Annotated bibliography of Nino Cocchiarella 1966-1977

Contents of this Section

This part of the section Ontologists of 19th and 20th centuries includes the following pages:

The Conceptual Realism of Nino Cocchiarella

Books and studies about the work of Nino Cocchiarella

Essays 1966-1977 (Current page)

Essays 1978-1992

Essays 1993-2019

Essays 1966-1977

1. Cocchiarella, Nino. 1966. "A Logic of Actual and Possible Objects." *Journal of Symbolic Logic* no. 31:688.
See note in 1966c.
2. ———. 1966. "A Completeness Theorem for Tense Logic." *Journal of Symbolic Logic* no. 31:689.
See note in 1966c.
3. ———. 1966. "Modality within Tense Logic." *Journal of Symbolic Logic* no. 31:690.
Note to the reprint of these three items in Karel Lambert (ed.) *Philosophical Applications of Free Logic*, New York: Oxford University Press 1991): " The abstracts are summaries of lectures given at the December, 1965 meetings of the Association for Symbolic Logic.
(A preliminary version of those lectures was given at UCLA in 1963, and a final version was given at UCLA in the spring of 1965 at a public lecture constituting the defense of my doctoral dissertation.)"
4. ———. 1968. "Some Remarks on Second Order Logic with Existence Attributes." *Noûs* no. 2:163-175.
"In *Past, Present and Future* A. N. Prior bas suggested an approach towards the concept of existence where, following medieval logicians, we are to distinguish "between predicates (like 'is red', 'is hard', etc.) which entail existence, and predicates (like 'is thought to be red, 'is thought of', etc.) which do not" (p. 161). Let us refer to attributes (including relational attributes) which are designated by the former kind of predicate as existence attributes, or for brevity, e-attributes. It is suggested then that "x exists" is to be defined as "there is some e-attribute which x

possesses". A formalization of this (at least) second order logic of existence was recently brought about and reported on by the present author in [6]. The formalization was shown to be complete in the sense corresponding to the completeness of standard second order logic, i.e. in the sense which encompasses normal, non-standard as well standard models. (Cfr. A. Church, *Introduction to Mathematical Logic* (1956) - § 54). I should like in the present paper to discuss some of the philosophical issues involved in this formalization as well as some issues concerning the general notion of e-attribute." (p. 163)

5. ———. 1969. "A Substitution Free Axiom Set for Second Order Logic." *Notre Dame Journal of Formal Logic* no. 10:18-30.
"In what follows we present an adequate formulation of second order logic by means of an axiom set whose characterization does not require the notion of proper substitution either of a term for an individual variable or of a formula for a predicate variable. The axiom set is adequate in the sense of being equivalent to standard formulations of second order logic, e.g., that of Church [1]. It is clear and need not be shown here that every theorem of the present formulation is a theorem of the formulation given by Church. It of course will be shown here, however, that each of Church's axioms are theorems of the present system and that each of his primitive inference rules is either a primitive (and only modus ponens is taken as a primitive rule here) or a derived rule of the present system.
The importance of obtaining an axiomatic formulation such as herein described lies partly in the significance of reducing any axiom set to an equivalent one which involves fewer metalogical notions, especially such a one as proper substitution. However, of somewhat greater importance, it is highly desirable that we possess a formulation of both first and second order logic which can be extended without qualification to such areas as tense, epistemic, deontic, modal and logics of the like. Now proper substitution especially has been the main obstacle to such unqualified extensions of standard logic, and we take it to be of no little significance that at least for first order logic (with identity) a substitution free axiomatic formulation has been provided. (1) The present system extends this earlier result to the level of second order logic. (2)
A second difficulty in unqualified extensions of standard logic concerns the form which Leibniz' law, i.e., the law regarding interchangeability salva veritate, is to take. Generally, in the extensions of standard logic to modal logic, this law has been formulated in an unqualified form applicable to all contexts, thereby lending credence to the questionable view that only "intensions" or the like can serve adequately as values of the variables for such systems. In the substitution free formulations of first order logic cited, however, Leibniz' law is axiomatically formulated only for atomic contexts, and the qualified form or forms the law takes for contexts involving non-standard formula operators is given in the statement of metatheorems. (3) But again, it is a far different matter having such qualifications stipulated in the form of metatheorems as opposed to having them built directly into the characterization of the logical axioms. As we have said, it is desirable that the standard logical axioms for either first or second order logic be so that axiomatic extensions of standard logic can be made without qualification.4 This desirable feature of the substitution free formulations of first order logic mentioned is retained in our present second order system." (pp. 18-19)
(1) Such a formulation is given by A. Tarski in [2] and developed byD. Kalish and R. Montague in [3]. The present author in [4] and [5] has also formulated a substitution free axiomatization of first order logic without "existential presuppositions."
(2) Of course, when extending either first or second order logic to tense, epistemic, deontic, or modal logic, qualifications in metatheorems regarding principles of proper substitution will be required. Nevertheless, it is a far different matter having such qualifications stipulated in the form of metatheorems than it is having them built directly into the characterization of the logical axioms themselves.

(3) cf. [4], lemma 4.27 (p. 108) and the discussion on page 106. The objections against an unqualified, general version of Leibniz' principle (or interchangeability *salva veritate*) are applicable when certain special 'Opaque" contexts are involved, be they modal or otherwise. But all such contexts are—or should be when properly formalized—other than atomic, their "opacity" being generated within the scope of special formula operators. Atomic formulas, because they are atomic, will contain no occurrences of such operators and consequently will uphold par excellence the Leibnizian principle unqualifiedly."

References

[1] Church, A., *Introduction to Mathematical* Logic, Vol. I, Princeton, 1956.
[2] Tarski, A., "A Simplified Formalization of Predicate Logic with Identity," *Arch. f. Math. Logik u. Grundl*., 7 (1965), 61-79.
[3] Kalish, D., and Montague, R., 'On Tarski's Formalization of Predicate Logic with Identity," *Arch. f. Math. Logik u. Grundl*., 7 (1965), 81-101.
[4] Cocchiarella, N., Tense Logic: A Study of Temporal Reference, Doctoral Thesis, UCLA, 1966.
[5] Cocchiarella, N., "A Logic of Actual and Possible Objects," *The Journal of Symbolic Logic*, Vol. 31 (1966), 688f.

6. ———. 1969. "A Second Order Logic of Existence." *Journal of Symbolic Logic* no. 34:57-69.

"A. N. Prior in [9] has suggested an approach towards a second order logic of existence where, following medieval logicians, we distinguish "between predicates (like 'is red', 'is hard', etc.) which entail existence, and predicates (like 'is thought to be red', 'is thought of', etc.) which do not."(2) Let us refer to attributes (including relational attributes) which are designated by the former kind of predicate as existence attributes, or for brevity, e-attributes. It is suggested then that *x exists' be defined as 'there is some e-attribute which x possesses'. In what follows, this approach regarding the concept of existence is formalized semantically as well as syntactically, and a completeness theorem is established corresponding to the completeness (in a secondary sense, i.e., as including normal, nonstandard models) of standard second order logic (as formulated, for example, in Church [1])."

[For a more philosophical discussion of the present system, especially of the substitution free form of its axiom set, cf. *Some Remarks on Second Order Logic with Existence Attributes*].

(2) p. 161

References

[1] A. Church, *Introduction to mathematical logic*, vol. I, Princeton, N.J., 1956.
[9] A. N. Prior, *Past, present and future*, Oxford Univ. Press, Oxford, 1967.

7. ———. 1969. "Existence Entailing Attributes, Modes of Copulation, and Modes of Being in Second Order Logic." *Noûs* no. 3:33-48.

"Recently, in [5], I formulated a second order logic of existence which centered around the distinction between those attributes that entail existence and those that do not. (1) The distinction provides
an especially apt explication of the concept of existence and is for this reason especially pertinent to pragmatics and intensional logic, encompassing as they do such areas as tense, epistemic, deontic and modal logic.(2) For example, apropos of tense logic some attributes, such as being red, being round, being hard, etc., cannot be possessed at a time except by objects existing at that time. Other attributes, especially relational attributes between objects whose "lifespans" need not overlap, such as being an ancestor of everyone (someone) now existing, being remembered by someone now existing, (3) etc., may very well be possessed by objects which no longer exist; others, e.g., being a future descendant of everyone (someone) now existing, may be possessed by objects which have yet to exist. Still other attributes such as being believed to be a flying horse may be possessed by objects which never exist. Those attributes which entail existence (at the time of their possession) I shall call existence attributes, or for brevity, e-attributes. By a relational e-attribute I

mean an attribute which entails existence with respect to each of its argument places.

In the present paper I shall discuss some of the motivation for distinguishing e-attributes from attributes in general. As indicated, this motivation depends essentially on the desire to use logistic systems in which we are allowed to recognize modes of being other than that of actual existence, e.g., such modes in tense logic as past and future existence, or, in the logic of belief, the mode of "intentional inexistence". As also indicated, the concept of existence is central to this discussion and I shall here examine informally its explication in terms of e-attributes. In a sequel to the present paper I shall present and discuss a formal analysis of this explication in the context of a semantics for standard second order logic, with quantification over e-attributes distinguished from quantification over attributes in general. The focal point of the formal discussion will be the issue of the logical priority of the notion of e-attribution over that of being an existing object, a priority which exemplifies that of the intensional over the extensional and which, for its clarification, requires some observations on the class-attribute distinction." (pp. 33-34)

(1) For a more philosophical discussion of the axiom set for this formalization, see [4]. I follow Carnap [1], p. 5, in using attribute' to comprehend both properties and relations (in-intension). Properties are 1-place or unary attributes and are designated by 1-place predicate expressions. Extending Carnap's terminology, propositions are understood to be designated by 0-place predicate expressions and are therefore construed as 0-place attributes (whose extensions are truth-values).

(2) Cf. R. M. Montague [9] and [10] for an elegant and philosophically stimulating formulation of pragmatics and intensional logic. Montague's formulation of intensional logic, supplemented by the distinction between existence entailing and other kinds of attributes, is perhaps the most appropriate general logical framework to which the discussion and observations of the present paper lead.

(3) This example is given by R. M. Montague in [9] and [10].

References

[1] Carnap, R., *Introduction to Symbolic Logic and Its Applications* (New York: Dover Press, 1958).

[4] Cocchiarella, N., “Some Remarks on Second Order Logic with Existence Attributes,” Noûs, II, 2 (1968): 165-175.

[5] Cocchiarella, N., “A Second Order Logic of Existence,” JSL, forthcoming [1969].

[9] Montague, R. M., “Pragmatics and Intensional Logic,” forthcoming in *Dialectica*. [published in *Synthese,* Vol. 22, No. 1/2, Semantics of Natural Language, II (Dec., 1970), pp. 68-94]

[10] Montague, R. M., “Pragmatics,” in Raymond Klibansky (ed.), *Contemporary Philosophy—La Philosophie Contemporaine* (Florence: 1968).

8. ———. 1969. "A Completeness Theorem in Second Order Modal Logic." *Theoria.A Swedish Journal of Philosophy* no. 35:81-103.

"In what follows we present a second order formulation of S5 which is shown to be complete relative to a secondary sense of validity corresponding to that relative to which standard second order logic is known to be complete.(1) In our semantical metalanguage we consider various indexed sets of possible worlds and allow that not all objects existing in one indexed world need exist in another. However, as we have therefore confessed in the metalanguage our ontological commitment to all the objects that exist in one world or another, we acknowledge and formalize this confession in our object languages through allowing for quantification over *possibilia*.

Our means for distinguishing the existent from the mere possible is through a distinction between those attributes that entail existence (with respect to each of their argument places), referred to hereafter as e-attributes, and those attributes that do not.2 Accordingly, we understand 'x exists" to mean There is some e-attribute which x possesses", thus rendering existence essentially impredicative. An

alternative and equivalent route—but which we shall not follow here—is possible through taking existence as primitive in the form of quantification over existing objects and defining e-attributes as those attributes which necessarily are possessed only by existing objects." (p. 81)
(1) Cf. Henkin [9].
(2) For a modal free complete (in a secondary sense) formulation of second order logic with existence attributes, see Cocchiarella [5]. For a more philosophical discussion of this approach toward existence, see Cocchiarella [6] and [7].
References
[1] Church, A., *Introduction to Mathematical Logic*, Vol. I, Princeton, 1956.
[5] Cocchiarella, N., "A Second Order Logic of Existence", Journal of Symbolic Logic., vol. 34 (1969).
[6] Cocchiarella, N., "Some Remarks on Second Order Logic with Existence Attributes", *Noûs*, 2, 1968, 165-175.
[7] Cocchiarella, N., "Existence Entailing Attributes, Modes of Copulation and Modes of Being in Second Order Logic", *Noûs*, 3, 1969, 33-48.
[9] Henkin, L., "Completeness in the Theory of Types", *Journal of Symbolic Logic*, vol. 15 (1950), 81-91.

9. ———. 1972. "Properties as Individuals in Formal Ontology." *Noûs* no. 6:165-187.
"Russell's supposed paradox of predication has occasionally been cited as a source for lessons in ontology. So, for example, Grossmann in [6] has argued that one of the lessons of Russell's paradox is that there are no complex properties. A recent re-evaluation of the supposed paradox, however, has led me to the conclusion that there is no paradox (cf. [3]). And of course where there is no paradox, there are no lessons of paradox.
There may, however, be lessons of non-paradox, especially if instead of contradiction what results is a highly instructive ontological oddity. In what follows I shall briefly review the considerations that led me to conclude that there is no paradox but instead only this ontological oddity with instructive lessons of its own, relative of course to the ontological framework within which it occurs. I shall then briefly consider several ways of responding to this oddity, where each response presupposes an alternative ontological framework relative to which the response accounts for the oddity by either showing it to rest on an ontological error, as with Grossmann's response, or by mitigating its effect through what purports to be a deeper or wider framework than the original one in which the oddity occurs." (p. 165)
References
[3] Cocchiarella, N., "Whither Russell's Paradox of Predication," forthcoming in *Logic and Ontology*, vol. 2 of *Studies in Contemporary Philosophy*, edited by M. K. Munitz and to be published by New York University Press [1973].
[6] Grossmann, R., "Russell's Paradox and Complex Properties," Noûs, Vol. 6, No. 2 (May, 1972), pp. 153-164.

10. ———. 1973. "Whither Russell's Paradox of Predication?" In *Logic and Ontology*, edited by Munitz, Milton K., 133-158. New York University Press.
Contributions to a seminar on ontology held under the auspices of the New York University Institute of Philosophy for the year 1970-1971.
"Russell's paradox has two forms or versions, one in regard to the class of all classes that are not members of themselves, the other in regard to "the predicate: to be a predicate that cannot be predicated of itself."(1) The first version is formulable in the ideography of Frege's *Grundgesetze der Arithmetik* and shows this system to be inconsistent. The second version, however, is not formulable in this ideography, as Frege himself pointed out in his reply to Russell. (2) Nevertheless, it is essentially the second version of his paradox that leads Russell to avoid it (and others of its ilk) through his theory of types.
The first version is of course the relevant version with respect to any formulation of the theory of types in which membership in a class is the fundamental notion, that is, a formulation utilizing 'ε' as a primitive binary predicate constant.(3) However,

Russell's theory of types (even ignoring its ramification) is essentially concerned with the notion of predication, and only indirectly through the (philosophically questionable) interpretation of predication as the membership relation is the first version of his paradox relevant to this formulation.
Apparently, Russell saw his paradox as generating an aporetic situation in regard to two fundamental “notions,” namely, the notion of membership (in a class) and the notion of predication (of an attribute).(4) In regard to the notion of membership, the application of Russell’s paradox is not here brought into question. However, in regard to the notion of predication, the applicability of the reasoning grounding Russell’s paradox will here be very much brought into question. Indeed, I shall claim that in this case the paradox fails.(5)" (pp. 133-135)
(1) “Letter to Frege,” reprinted in [10], p. 125.
(2) “Letter to Russell,” ibid., p. 128.
(3) Cf. [5], p. 140 for a specific formulation of this kind of type theory.
(4) Gödel (cf. [6], p. 131f.) distinguishes these two forms of Russell’s paradox by referring to them as the “extensional” and the “intensional” forms, respectively. For the purposes of the present paper, this distinction is preferable to Ramsey’s different but better known distinction between “logical” and “semantical” paradoxes.
(5) With this failure of course goes a primary if not sole motivation for the simple theory of ontological types of third and higher order. The ontological scheme of second-order logic remains unaffected, having as it does a natural motivation of its own. Ramification also has its own motivation, and it may be appended to second-order logic (cf. [2], §58.) even though historically it was first appended to the simple theory of types.
References
[2] Church, A., *Introduction to Mathematical Logic*. Princeton, N.J.: Princeton University Press, 1956.
[5] Fraenkel, A., and Y. Bar-Hillel, *Foundations of Set Theory*. Amsterdam: North-Holland Publishing Company, 1958.
[6] Gödel, K., “Russell's Mathematical Logic,” *The Philosophy of Bertrand Russell*. P. A. Schilpp (ed.). Chicago: Northwestern University Press, 1944.
[10] Van Heijenoort, J., *From Frege to Gödel*, Cambridge: Harvard University Press, 1967.

11. ———. 1974. "Fregean Semantics for a Realist Ontology." *Notre Dame Journal of Formal Logic* no. 15:552-568.
"T* is a logistic system1 designed to represent the original ontological context behind Russel's paradox of predication. It encompasses standard second order logic, hereafter referred to as T, but goes beyond it by allowing predicate variables to occupy subject positions in its formulas.
Because of a violation of the restrictions imposed for the proper substitution of a formula for a predicate variable, RusselΓs argument fails in T*. Indeed, not only is T* consistent but it is also a conservative extension of T.
It has been suggested that one way of understanding this result is to construe occurrences of predicates in subject positions as referring, not to the properties which occurrences of the same predicates in predicate positions designate, but instead, to individual objects associated with these properties.4 Such a suggestion of course is reminiscent of Frege's ontology. And were it not that Frege is quite insistent in viewing predicates as "unsaturated" expressions and therefore not qualified as substituends for subject positions which can be occupied only by "saturated" expressions, it might be tempting to construe T* as representative of Frege's ontology. Be that as it may, the disproof of the principle that indiscernible properties are co-extensive, which is all that RusselΓs paradox comes to in T*, is reinterpreted according to this suggestion so as to show merely a variant of Cantor's theorem. And that after all is rather appropriate, since Russell's argument for his supposed paradox is really but a variant of Cantor's argument for his theorem.
In what follows we formulate the suggestion semantically and show that although the semantics thus provided does not characterize T*, it does characterize a certain

rather interesting subsystem T** of T* supplemented by the extensionality principle that co-extensive properties are indiscernible.(5) The supplemented system, T**+(Ext*), no doubt appears bizarre from the point of view of the original ontological background represented by T*—since in this ontology not all indiscernible properties are co-extensive whereas, according to the supplement, all co-extensive properties are indiscernible, thus suggesting co-extensiveness to be a stronger connection between properties than is the indiscernibility relation.
On the other hand, from the point of view of its quasi-Fregean semantics, the supplement seems rather natural—for according to this semantics the supplement amounts to the stipulation that the same individual object is to be associated with co-extensive properties. Fregean naturalness aside, it should perhaps be noted that the existence of a model-set-theoretic semantics characteristic for T*—or of T* supplemented with principles natural to the ontology of T*—remains yet an open problem." (pp. 552-553)
(2) Cf. [2], §6.
(3) Ibid., §5. We should avoid using 'identical' in place of indiscernible' here. In [3], Meyer has shown that according to T* there exists no relation which satisfies full substitutivity, and, accordingly, insofar as full substitutivity is taken to be a necessary feature of identity, there is and can be no identity relation in the ontology of T*.
(4) This suggestion is implicit, though only in a partial way, in the argument independently arrived at by Zorn and Meyer that T* is a conservative extension of T. It is explicit in the type of model defined below as quasi-Fregean and first recommended to the author as characteristic of T* by N. Belnap.
(5) It is easily seen from the proof in [2] that T* is a conservative extension of T, that this extensionality principle is not a theorem of T*—nor for that matter is its negation.
References
[2] Cocchiarella, N., "Whither Russell's Paradox of Predication?," in *Logic and Ontology*, edited by Milton K. Munitz, New York University Press (1973), pp. 133-158.
[3] Meyer, R., "Identity in Cocchiarella's T*" *Nou*s, vol. VI (1972), pp. 189-197.

12. ———. 1974. "Formal Ontology and the Foundations of Mathematics." In *Bertrand Russell's Philosophy*, edited by Nakhnikian, George, 29-46. London: Duckworth.
"In his paper, 'The Undefinability of the Set of Natural Numbers in the Ramified *Principia'*, [*] Myhill has shown that the general concept of a natural number or finite cardinal - general enough, that is, to yield the induction schema - is not definable in terms of ramified type theory in essentially its original form and without the axiom of reducibility. In my commentary I shall examine Myhill's concluding philosophical remarks within the context of general metaphysics or what below I call formal ontology. I shall especially be concerned with the sense in which ramified type theory (without the axiom of reducibility) purports to represent a constructive philosophy of mathematics. In addition, I shall sketch several forms of realism according to which the claim that "impredicativity is present in mathematics from the beginning" is true in an especially apt and interesting sense that goes beyond that intended by Myhill." (p. 29)
[*] In George Nakhnikian (ed.), *Bertrand Russell's Philosophy*, pp. 19-27.

13. ———. 1974. "Logical Atomism and Modal Logic." *Philosophia.Philosophical Quarterly of Israel* no. 4:41-66.
Reprinted as Chapter 6 in *Logical Studies in Early Analytic Philosophy*, pp. 222-243.
"Logical atomism has been construed as both a realist and a nominalist ontology. Despite their different ontological commitments, proponents of both types of atomism have tended to agree that modal operators for necessity and possibility, and thereby strict entailment too, are totally alien to the ontological grammar of logical atomism. The reason for this, apparently, is that any inclusion of modal operators in the ontological grammar of logical atomism, whether that grammar be of the

nominalist or realist variants, would represent a commitment to internal properties and relations with material content. And in logical atomism, of course, all internal properties and relations, be they of objects or of situations, are formal and not material in nature. (Cf. Wittgenstein, *Tractatus Logico-Philosophicus*, ([TR]), 4.122).

However, to the contrary, we shall argue that not only are propositional connectives for logical necessity and possibility, and thereby strict entailment too, no more alien to the ontological grammar of logical atomism than are connectives for conjunction and disjunction, but, moreover, that the formal or internal properties and relations of objects and situations in the ontology cannot be adequately represented by the propositional forms of that grammar unless connectives for logical necessity and possibility are included (or definable by others so included) therein.

That is, we shall argue that connectives for logical necessity and possibility, together with their proper "logico-syntactical employment" ([TR], 3.327), represent formal, and not material, internal "properties," and, moreover, that these formal, internal "properties" are part of the ontology of logical atomism and cannot be adequately represented without the inclusion of such connectives in the ontological grammar of any formal system purporting to represent that ontology.(1)

Our position and argument, incidentally, applies only to modal operators for logical necessity and possibility. All other modal operators, we agree, because they purport to represent internal "properties" or "relations" with real material content (e.g., causality, and even temporality via tense logic), are strictly prohibited within the metaphysical framework of logical atomism. "Superstition is nothing but belief in the causal nexus" ([TR], 5.1361). "The only necessity that exists is logical necessity" ([TR], 6.37).

Moreover, our concern here shall be with logical atomism as the metaphysical framework for a type of formal ontology. Our concern will not be with logical atomism as the framework for either a theory of meaning or a theory of knowledge. Accordingly, neither the Carnapian theory of *Protokolsätze* nor the Tractarian picture theory of meaning are essential to our present purely ontological considerations. We should note, however, that the Tractarian theory of elementary propositions as pictures contains both a theory of predication and a theory of meaning. It is the theory of predication that is an essential part of the ontology of logical atomism.

In the present chapter we shall limit our formal developments to the level of analysis dealing solely with propositional connectives. Our next chapter will deal with nominalist logical atomism where only individual variables are bindable but where atomism's theory of predication enters the ontological grammar in a fundamental way. That chapter will also contain a description of several variants of realist logical atomism, one in which material properties and relations of objects are themselves objects, and another where material properties and relations of objects, though indicated by bound predicate variables (as in the first variant of realism), are not themselves objects (values of individual variables) but are nexuses or modes of configurations of objects (as they are in nominalism where they are not indicated by bound predicate variables)." (pp. 222-223 of the reprint)

14. ———. 1974. "La Semantica della Logica del Tempo." In *La Logica del Tempo*, edited by Pizzi, Claudio, 318-347. Torino: Boringhieri.
Italian translation of the third chapter of the unpublished Ph. D. Thesis: *Tense Logic: A Study of Temporal Reference*, (1966).

15. ———. 1974. "A New Formulation of Predicative Second Order Logic." *Logique et Analyse* no. 65-66:61-87.
"In what follows, a predicative second order logic is formulated and shown to be complete with respect to the proposed model theoretic semantics. The logic differs in certain fundamental ways from the system formulated by Church in [1] § 58. The more important differences are noted and discussed throughout the present paper. A more specialized motivation for the new formulation is outlined in § 2.

In regard to the motivation for Church's formulation, this will be found in its natural extension to ramified type theory (without the axiom of reducibility). Within this larger framework, the theory of predication represented by such a formulation can be seen to be constructive: higher order entities are constructible from entities of lower order, with real, nonconstructed individuals as the entities of lowest order. Set theory, to whatever extent it is representable in the framework, appears in the ramified hierarchy only after propositional functions are allowed to be arguments of third and higher order predicates. To introduce sets as real, non-constructed individuals of lowest order would be antithetical to the framework's constructive theory of predication and in violation of its philosophical motivation."
(...)
"Essential to this proposal, however, is a view of the predicative/impredicative distinction radically different from that found in ramified type theory — and hence in Church's formulation of predicative second order logic. The latter framework (barring the axiom of reducibility) represents a constructive theory of predication that rules out all manner of categorial content (indicated by bound predicate variables) or logistic efficacy for impredicative contexts. In the proposed, modified Fregean theory, however, impredicative contexts (wffs) are allowed to have logistic efficacy — and perhaps even categorial content if standard quantifiers ranging over all properties and relations are retained as well.
If, on the one hand, only quantifiers for predicatively specifiable properties and relations are allowed, then in this new formulation of predicative second order logic impredicative contexts — which in general will contain free (schematic) predicate variables or certain predicate constants — will be syncategorematic expressions, since they will not then be permissible substituends of generalized predicate variables. This does not mean that they must then be accorded null content. They may instead represent logical or formal content variant to what Frege calls second and third level "concepts". (4)
This logical content would in effect be the basis of their logistic efficacy. (5)
If, on the other hand, these impredicative contexts are to be given categorial content by retaining standard quantifiers, then care must be made to distinguish these quantifiers from those ranging over only predicatively specifiable properties and relations. Both kinds of quantifiers will bind the same variables, but impredicative wffs will be permissible substituends only of variables bound by the one quantifier. (6)
They remain impermissible substituends of variables bound by the quantifiers for predicative properties and relations.
In the system to be formulated here we are concerned only with the first of the above alternatives, although once formulated it is easily extended to the richer framework. (7)" (pp. 61-64)
(4)We should distinguish at least two kinds of content that expressions of a formal system might have. The first is generally called descriptive, but historically has been called categorial, which we prefer here since even without (applied) descriptive constants the content is still indicated by bound variables, (Hence our reference to categorial content.) The second is generally called logical or formal, or, traditionally, syncategorematic and is understood to be immanent to the logistic system in question. This latter content is usually said to be null or non-existent because it is not denoted or designated by corresponding constants, or, equivalently, because it is not indicated by any type of bound variable. (It may however be «indicated» in a secondary sense by free or schematic variables, and therefore also by constants that are substituends of these free or schematic variables.)
This rather standard view is untenable, however; for if the corresponding or associated expressions have logistic efficacy in the system, that fact can be accounted for only in terms of their representing content of some sort.
On the other hand, because of its immanency, this content need not be therefore accorded categorial existence, i.e., it need not be indicated by bound variables. Our point here, however, is that categorial existence is not the only philosophically viable notion of existence. In ramified typetheory (without the axiom of

reducibility), impredicativity has neither categorial nor syncategorial existence. In the new predicative second order logic, impredicativity has syncategorial but not categorial existence. In standard second order logic, impredicativity has categorial existence.
(5) A perspicuous representation of this logistic efficacy is the rule (S) of substitution of wffs for free (schematic) predicate variables or constants occurring in theorems. (Cf. 4 below for a description and derivation of (S).) This rule, though derivable in the predicative second order logic formulated here, is not derivable in Church formulation. Indeed, its addition there as a new rule results in standard, and not predicative, second order logic. This is not the case in the new formulation given here.
(6) The principle of universal instantiation, (UI), of wffs those containing as well as those not containing bound predicate variables — for a generalized predicate variable is now both formulabale and valid when the generalized predicate variable is bound by the standard quantifier. This principle implies the weaker rule (S) and therefore contains, and goes beyond, the logistic efficacy of that rule. (7)" (pp. 61-64)
References
[1] Church, A., *Introduction to Mathematical* Logic, Princeton University Press, 1956.

16. ———. 1975. "On the Primary and Secondary Semantics of Logical Necessity." *Journal of Philosophical Logic* no. 4:13-27.
"The semantical development of modal logic over the past fifteen years has incorporated a particular model-theoretic artifice which has received little or no critical attention. It is our contention that this artifice introduces, at least within conceptual frameworks typified by logical atomism, a subtle form of descriptive as opposed to merely formal content into the semantics of modal operators. This is particularly noteworthy at least for systems containing operators for the so-called logical modalities, e.g., logical necessity or possibility, or their cognate binary modality, strict implication; for, if any modal operators or connectives had ever been conceptually ordained to represent merely logical or formal operations with no material or descriptive content, it is such as these. Yet, as a result of this model-theoretic artifice, that is precisely what they fail to do.
Relative to a given non-empty universe of objects and a set of predicates of arbitrary (finite) addicity (representing the nexuses of atomic or basic states of affairs), the artifice in question concerns allowing modal operators to range (in their semantical clauses) over arbitrary non-empty subsets of the set of all the possible worlds (models) based upon the given universe of objects and the set of predicates in question. The intuitive and natural interpretation of modal operators for logical modalities, however, is that they range over all the possible worlds (models) of a logical space (as determined by a universe of objects and a set of predicate-nexuses) and not some arbitrary non-empty subset of that totality. The latter interpretation, by allowing the exclusion of some of the worlds (models) of a logical space, imports material conditions into the semantics of modal operators. This exclusion, however appropriate for the representation of non-logical (e.g., causal or temporal) modalities, is quite inappropriate for the representation of what are purported to be merely formal or logical modalities.
This model-theoretic artifice of allowing the exclusion of some of the worlds (models) of a logical Space goes back to Kripke [5] where the notion of universal validity is used instead of the intuitive and primary notion of logical truth. Later semantical developments, by Kripke and others, retained the artifice and supplemented it with additional model-theoretic features, e.g., special accessibility relations between the non-excluded worlds, or semantical clauses allowing objectual quantifiers to range over arbitrary subsets of the universe of objects (thereby importing material content into the semantics of these operators as well). Such additions only deepened and supplemented the type and variability of the material content already induced by modal operators as a result of the artifice in its simplest

form. And in that regard, however appropriate these additions may be for the representation of particular non-logical modalities, they only mark a further departure from the supposed purely formal content of operators for logical modalities. For this reason we shall ignore these later developments here and restrict our observations to some of the implications of the artifice in its original and simplest form. It should be kept in mind, however, that our discussion pertains only to operators for the so-called logical modalities." (pp. 13-14)
(...)
"Concluding remarks.
It is not our contention that we should eschew either the model-theoretic artifice of allowing modal operators to range over only some and not all of the worlds (models) of a given logical space or the related artifice of allowing («-place) predicate quantifiers to range over only some and not all of the sets (of «-tuples) of objects in the universe of that space. Indeed, we agree that such artifices are quite appropriate and may in fact be required for operators purportedly representing non-logical modalities (e.g., temporal or causal modalities) or for quantifiers which purportedly range over attributes of a restricted form of content (e.g., perceptual content, or existence-entailing content where past- and future-existence are distinguished from present-existence).
It is our contention, however, that the employment of such an artifice is inappropriate in the semantics of what one considers to be a purely formal or syncategorematic sign. The fact that a secondary semantics which includes such an artifice yields a proof of completeness where the primary semantics showed incompleteness instead does not itself justify employment of the artifice. Rather, to adopt a secondary semantics for this sort of reason is, in our view, to already call into question the sense in which the sign is said to be syncategorematic or the sense in which the content purportedly represented is said to be of a purely formal nature. That of course may in the end be the appropriate question to raise in regard to all our so-called syncategorematic or logical constants. But to raise the question and to answer it adequately are two entirely different enterprises." (p. 28)
References
[5] Kripke, S., ‘A Completeness Theorem in Modal Logic’, *Journal of Symbolic Logic* 24 (1959), 1-14.

17. ———. 1975. "Second Order Theories of Predication: Old and New Foundations." *Noûs* no. 9:33-53.
"Second-order theories of predication are based on the assumption that a semantical or ontological interpretation of the forms of predication found in first-order languages will be philosophically adequate only if within the framework of the interpretation there exist entities corresponding to (some if not all of) the predicates occurring in these forms. These entities, depending on the theory in question, may or may not be projected as existing in reality independently of the structure of thought. For convenience, however, we shall refer to them in either case as properties when they are projected as corresponding to monadic predicates and as n-ary relations when they are projected as corresponding to n-place predicates, for n > 1.
Now the nature of the correspondence in which properties and relations are purported to stand to predicates in second-order theories is such that it cannot be identified with or reduced to the relation of denotation between singular terms, e.g., individual constants or variables, and the individuals or objects which they are understood to denote. In some second-order theories it cannot be understood as a relation at all, though in others it will (properly) include the singular-term denotation relation (in the sense that properties and relations can also be denoted therein by singular terms) while still going beyond it in ways that are peculiar to predicates. For this reason, quantification over the theoretically projected or posited properties and relations is primarily effected through quantified predicate variables and not, as it were, through a form of restricted quantification over one or another kind of individual. Informally, we say that properties and relations have in this

regard a *predicative nature*, though in some theories they may have a *nominative* nature as well.

In what follows we shall be concerned, though somewhat unevenly, with this distinction between second-order theories in which properties and relations have only a predicative nature as opposed to those in which they are purported to have a nominative nature as well. The two general types of second-order theory we have in mind, then, are distinguished according to (1) whether the nature of the correspondence between predicates on the one hand and properties and relations on the other is to (properly) include the singular-term denotation relation so that predicates, within the framework of the theory, are allowable substituends of individual variables; or (2) whether the purported mode of being of properties and relations is strictly of a predicative nature which excludes their being arguments or logical subjects of predication in any sense which is logically similar to that in which individuals in general are. In the first type of theory, properties and relations are themselves individuals, i.e., have a nominative as well as a predicative nature, whereas in the second the categories or modes of being purportedly indicated by quantified predicate and individual variables are ontologically disjoint. Following Frege, we shall speak of properties and relations as *unsaturated* entities when they are projected entities of a theory of the latter sort." (pp. 33-34)

18. ———. 1975. "Logical Atomism, Nominalism, and Modal Logic." *Synthese* no. 31:23-62.

Reprinted as Chapter 7 in *Logical Studies in Early Analytic Philosophy*, pp. 244-275.

"Logical atomism, through its theory of logical form, provides one of the most coherent formal ontologies in the history of philosophy. It is a coherence which, whether we agree with the ontology or not, renders the framework important and useful as a paradigm by which to compare and better evaluate the coherence of alternative systems based upon alternative theories of logical form and especially alternative theories of predication.

As the basis of a formal ontology, logical atomism, aside from the differences between its realist and nominalist variants, specifies not only a 'deep structure' ontological grammar within which all analysis must ultimately be resolved, but determines as well a logistic for that grammar. Both together constitute the formal ontology and serve to indicate how logical atomism views the fundamental structure of reality. Thus, for example, the grammar serves to indicate the formal as well as the material categories of being acknowledged by the ontology, while the logistic, by regulating the proper 'logico-syntactical employment' ([TR], 3.327) of the expressions of that grammar serves to indicate not only the logical 'scaffolding of the world' ([TR], 6.124) but supplements the grammar in its presentation of the ontological structure of reality.

The distinction between logical scaffolding and ontological structure is fundamental to atomism and pertains to a distinction between material and formal content that grammar alone is insufficient to represent. It is a distinction that any proposed formalization of logical atomism must account for (through the Doctrine of Showing) in order to be an adequate formal representative of that ontology. It is a distinction, however, or so it will be argued here, that cannot be made without the introduction of modal operators for logical necessity and possibility.

The argument for this last claim was already given in chapter 6, but it was there restricted to the level of logical analysis dealing solely with propositional connectives."

(...)

"In what follows we shall be concerned with the problematic extension of these results to the level of analysis involving quantifiers for objects as concrete particulars along with some means for expressing their self-identity and mutual difference. On this level, logical atomism's theory of predication enters our considerations in a fundamental way. For according to that theory, only elementary predications represent or 'picture' a structure with material content, and that content is in all cases external to the constituents of the structure. Such a structure is an atomic situation (*Sachlage*) and the externality of its content to its constituents consists in both it and its complement being logically possible. The difficulty here is that since objects are quantified over, they are part of the world and therefore contribute to the ontological content of the world (cf. [TR] 5.5561); and in that regard their self-identity and mutual difference or nonidentity, and thereby their total number, would prima facie seem to involve material content. Yet, in atomism, an object's self-identity or nonidentity with any other object is not an external condition of that object, (3) and, as a consequence of the dependence of logical space on reality, it is logically impossible for the totality of objects, no less the number of that totality, to differ from world to world. In other words, in logical atomism, if not in other ontologies, identity and difference, as well as objectual quantification, are formal and not material aspects of reality. Here already we begin to see the paradigmatic role of logical atomism, for in most other systems identity and difference, as well as objectual quantification, are also said to be formal in content, though propositions regarding that content are not also said to be either logically necessary or logically impossible.

Because our considerations will be restricted to quantifying over objects as concrete particulars and not, for example, over material properties and relations as well, the variant of logical atomism we shall discuss here is nominalistic. Several realist alternatives are sketched in order to highlight the significant theses and/or difficulties of nominalism, though it should be noted that not all forms of nominalism need agree with the special ontological theses of nominalist logical atomism.

Finally, it should also be noted that our concern in this chapter is with an adequate formal representation of the ontology of logical atomism and not with its theory of thought, meaning, or philosophy of language. We wish to leave open how these might or must be developed with respect to the system constructed here, especially with regard to how they might or must pertain to the question of its logistic completeness." (pp. 244-247 of the reprint)

(1) The convention adopted here is to use scare-quotes when speaking of what connectives represent as 'properties' or 'relations'. This is done to mark a special philosophical use which is convenient in our informal discussion but which strictly speaking is ontologically misleading. A similar convention applies throughout when we refer to existence (being-the-case) and nonexistence (being-not-the-case) as material 'properties' of atomic situations.

(3) That is, an object's self-identity or nonidentity with any other object is invariant through all the possible worlds of a logical space containing that object. We must distinguish this ontological invariance from the varying semantical relation of denotation (*Bedeutung*) between an object and a (non-Tractarian) name or definite description of that object. The former must be accounted for within the formal ontology, the latter only within its applications.

References

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19. ———. 1975. "A Second Order Logic of Variable-Binding Operators." *Reports on Mathematical Logic* no. 5:13-18.

"It is well-known that Prego distinguished between first- and second-level concepts or functions. First-level concepts he associated with properties and relations between objects. These concepts Frege characterized as functions which assigned truth-values (the true or the false) to (n-tuples of) object(s) (1). An (n-tuple of) object(s) was said to fall under such a concept if it was assigned the true by that concept. In his *Begriffsschrift* these concepts were indicated by predicate variables. Second-level concepts or functions, on the other hand, correspond to variable binding operators, e.g., the universal quantifier or, as in Frege's later development of the Begriffsschrift, the course-of-values abstraction operator. The latter assigns to a monadic concept the class which is its extension while the former assigns a truth-value. Second-level concepts, i.e., second-level functions corresponding to variable-binding operators of the quantifier type, accordingly, can be associated with properties or relations between properties and relations of objects in a sense analogous to (but also different from) that in which first-level concepts are associated with properties or relations between objects. In Frege's terminology, while an (w-tuple of) object(s) is said to fall under a first-level concept, the latter is said to fall within, not under, a second-level concept if it is assigned the true by that concept.

Third-level concepts corresponding to quantifiers binding predicate variables were also introduced into the *Begriffsschrift*, but Frege seems to have had some doubts regarding their ontological or objective significance. Indeed, Frege's attitude toward third-level concepts seems in general to resemble the nominalists' attitude toward second-level concepts, viz., that they are merely formal or syncategorematic concepts which are immanent to the *Begriffsschrift* and correspond to nothing objective in reality.

The objectivity of first- and second-level concepts, however, was said by Frege to be "founded deep in the nature of things" (2). These concepts, in other words, have an objective and not merely a formal or syncategorematic content according to Frege. Accordingly, from the point of view of rendering one's ontological commitments explicit by means of appropriate quantifiers, this indicates that in a framework such as the *Begriffsschrift* we should allow not only for third-level quantifiers binding predicate-variables (having first-level concepts as their values), as Frege explicitly did allow, but also for fourth-level quantifiers binding second-level quantifier variables (having second-level concepts as their values), as Frege only implicitly allowed. This he did in effect by allowing free or schematic occurrences of second-level quantifier variables (as affixed to schematic individual variables)."

(...)

"Finally, we should perhaps point out that not all second-level concepts need be quantifier concepts. E.g., Frege himself took the "property" of being a property of the number 2 to be a second-level concept ([4], p. 75), and no doubt he intended there to be such a second-level concept corresponding to each and every object. In the present system we remain faithful to Frege's intentions. Indeed, by (CP-2), it is valid here that for each object x there exists a second-level concept within which fall all and only those first-level concepts under which x falls.

Our approach to the semantics of variable-binding formula operators differs in this regard from that of Mostowski [8], Thomason and Johnson [9], and Issel [5], [6], [7], all of whom, aside from restricting their considerations to first order languages (and, generally, to 1-ary 1-place quantifiers), interpret such operators as designating "quantities" of first-level concepts, i.e., they restrict their considerations to quantifier concepts.

The present system includes these sorts of second-level concepts but goes beyond them to include others as well. However, since the 'quantifier' terminology is simpler and more convenient than referring to variable-binding formula operators, we shall hereafter conflate the latter with the former and speak only of "quantifiers", though of course now quantifiers do not in all cases represent "quantities" (6)." (pp. 13-15)

(1) Frege apparently allowed only for binary relations. We extend his framework to include n-ary relations for arbitrary finite n ^ 2. In addition, we refer to all these relations as (n-ary) concepts. (Frege referred only to properties as concepts.)

(2) *Function and Concept*, p. 41 of [3].

(6) So-called branched quantifiers represent second-level concepts that are somewhat anomalous to quantifier concepts in general, i.e., to "quantities". It is well-known, however, that the semantic content of these quantifiers is representable in second order logic, and, accordingly, these concepts too are included among those represented in the system formulated below.

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20. ———. 1976. "A Note on the Definition of Identity in Quine's New Foundations." *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* no. 22:195-197.

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"A minimal second order modal logic of natural kinds is formulated. Concepts are distinguished from properties and relations in the conceptual-logistic background of the logic through a distinction between free and bound predicate variables. Not all concepts (as indicated by free predicate variables) need have a property or relation corresponding to them (as values of bound predicate variables). Issues pertaining to identity and existence as impredicative concepts are examined and an analysis of mass terms as nominalized predicates for kinds of stuff is proposed. The minimal logic is extendible through a *summum genus*, an infima species or a partition principle for natural kinds."

"A standard objection to quantified modal logic is that it breeds such reptiles of the mind as Aristotelian essentialism, "the doctrine that some of the attributes of a thing (quite independently of the language in which the thing is referred to, if at all) may be essential to the thing, and others accidental" ([5], p. 173f.). This objection has been criticized on one front by pointing out that none of the standard systems of quantified modal logic commit us to more than the meaningfulness of the non-trivial versions of the doctrine and that indeed we can, if we so choose, actually deny such versions in these systems (cf. [4]). A more heroic response, however, accepts these versions of the doctrine, at least when properly stated, and finds quantified modal logic the appropriate medium for its formulation. In what follows I shall attempt to formulate one such response, at least for the purpose of clarifying the general sort of logistic framework it presupposes if not also for exposing some of the more fascinating serpents that breed therein." (p. 202)

(...)

"Concluding Remarks. The above are only some of the more obvious principles that come to mind in the development and application of a logic of natural kinds. My objection to including them within the minimal system is based solely upon the rather strong sense of independence from the structure of thought (and therefore of "logic") which I assume natural kinds to have. Of course, in certain restricted contexts or for specialized applications these principles, and perhaps others as well, may be fully justified and used accordingly.

There are of course other developments and applications which I have not touched upon at all in this paper, e.g., the analysis of causal counterfactuals as based upon natural kinds or of a comparative similarity relation between individuals in terms of the natural kinds they share, etc. Our interests in these sorts of developments or applications should, it is hoped, vindicate at least to some extent the ontology of natural kinds as causal or nomological essences. In any case, such reptiles of the mind as these are taken to be by some philosophers seem hardly poisonous or deadly at all.

Finally, there is the sort of application suggested in section 4 for extending the logic of natural kinds to include nominalized predicates so as to provide a general analysis of the logic and ontology of mass terms. I have only hinted throughout this essay at how this richer framework might be developed, and though I do have some further suggestions which I have not gone into here, it is hoped that perhaps others will also take up the clarion call to defend this rather fascinating serpent of the mind." (. 220)

References

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Reprinted in Lennart Aqvist and Franz Guenthner (eds.), *Tense Logic,* Louvain: Nauwelaerts, 1978.

Abstract: "There are different ways in which we might investigate and represent the successive stages of the development of our common-sense and scientific conceptual frameworks. Jean Piaget’s “fundamental hypothesis” regarding this development is that there is a parallelism between the progress made in the logical and rational organization of knowledge and the corresponding formative psychological processes” ([9], p.13). Piaget’s approach has been a general inquiry into our formative psychological processes, a type of inquiry that requires us “to take psychology seriously” (ibid., p.9). There is an alternative for philosophical logicians, however. For while it is not within our expertise to investigate formative psychological processes, we can nevertheless contribute to the study and representation of “the logical and rational organization of knowledge” through the construction of theories of logical form that are characteristic of at least some of the more important stages in the development of our common-sense and scientific frameworks. We adopt the methodology of such a construction in this paper where our primary concern will be the logical structure of our referential devices for quantifying, identifying and classifying things.

We will be concerned in particular with how this structure is to bear upon the problem of cross-world and cross-time re-identification."

"Investigations into the logical structure underlying ordinary language and our common-sense framework have tended to support the hypothesis that there are different stages of conceptual development and that while the structures elaborated at a later stage are in general not explicitly definable or reducible to those at the earlier they nevertheless presuppose them as conceptually prior bases for their own construction and elaboration— even when the conceptually prior structures are somehow eliminated or completely reconstructed at the later stages. This applies, moreover, not just to the conceptual structures underlying our common-sense framework but to those underlying the development of logic, mathematics and the different sciences as well.

Jean Piaget, for example, as a result of his investigations into genetic epistemology has found that our knowledge of logico-mathematical structures is obtained through a process of “constructive” or “reflective” abstraction that proceeds through a hierarchy of successive stages at which the structures acquired at a previous stage are reconstructed before they are integrated into the new structures elaborated at later stages (cp. [10], p.159). But, as Piaget has also shown, it is not just in logic and mathematics that cognitive activity develops through successive stages of progressive structuration; for the development of intelligence and knowledge in general, whether as represented in our common-sense or our scientific framework, proceeds in essentially the same way. Indeed, the construction of our scientific framework on the basis of our common-sense framework is itself a prime example not only of how conceptual structures acquired at a previous stage are completely reconstructed before they are integrated into those elaborated at a later stage but also of how the later structures, though built upon the earlier, cannot be reduced to or defined in terms of them (cf. Sellars [11]).

Now there are different ways in which we might investigate and represent the successive stages of the development of our commonsense conceptual framework. E.g., because of his "fundamental hypothesis" that there is a parallelism between the progress made in the logical and rational organization of knowledge and the corresponding formative psychological processes" ([9], p.13), Piaget's approach has been a general inquiry into our formative psychological processes. The first principle of genetic epistemology, according to Piaget, is "to take psychology seriously" (ibid., p. 9).

There is an alternative for philosophical logicians, however. For while it is not within our expertise to investigate our formative psychological processes, we can nevertheless contribute to the study and representation of "the logical and rational organization of knowledge" through the construction of theories of logical form that are characteristic of at least some of the more important stages in the development of our common-sense and scientific frameworks.

One thing in particular that the construction of such a theory would help explain is the sense in which the operations and co-ordinations of concepts that characterize a given stage of conceptual involvement constitute a self-sufficient structured whole which purports to have limits beyond which there is nothing for thought. It would also help explain how the formalization of these operations and the clarification of their limits can be the basis for new and more elaborate operations whose structuration transcends those same limits and leads to a new stage of conceptual involvement.

It is this methodology that we shall adopt in what follows where our primary concern will be the logical structure of our referential devices for quantifying, identifying and classifying things. We shall particularly be concerned with how this structure is to bear upon the problem of cross-world and cross-time re-identification." (pp. 439-441)

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