano’s reasoning against unknowability in § 314.(11) The *Wissenschaftslehre* which Hartmann had thoroughly studied is that of Fichte (cf. *Die Philosophie des deutschen Idealismus*, pp. 45—80).

This is not only a scholarly point, because Bolzano's analysis of the knowability problem is one of the most remarkable ever presented of it and his arguments present a severe challenge to anybody who thinks that human knowledge has absolute limits. Bolzano's contribution was unknown to Hartmann — but surely not unknowable.

According to Bolzano, there is an infinite number of true propositions. But, because we have a finite capacity of comprehending these propositions, it may be asked whether our ability to know has definite limits and whether we are able to determine these. Before answering this question, says Bolzano, we first have to clarify what we understand by such limits to our ability to know and by their determination.(12) One may determine the limits of a certain power either completely or incompletely. Complete determination concerns everything that the power can and cannot bring about, whereas incomplete or partial determinations concern only something the power can or cannot accomplish. A complete determination of our ability to know should characterize in a perfect way the totality of the truths which we know and of the truths we are not able to know; otherwise a determination of this ability is incomplete. It is the complete determination that is needed: certain sentences, which truthfully state what we are able to know and what not. 'Knowledge' here means true justified belief. One may speak of the (public) knowledge of the whole of humanity or of all finite minds in the cosmos, or of the (personal) knowledge of some individual. The question of the determination of the limits of knowledge concerns *criteria* which allow us to decide whether *answering* a given *question* does not exceed our ability to know. If such criteria could be given, we would be able to abstain from a spurious search. The criteria should concern knowability in principle and not only what is presently or up to now unknowable. The fact that we have not been able to know a certain truth up to the present time does not imply that this will also be impossible for us in the future. The criteria should characterize a class of truths which we not only do not know at present, but will never know, at least as long as we are human beings.(13)

After these considerations, it should be clear enough what is being searched for. There are three possible ways of proceeding:

1) *enumerating* all unknowable truths
2) *indicating* unknowables by giving a property which characterizes them
3) *looking* for such propositions the truth or falsity of which is not only unknown but will never be found out.(14)

This means that we may use either an *extensional* or an *intensional* procedure. It is clear that procedure 1) is self-refuting, Bolzano claims: if we know that the propositions p_1..., p_n are true and unknowable to us, then they must be knowable to us.(15) It is more difficult to see that procedure 2) is also a dead end. In indicating unknowables, we have to use such characterizations as

All A's are unknowable to us.
No truths of the form 'A is X' are knowable to us.(16)

But these kinds of statements are illegitimate and self-contradictory, according to Bolzano. If we can characterize a thing by a property (say 'A'), then we know something of it, at least what it has in common with other things. However, the sentence "We cannot know anything of A" is a sentence about A, so that it is a contradiction to say that one does not know of A at all, i.e., that one cannot express any true proposition about it.

It may be said that, by claiming something to be unknowable, one does not mean that none of its properties are known, or that no proposition can be expressed about it. It may be claimed that a thing is unknowable to us if we cannot specify any of its *real* properties. Bolzano refutes this claim as follows: the property 'A' by which we comprehend a thing, is a real property of that thing, because the features of that thing are such as belong only to things which are subsumable under 'A.' For instance, colour belongs to the properties of a star. Somebody could claim that it is not a real property, because only physical properties (mass, density, heath) are real. If nothing else than the colour were known of a star, it would then be unknown, and might remain unknowable as to its real properties. Bolzano's argument means that the claim "All stars whose properties other than colour cannot be known are unknowable to us" is self-refuting. If we identify a distant star by virtue of its colour (e.g., Vega as being blue), then that colour is a genuine feature of the star, because it belongs only to those things which have the same colour. Accordingly, the property "stars whose properties other than colour cannot be known" (A) *fails to characterize* something genuinely unknowable.

A further claim would be to say that an unknowable thing is unknowable due to such real properties which are not subsumable under 'A.' This claim may be expressed as follows:

All such properties ('X') as do not belong to the property 'A,' by which we characterize the ' A'-things, are unknowable to us.
No truths of the form 'All A's are X' are knowable to us, to the extent that 'X' is not included in the content of 'A.'(17)
Thus, the adjectives of our language might in principle fail to attain the richness of the properties characteristic of things in the world, and we would be unable to acquire knowledge of all objective properties. Admittedly, says Bolzano, there may be properties of things which are not included in our designations. But then the truth of a proposition concerning these properties should be proved on the basis of the special character of these designations. Can such a special character of designations be known? If it can be known, then it is a contradiction to maintain that it is unknowable.(18) On the other hand, if one does not come to know those properties which are not included in the content of 'A,' then one has no reason to assume their existence.(19) For instance, there is no reason to assume that there may be colours outside the reach of our colour-words.
Bolzano applies the above considerations to a criticism of Kant's hypothesis that there are things in themselves which are in principle unknowable. According to him, that hypothesis is self-contradictory: the proposition that insensible things cannot be the contents of our synthetic judgments *is itself a synthetic judgment* about them.(20) Bolzano also criticizes Kant's distinction between sensible and insensible objects, because he thinks that it is in some respects misleading.(21) Furthermore, he criticizes Kant's antimonies. According to Bolzano, the proofs which Kant presented for the theses and antitheses contain many errors.(22)
The third possibility of showing that knowledge has absolute limits is not self-refuting, as the other two are in Bolzano's analysis. This possibility is based, like the first one, on extensional considerations. The main idea is to *suggest candidates* — propositions the truth or falsity of which can never be discovered. Bolzano's objection to this procedure is that it is *difficult to prove* statements concerning unknowability. The fact that we have up to now been unable to decide whether the property 'B' belongs or does not belong to the things of class 'A' does not entitle us to conclude that we will never have any reason for such a decision. In order to make the difficulty clearer, Bolzano studies two different possibilities: the connexion between the concepts 'A' and 'B' may be
(i) purely conceptual, or
(ii) empirical.
In the first case, the history of mathematics can teach us about many questions which could not be decided for centuries but in the end were decided by suitable means. Nobody can be sure that a conceptual connexion not yet known cannot be discovered by the painstaking analysis of the concepts involved and their comparison not only with each other but also with related concepts.(23) In so far as empirical questions are concerned, nobody can anticipate the rich possibilities of future experience. Bolzano's example is an especially happy one: it is not known what the moon's inhabitants look like (the *Wissenschaftslehre* appeared in 1837). A modem example would be the question whether there are intelligent beings in other planetary systems. According to Bolzano, all possible "proofs" of unknowability are in fact based on a questionable argument from the limitations of the present state of knowledge to an unsurpassable limitation.(24)
Bolzano then considers a possible objection: because his analysis results in the impossibility of giving a limitation of our ability to know, this result itself seems to be a kind of limitation of this ability. His answer to the objection is the following: he does not maintain that it will never be possible to set a limit to knowability — only that he does not himself know of such a limit. Furthermore, the result that it is impossible to differentiate knowability from unknowability can be considered as an indication of a truth concealed for ever *only under the presupposition* that there really *is* an absolute limit of knowledge. Bolzano does not subscribe to this presupposition; he thinks rather that we cannot draw the limit between knowability and unknowability simply because there *is no such limit*. Human knowledge can be enriched ad *infinitum*.(25)" (pp. 103-106)

Notes

(11) Hartmann knew Bolzano's theories of sentences in themselves and truths in themselves, and eagerly accepted them; cf. *Grundzüge einer Metaphysik der Erkenntnis*, p. 25 and "Die Erkenntnis im Lichte der Ontologie," *Kleine Schriften* I, p. 134. Cf. also J. N. Mohanty *Nicolai Hartmann and Alfred North Whitehead. A Study in recent Platonism* (1957), p. 42: "Bolzano's doctrine of 'Satz an

sich' is accepted with admiration ..." (Mohanty points to *Grundzüge einer Metaphysik der Erkenntnis* p. 25).

(12) B. Bolzano, *Wissenchaftslehre* III, § 314, p. 232.
(13) *Ibid.*, pp. 233 ff.
(14) *Ibid.*, pp. 234 ff. The expressions "enumerate" and "indicate" have been used by J. Berg *Bolzano's Logic* (1962), p. 70.
(15) B. Bolzano, *Wissenchaftslehre* III, § 314, p. 234: "Wenn wir die Grenzen, die unser eigenes oder die das Erkenntnisvermogen der ganzen Menschheit hat, zu bestimmen suchen: so leuchtet ein, dies konne nicht dadurch geschehen, daB wir die Wahrheiten, die fur uns Einzelne oder fur alle Menschen unerreichbar sind, namentlich angeben; denn um dieB zu vermogen, miiBten sie uns nicht unbekannt, sondem bekannt seyn."
(16) Cf. the reconstruction of Bolzano's argument by A. Wedberg *Filosofins historia III. Fran Bolzano till Wittgenstein* (1966), p. 96. [English translation: *A History of Philosophy: From Bolzano to Wittgenstein*, Oxford: Clarendon Press, 1984]
Cf. also a similar reconstruction by J. Berg (1962), p. 70.
(17) Cf. A. Wedberg (1966), p. 96; J. Berg (1962), p. 70.
(18) B. Bolzano, *Wissenchaftslehre* III, § 314, p. 236.
(19) *Ibid.*: "Ich nun fur meinen Theil gestehe, von einer solchen Eigenthiimlichkeit gewisser Vorstellungen keine KenntniB zu haben."
(20 B. Bolzano, *Wissenchaftslehre* III, § 315, p. 247 f.
(21) *Ibid.*, pp. 246 ff.
(22) *Ibid.*, pp. 250 ff.
(23) B. Bolzano, *Wissenchaftslehre* III, § 314, p. 237. Cf. D. Hilbert (1965), p. 298: "[I]n der Mathematik gibt es kein Ignorabimus!"
(24) B. Bolzano, *Wissenchaftslehre* III, § 314, pp. 236 ff.
(25) *Ibid.*, p. 238.

From: Arto Sitonen, *Problems of Aporetics*, Helsinki: Suomlainen Tiedeakademia 1989.

Pages in PDF format

Detailed Index of the first important study of Brentano's philosophical work (PDF):

Hugo Bergmann, *Das philosophisches Werk Bernard Bolzano mit Benutzung ungedruckter Quellen kritisch untersucht.* Halle: Max Niemeyer 1909 (Table of Contents in German).