Bernard Bolzano's Logic: Annotated bibliography of the studies in English

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by Raul Corazzon

website: https://www.ontology.co

e-mail: rc@ontology.co

Bernard Bolzano's Logic and Ontology. Bibliography of the studies in English

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Biographies

1. Morscher, Edgar. 2008. *Bernard Bolzano's Life and Work*. Sank Augustin: Academia Verlag.
Table of contents: Preface 9; Introduction 13; 1. Bolzano's life and scientific career 17; 2. Bolzano's removal from Office and the "Bolzano Trial" 23; 3. A short survey of Bolzano's work 29; 4. Logic 33; 5. Epistemology and philosophy of science 75; 6. Ethics 89; 7. Aesthetics 107; 8. Political and social philosophy 113; 9. Philosophy of religion and theology 125; 10. Metaphysics 135; 11. Philosophy of nature and of physics 139; 12. Philosophy of mathematics 141; 13. Metaphilosophy and history of philosophy 149; 14. The so-called Bolzano Circle and Bolzano's influence on the development of the sciences and on intellectual history 151; Appendix: A formal reconstruction of Bolzano's method of idea-variation and of his definitions of logical truth and of logical consequence 159; Bibliography 169; Index of names 207.
"Despite the enormous increase of interest in Bolzano's philosophy during the last decades, an up-to-date monograph on Bolzano's philosophy is still a desideratum. The last book that might be called a monograph on Bolzano's philosophy dates from almost 100 years ago; it is Shmuel Hugo Bergmann's *Das philosophische Werk Bernard Bolzanos* (Halle/S. 1909), written in the spirit of the Brentano school, in particular of Bergmann's teacher Anton Marty.
When I was invited by the Editors of the *Stanford Encyclopedia of Philosophy* to contribute the entry on Bernard Bolzano, I took it as a challenge for starting my long-standing plan to write a monograph on Bolzano's philosophy. The present book is, to be clear, merely the first step toward this end. In this respect I can benefit from the generous copyright regulations of the Stanford Encyclopedia of Philosophy which allow the entries to appear also in print. The author welcomes any kind of comments and criticism to the present printed version of the Internet article in order to take them into consideration in his projected monograph on Bolzano's philosophy.
(...)
I dedicate this book to the greatest and most meritorious Bolzano scholar ever, Jan Berg, without whom Bernard Bolzano would not be seen as the outstanding philosopher as we now know him to be." (From the Preface)

2. Rusnock, Paul, and Šebestik, Jan. 2019. *Bernard Bolzano: His Life and Work*. New York: Oxford University Press.
Contents: A Note on Citations XIII; Acknowledgements XV; Preface XVII; Chronology XX; Extracts XXXI-XXXII; Introduction 1; 1. Bolzano’s Life 5; 2. Ethics 83; 3. Political Philosophy 105; 4. Philosophy of Religion 139; 5. Catholicism and the Catholic Church 167; 6. Logic 187; 7. Theory of Knowledge 337; 8. Ontology and Metaphysics 405; 9. 9 Mathematics 502; 10. Aesthetics, the Science of Beauty 544; Afterword 595; Bibliography 599; Index of Persons 647; Index of Subjects 653-667.
"In the English-speaking world, Bolzano is best known for his work in logic and mathematics. There are certainly things of great importance and beauty in these parts of his work. We have already written, each of us, on these matters, and will have more to say about them in this book. But a faithful portrait of Bolzano cannot limit itself to this, for until he was 40 years old, he was only able to pursue these subjects in his spare time. With his considerable gifts in these non-controversial areas, he certainly might have led a distinguished life of speculation as a mathematician or philosopher. Instead he chose quite deliberately to plunge into the turbulent political life of his homeland, applying his formidable intelligence, energy, and determination to the reform of his society and its institutions. It is here that we shall begin." (p. 3)

Bibliographies

1. Lapointe, Sandra. 2019. Bernard Bolzano. *Oxford Bibliographies Online*: 1-13.
"Introduction
Bernard Bolzano's (b. 1781-d. 1848) originality and numerous anticipatory insights have deserved him a unique position in the history of philosophy. While scholarship trudged for more than a hundred years after his death, in the second half of the 20th century, Bolzano emerged at once as the most significant logician between Leibniz and Frege, one of Kant's most scrupulous and formidable critics, and what may have been one of the greatest single influences on Brentano's students, in particular Twardowski and Husserl, beside Brentano himself. For a variety of reasons—e.g. methodological and thematic proximity—analytic philosophers have found in Bolzano a congenial interlocutor. As a result, most commentaries and discussions tend to focus on aspects of Bolzano's views on logic and its philosophy, in particular his treatment of questions relating to analyticity, deducibility, and grounding in his *opus magnum*, the *Theory of Science* (1837). But the wealth of ideas we find throughout his work is far from exhausted. Because Bolzano research is young, there still subsist substantial gaps in the literature. More importantly, perhaps, there is ample space for reassessments of standard interpretations. The present bibliography is designed so as to both provide interested researchers and prospective scholars with a sense of those issues that constitute the poles of current discussions and leave room for ulterior updates."

Studies on Bolzano's Logic and Ontology

Abbreviations: WL = Bolzano's Wissenschafslehre (1837)

1. *Bernard Bolzano, 1781-1848 Bicentenary. Impact of Bolzano's Epoch on the Development of Science*. 1982. *Acta Historiae Rerum Naturalium Necnon Technicarum*. Prague: Institute of Czechoslovak and General History CSAS
Proceedings of the Bernard Bolzano's Bicentenary Conference, Prague, September 7-12, 1981.
Contents: (only the contributions on Bolzano are cited): Zdenèk Ceska: B. Bolzano and the Charles University 3-8; Lubos Novy: Bolzano's contribution to science and society 9-23; Günter Kröber: Bernard Bolzano und das Problem des Wissenschfatlich-technischen und sozialen Fortschrittes 103-128; Jan Janko: Veränderungen der Naturgeschichte und ihre Differenziation in der I. Hälfte des 19. Jahrunderts 129-154; Joseph W. Dauben: Progress of mathematics in the early 19th century: Context, contents and consequences 223-260; Jan Berg: 415-425; Karel Berka: B. Bolzano's philosophy of science 427-442; Jonathan Cohen: Bolzano's theory of induction 443-457; Helmut Metlzer: Bernard Bolzano Beitrag zum Gestaltwandel der Logik 479-489.
2. "Bolzano Studien." 1987. *Philosophia Naturalis* no. 24:351-499
Content (Essays in English):
II. Basic Questions of Logic and Semantics.
Jan Berg: Bolzano and Situation Semantics: Variations on a Theme of Variation 373-377; Peter Simons: Bolzano, Tarski, and the Limits of Logic 378-405;
III. On the Problem of Paradoxis.
Jan Berg: Is Russell's Antinomy Derivable in Bolzano's Logic? 406-413;
IV: Probability, Induction and Syllogistic.
Jan Berg: Bolzano on Induction 442-446;
V. Contributions on Bolzano's Metaphysics.
Rolf George: Bolzano on Time 452-468.

3. *Bolzano's Wissenschaftslehre 1837-1987. International Workshop (Firenze, 16-19 September 1987)*. 1992. Firenze: Olschki
Content: Premessa V; Rolf George: Concepts of Consequence 3; Detlef D. Spalt: Bolzano's Zahlbegriffe. Bislang Übersehene Marksteine Feudal-absolutistischer Mathematik 27; Ettore Casari: An Interpretation of Some Ontological and Semantical Notions in Bolzano's Logic 55; Jan Berg: The Connection Between Bolzano's Logic of Variation and His Theory of Probability 107; Benson Mates: Bolzano and Ancient Pyrrhonism 121; Karel Berka: Bolzanos Lehre vom naturlichen Schliessen 141; Jan Sebestik: The construction of Bolzano's Logical System 163; Carlo Cellucci: Bolzano and Multiple-Conclusion Logic 179; Rudolf Haller: Bolzano and Austrian Philosophy 191; Massimo Mugnai: Leibniz and Bolzano on the "Realm of Truths" 207; Bob van Rootselaar: Axiomatics in Bolzano's Logico-Mathematical Research 221-230.

4. "Bolzano and Analytic Philosophy." 1997. *Grazer Philosophische Studien* no. 53:1-266
Edited by Wolfgang Künne, Mark Siebel, Mark Textor.
Proceedings of the International Symposium held in Hamburg 3rd-t5th January 1997.
Contents: Preface; VI; Dagfin Føllesdal: Bolzano's Legacy 1; Jan Berg: Bolzano, the Prescient Encyclopedist 13; Jan Šebestik: Bolzano, Exner and the Origins of Analytical Philosophy 33; Paul Rusnock: Bolzano and the Traditions of Analysis 61; Peter Simons: Bolzano on Collections 87; Ali Behboud: Remarks on Bolzano's Collections 109: Mark Siebel: Variation, Derivability and Necessity 117; Edgar Morscher: Bolzano's Method of Variation: Three Puzzles 139; Rolf George: Bolzano's Programme and Abstract Objects 167; Mark Textor: Bolzano's Sententialism 181; Wolfgang Künne: Propositions in Bolzano and Frege 203: Michael Dummett: Comments on Wolfgang Künne's Paper 241; Carsten Uwe Gieske: Bolzano's Notion of Testifying 249-266.
"From January 3rd to January 5th 1997 the international symposium *Bolzano and Analytical Philosophy* took place in Hamburg.
(---)
Michael Dummett once called Bernard Bolzano the "great-grandfather of analytical philosophy".[*] The aim of the symposium was to explore whether Bolzano's analytical great-grandchildren can still learn from their Bohemian ancestor. We hope the symposium will stimulate further systematic and exegetical research in thisarea." (from the *Preface*)
[*] Michael Dummett, *Origins of Analytic Philosophy*, Cambridge (MA): Harvard University Press 1993, p. 171.

5. "Bernard Bolzano." 1999. *Revue d'Histoire des Sciences* no. 52:339-506
The following contributions are in English:
Charles Chihara: Frege's and Bolzano's rationalist conceptions of arithmetic 343; Joëlle Proust: Bolzano's theory of representation 363; Johannes Hafner: Bolzanos' criticism of indirect proofs 385; Paul Rusnock: Philosophy of mathematics: Bolzano's responses to Kant and Lagrange 399.

6. "Bolzano and Kant." 2012. *Grazer Philosophische Studien* no. 85
Guest Editor: Sandra Lapointe.
Table of Contents: Sandra Lapointe: Introduction 1; Sandra Lapointe: Is Logic Formal? Bolzano, Kant and the Kantian Logicians 11; Nicholas F. Stang: A Kantian Reply to Bolzano's Critique of Kant's Analytic-Synthetic Distinction 33; Clinton Tolley: Bolzano and Kant on the Place of Subjectivity in a *Wissenschaftslehre* 63; Timothy Rosenkoetter: Kant and Bolzano on the Singularity of Intuitions 89; Waldemar Rohloff: From Ordinary Language to Definition in Kant and Bolzano 131-149.

7. Adair-Toteff, Christopher. 2002. "Bolzano's *Gesamtausgabe*." *British Journal for the History of Philosophy* no. 10:127-133

"Shortly after Bolzano's death there was an attempt to collect his works as a *Gesamtausgabe*, but there was little interest. Another attempt was made to honour the sixtieth anniversary of his death in 1908 but that also failed.
The *Wissenschaftslehre* was republished in four volumes in the early 1930s, but it was not until the late 1960s that a number of international Bolzano scholars succeeded in planning the Bernard Bolzano *Gesamtausgabe*. The first of a projected 100+ volumes appeared in 1969 – an introductory book that was a biography. The *Gesamtausgabe* is composed of four series:
I Writings (*Schriften*);
II Posthumous writings (*Nachlaß*);
III Correspondence (*Briefwechsel*);
IV Documents (*Dokumente*)." (p. 128)
(...)
"Bolzano's *Wissenschaftslehre* was published in four volumes in Germany in 1837. It consists of five Books: Theory of Fundamentals, Theory of Elements, Theory of Knowledge, Heuretic, and Theory of Science Proper.
In the first book Bolzano defines *Wissenschaftslehre* as the attempt to provide an account of science in general. Its function is not to discuss any individual sciences but rather to determine the rules by which all truths can be determined to belong to the individual sciences. He acknowledges that he is using science and doctrine in slightly unusual terms but insists that his account is superior to either Fichte's or Hegel's. Bolzano maintains that Fichte's *Wissenschaftslehre* offers a mistaken attempt to provide a doctrine of knowledge in general. Bolzano does admit that he does not really understand Fichte's philosophy. Bolzano also admits to a similar lack of comprehension of Schelling and Hegel.
Bolzano uses a number of technical terms. One is a 'proposition in itself' (*Satz an sich*) which he takes to be an assertion that something is or is not the case. A proposition in itself need not be uttered or even thought. In a similar vein Bolzano speaks of 'truth in itself' (*Wahrheit an sich*) or 'objective truth' that does not have real existence in contrast to recognized truths that do exist. And, there are 'representations in themselves' (*Vorstellungen an sich*). Examples of ideas in themselves would be 'Caius' and 'wisdom' in the proposition 'Caius has wisdom'. Representations in themselves have neither truth nor existence. Much in the first three books appears to be an attack on Kantian philosophy – he has no use for Kant's psychology and his *Ding an sich*. But, these books are not simply negative. He claims to have shown that there are truths and that we can recognize them and he sets out the conditions under which we can recognize them." (p. 129)

8. Bar-Hillel, Yehoshua. 1950. "Bolzano's Definition of Analytic Propositions." *Methodos*:32-55
Published also in *Theoria* 16, 1950, pp. 91-117.
Reprinted in: Y. Bar-Hillel, *Aspects of language. Essays and lectures on philosophy of language, linguistic philosophy and methodology of linguistics*, Jerusalem: The Magnes Press - The Hebrew University, 1970, pp. 3-28.
"In view of recent discussions on the nature of analytic truth, it should be rather interesting to inquire into the treatment which this subject received by the most outstanding logician of the first half of the 19th century, the Austrian philosopher, theologician, and: physicist Bernard Bolzano.
Our investigation will turn upon section 148 of Bolzano's four volumed masterwork *Wissenschaftslehre* (1837). Only occasionally shall we need to refer to other parts of this work. This section, headed "Analytic or Synthetic Propositions", comprises pages 83-89 of the second volume and is divided into three subsections of less than two pages altogether, followed by four annotations, filling the next five pages. I dwell so long upon these bibliographical particulars only to bring into full light the wealth of systematic and historic material contained in these few pages.
1. Pre-History.
Bolzano's aim, in § 148, was to define a concept which could serve as an adequate explication for what is now commonly termed 'logical truth'. Though this aim is

nowhere explicitly stated, there can be no doubt about it, just as Kant before him and many logicians after him doubtless aimed at the same target when they proposed their respective definitions.
Bolzano devotes the greater part of his fourth annotation the discussion of many such attempts made by his predecessors and contemporaries. He mentions *Aristotle, Locke, Crusius* (the German logician of the first half of the 18th century who was probably the first to use the terms 'analytic' and 'synthetic' in their Kantian senses), Kant and many other minor philosophers. He easily succeeds in proving the inadequateness of Kant's two definitions for analytic', the one given in his *Logik* and equating, in effect, Analytic (1) with Identical, the other much better known in the introduction to the Critique of Pure Reason, where he proposes to call propositions 'analytic', whose predicate-concept is contained (perhaps in a hidden manner) in the subject-notion. Bolzano points out (p. 87) the vagueness of the term 'contained' and argues that, according to a quite natural interpretation of this term, the proposition « The father of Alexander, King of Macedonia, was King of Macedonia » ought to be analytic, a consequence which Kant certainly did not intend to be drawn.
But to even more refined versions of Kant's definition, given by some of his followers, replacing the vague 'contained' by more concise terms, such as those making use of 'essential characteristics', Bolzano objects that only one type of proposition conforms to them, namely 'A (which is B) is B'. But should not, continues Bolzano, also propositions of the type 'Every object is either B or non-B' be counted among the analytic propositions?
Having thus convinced himself of the inadequateness of all prior approaches, he started to attack the subject along a new and highly original line." (pp. 3-4 of the reprint).

9. ———. 1952. "Bolzano's Propositional Logic." *Archiv für Mathematische Logik und Grundlagenforschung* no. 1:65-98
Reprinted in: Y. Bar-Hillel, *Aspects of language. Essays and lectures on philosophy of language, linguistic philosophy and methodology of linguistics*, Jerusalem: The Magnes Press - The Hebrew University, 1970, pp. 33-68.
"1848 is a remarkable year not only in general history; in the history of human culture and thought it will be remembered also as the birth year of *G. Frege*, "the greatest logician of the 19th century",(2) and should be remembered as the year in which the death of the greatest logician between Leibniz and Frege, the Czech Bernard Bolzano, occurred. So far, little has been done to evaluate his important contributions to logical theory,(3) and I hope that the present article will help to undo this undeserved wrong.
The purpose of this article is very restricted: only a small part of Bolzano's investigations will be dealt with, i.e. his propositional logic, and even this in a limited degree. This theory is in my opinion not only a master-work of outstanding historical interest, I also believe that it contains many features neglected even by modern symbolic logic and nevertheless worthy of close study. I am convinced that such a study will considerably enrich our logical technique and terminology.
Since our principal aim is to emphasize the impact which Bolzano's ideas should have on contemporary logic, I shall allow myself to depart, sometimes considerably, from his original account and even to disregard parts of his theory unacceptable to us which do not play any decisive role in its construction, all this, of course, after due warning shall have been given.
I shall summarize the contents of §§147, 154-160 of Bolzano's *Wissenschaftslehre* (1837), with which alone this study is concerned, in 28 definitions and 95 theorems. Most of these theorems will not be proved, for the sake of brevity, but the reader will, in general, be able to supplement the proofs by himself. Many definitions and a few theorems will be illustrated by simple examples. Major departures from Bolzano's original account will be specially mentioned and justified.
In the second part of the study I shall outline the place of Bolzano's contribution within the framework of modern semantics, by its detailed comparison with the

corresponding parts of R. Carnap's two volumes of Studies in Semantics. This comparison will give us a certain perspective on the bearing of Bolzano's highly original innovations for modern research, and on the other hand enable us to see clearly the precise nature of some of his shortcomings." (pp. 33-34)
(1) This article has been written as an outcome of conversations with Professor Hugo Bergman of the Hebrew University, Jerusalem, and a joint reading of the relevant passages of Bolzano's *Wissenschaftslehre*. It is to Professor Bergman that I owe the general ideas on which this paper is based.
(2) According to A. Tarski, *Introduction to Logic*, 1941, p. 19.
(3) The following is a list of the most important articles dealing mainly with Bolzano's contributions to logic which have appeared in the last two decades:
W. Dubislav Bolzano as Vorlaufer der mathematischen Logik", Philosophisches Jahrbuch der Görres-Gesellschaft, vol. 44 (1931), pp. 448-456.
H. Scholz, "Die Wissenschaftslehre Bolzanos", Semesterberichte, 9. Semester, 1936/37, pp. 1-53.
H. Scholz, "Die Wissenschaftslehre Bolzanos", Abhandlungen der Fries'schen Schule, n. s. vol. 6 (1937), pp. 399-472.
H. R. Smart, "Bolzano's Logic", The Philosophical Review, vol. 53 (1944), pp. 513-533.
I have not been able to get hold of Scholz's second article, but since it is, according to the *Journal of Symbolic Logic*, only a somewhat broader version of his first article, the loss is probably not too great. My quotations from Scholz will therefore refer always to his first article.

10. ———. 2006. "Bolzano, Bernard." In *Encyclopedia of Philosophy: Second edition. Vol. 1*, edited by Borchert, Donald M., 646-648. New York: Thomson Gale
First edition 1967.
"Bernard Bolzano, a philosopher, theologian, logician, and mathematician, was born in Prague, where his father, an Italian art dealer, had settled; his mother was a German merchant’s daughter. Bolzano studied mathematics, philosophy, and theology in Prague and defended his doctor’s thesis in mathematics in 1804; he was ordained a Roman Catholic priest the following year. Shortly thereafter he was appointed to a temporary professorship in the science of religion at Karlova University in Prague and two years later was given a newly established chair in this field. Some time later he was accused of religious and political heresy and was removed from his teaching position in December 1819. Bolzano spent much of his time thereafter with the family of his friend and benefactor, A. Hoffmann, at their estate in southern Bohemia. He had difficulty getting his later publications through the Metternich censorship. Some of his books were put on the Index, and many appeared only posthumously. Some manuscripts are yet to be published; the most important of these are in the National Museum and the University Library in Prague, others are in the Österreichische Nationalbibliothek in Vienna. In December 1848, Bolzano died of a respiratory disease from which he had suffered for most of his life." (p. 646)

11. Behboud, Ali. 1997. "Remarks on Bolzano's Collections." *Grazer Philosophische Studien* no. 53:109-115
"With his "zoology of general kinds of collective entities"[*], Peter Simons has sketched - in a very helpful way - some main alternatives for possible interpretations of Bolzano's collections. As he pointed out, we may not have more thanjust a best fit - and, in fact, he proposes that Bolzano' s account is "a distinct and distinctive theory of collections". I do agree with Simons that there are many difficulties we have to face when we try to fit Bolzano' s account into one of our theories. Also, ignoring the considerable historical interest for the moment, the price the technical inconveniences as weIl as the conceptual complexities which such a fit might require in the end could be too high to be of any practical interest. Nevertheless, I would like to try my luck for a best fit. (1)" (p. 105)
(1) Simons is clearly right that Bolzano does not develop a systematic theory of collections. However, collections play a fundamental role for Bolzano (even beyond

his mathematical theories), since anything whatsoever is either a collection or an "atom" (*einfach*). So it is no surprise thatthe notion of a collection is almost ubiquitous in Bolzano's works.
[*] P. Simons, *Bolzano on Collections*, (1997), p. 87.

12. Bellomo, Anna, and Massas, Guillaume. 2021. "Bolzanos' Mathematical Infinite." *The Review of Symbolic Logic*:1-55
Abstract: "Bernard Bolzano (1781–1848) is commonly thought to have attempted to develop a theory of size for infinite collections that follows the so-called part–whole principle, according to which the whole is always greater than any of its proper parts. In this paper, we develop a novel interpretation of Bolzano's mature theory of the infinite and show that, contrary to mainstream interpretations, it is best understood as a theory of infinite sums. Our formal results show that Bolzano's infinite sums can be equipped with the rich and original structure of a non-commutative ordered ring, and that Bolzano's views on the mathematical infinite are, after all, consistent."

13. Benoist, Jocelyn. 2002. "Husserl and Bolzano." In *Phenomenology World-Wide: Foundations, Expanding Dynamisms, Life-Engagements. A Guide for Research and Study*, edited by Tymieniecka, Anna-Teresa, 98-100. Dordecht: Kluwer
"Bolzano's influence on Husserl has recently come to be appreciated at its true worth. It is actually an extremely important one.
Husserl recalls that he attended a lecture given by Brentano on *The Paradoxes of the Infinite*. But he mayalso have heard of Bolzano from his mathematics professor Karl Weierstrass. Papers written by the Brentanist Benno Kerry (*Ueber Anschauung und ihre psychische Verarbeitung* 1885-1891) also had a certain bearing on Husserl's knowledge of Bolzano. The insightful discussion of the Bolzanian thesis of "representations without object" to be found in Twardowski's book *On Content and Object of Presentations* succeeded in interesting Brentano's pupil in that author definitively.
The psychological point of view adopted by Husserl in the *Philosophy of Arithmetic* does seem to be very far removed from that of Bolzano. However, on the other hand, Husserl's break with psychologism, which took place during the years 1894-1896, appears to have had a direct bearing on Husserl's better acquaintance with Bolzano's *Wissenschaftslehre* during that period. We now know that in 1896 Husserl gave a course which was not, as is commonly believed, a draft of the *Prolegomena*, but was rather a survey of Bolzano's *Wissenschaftslehre*. [*]
What really matters is that the break with Brentanian psychologism was indeed a Bolzanian move. Such a move allows us to speak of a "Bolzanian tum" in Husserl's thought, taking place around 1896. From that point of view, Husserl's thought, "phenomenology", may and must be understood as a (quite strange) kind of compromise between Brentanian descriptive psychology and Bolzanian propositionalism." (p. 98)
[*] E, Husserl, *Logik. Vorlesung 1896*, edited by Elisabeth Schuhmann, Dordrecht: Springer 2001.

14. Benthem, Johan van. 1984. *Lessons from Bolzano*. Stanford: Center for the Study of Language and Information, Leland Stanford Junior University
"Bernard Bolzano's contributions to logic, largely unnoticed in the 19th century, have been receiving ever more attention from modern logicians (cf. Scholz, 1937; Berg, 1962; Corcoran, 1975). As a result, it has already become something of a commonplace to credit Bolzano with the discovery of the notion of logical consequence in the semantic sense. Now, this particular attribution, whether justified or not, would at best establish a historical link between modern logical concerns and Bolzano's work. The purpose of the present note, however, is to bring out three important aspects of that work that are still of contemporary systematic interest. No detailed textual study of Bolzano is needed to substantiate our suggestions. We shall refer to well-documented 'public' aspects of the

'Wissenschaftslehre' (Bolzano, 1837), pointing out their more general logical significance." (p. 1).
References
Jan Berg, *Bolzanos's Logic*, Stockholm: Almqvist & Wiksell 1962.
John Corcoran, "Meanings of Implication." *Diálogos*, 9, pp. 59-76, 1975.
Heinrich Scholz, "Die Wissenschaftslehre Bolzano's: Eine Jahrhundert-Betrachtung." *Abhandlungen der Fries'schen Schule*, 6, pp- 399-472, 1937.

15. ———. 1985. "The Variety of Consequence, According to Bolzano." *Studia Logica* no. 44:389-403
Abstract: "Contemporary historians of logic tend to credit Bernard Bolzano with the invention of the semantic notion of consequence, a full century before Tarski. Nevertheless, Bolzano's work played no significant role in the genesis of modern logical semantics. The purpose of this paper is to point out three highly original, and still quite revelant themes in Bolzano's work, being a systematic study of possible types of inference, of consistency, as well as their meta-theory. There are certain analogies with Tarski's concerns here, although the main thrust seems to be different, both philosophically and technically. thus, if only obliquely, we also provide some additional historical perspective on Tarski's achievement."

16. ———. 2003. "Is There Still Logic in Bolzano's Key?" In *Bernard Bolzanos Leistungen in Logik, Mathematik und Physik*, edited by Morscher, Edgar, 11-34. Sankt Augustin: Academia Verlag
"Bolzano is widely seen as the philosopher of abstract propositions, far removed from psychological blemishes. Nevertheless, many themes in this paper suggest links with the actual reasoning performed by non-Platonic humans like us. We saw this with attention to diverse styles of task-dependent reasoning, with degrees of logicality for the expressions of natural language that we actually use, with inferences transferring information across discourse situations, with global architecture of reasoning styles, or with mixtures of such neatly compartmentalized logical activities as semantic evaluation and proof. When we take all this seriously, it becomes hard not to go one step further, and do something which Frege has forbidden - but probably also Bolzano: take the psychological facts seriously. All the above topics border on cognitive science and the experimental study of human reasoning, and the eventual agenda of modem logic will also have to come to better terms with that than the by now pretty stale slogan of 'anti-psychologism'.
Conclusions.
We have surveyed some aspects of Bolzano's logic from a modern standpoint, stressing in particular his different styles of consequence, the essential ternary nature of consequence when language is taken into account, and the mixed notion of consequence in a model. In all three cases we included some new technical observations to show that the issues are still alive. But the more general thrust is this.
Bolzano's work remains interesting for logic today, both in its general sweep, and in some of its details. Partly, it is attractive precisely because it is so non-mainstream, and hence valuable for modern agenda discussions. Its themes crossing logic and philosophy of science reflect current rapprochements, while its thrust also seems to fit with some themes from AI. Classical mathematical logic has had an Austrian icon in Kurt Gödel: modem logic might consider at least having a Czech-Austrian patron saint." (pp. 30-31).

17. ———. 2013. "Bernard Bolzano's *Wissenschaftslehre*." *Topoi* no. 32:301-303
"In this review, I will focus on Mr. Bolzano's thoughts about logic, even though he offers much more than that to readers interested in theory of science and general philosophy. Modern logic has become more and more technical, cutting itself loose from its broader origins as the study of reasoning, and philosophers of logic slavishly play up to this trend by devising ever more arcane criteria of 'logicality' that apply only to a small elite of 'logical constants', making it harder and harder for new themes to enter the field. Refreshingly, Mr. Bolzano does none of this. He

resolutely ignores received wisdom in logic textbooks, and deftly avoids entanglement in the scholasticism of our modern age. Instead, he just goes back to what logic is about, and rethinks it afresh." (p. 301)

18. Berg, Jan. 1962. *Bolzano's Logic*. Stockholm: Almqvist & Wiksell
Contents: Preface 5; Introduction 7; Abbreviations 12; I. Bolzano's life and work 13; II. A logical frame 33; III. Bolzano's fundamental notions 41; IV. Bolzano's logic of variation 92; V. Other logical theories 146; VI. Bolzano's logic of entailment 151; VII. Bolzano's philosophy of mathematics 165; Bibliography 179; Index of proper names 213-214.
"Bernard Bolzano made essential contributions to, *inter alia*, theology, logic, and mathematics. For political reasons, however, he was prevented from influencing to a full extent the age in which he lived. As a mathematician his name has survived, although many of his most remarkable results were not published until a century after their conception. As a logician he has begun to appear again in scattered articles and comments. In this study I have tried to give an exposition and evaluation of the main ideas of his logic from a modern viewpoint.
Important parts of Bolzano's theories of logic and semantics were new with him, and when these ideas reappear later they were independent of him. This position, in part outside of the historical development, makes it highly pertinent to compare Bolzano's theories directly with modern logic. When tracing the lineage of some of Bolzano's ideas I have even projected the earlier theories onto a modern scheme of reference.
In the first chapter I shall briefly mention the genesis of Bolzano's main works. But I am not primarily interested in the genetic aspect of Bolzano's theories. Therefore, as often as possible I shall consider Bolzano's various formulations of his ideas as if they were parts of a simultaneous whole.
My analysis of Bolzano's achievements in logic, semantics, and mathematical philosophy is based on his mature production after 1820, when he started writing his *magnum opus*, the *Wissenschaftslehre*. The works chiefly consulted are:
(1) *Wissenschaftslehre*, I-IV (Bolzano (15) mostly abbreviated "WL"). [Names followed by parenthesized numerals refer to the bibliography; see the last paragraph of this introduction.]
(2) *Der Briefwechsel B. Bolzano's mit F. Exner* (Bolzano (54), "BE").
(3) *Erste Begriffe der allgemeinen Grössenlehre* (Bolzano (78), "AG").
(4) *Reine Zahlenlehre. Erster Abschnitt. Von dem Begriffe, den allgemeinsten Beschaffenheiten und der Bezeichnungsart der Zahlen* (Bolzano (79), "RZ").
(5) *Unendliche Grössenbegriffe* (Bolzano (81), "UG").
(6) *Bolzano's Wissenschaftslehre und Religionswissenschaft in einer beurtheilenden Uebersicht* (Bolzano (z5), "WU").
(7) *Einleitung zur Grössenlehre* (Bolzano (83), "EG").
(8) *Paradoxien des Unendlichen* (Bolzano (45), "PU").
For (3)-(5) and (7) I have utilized Bolzano's unpublished manuscripts. In general I have presupposed that some editions of (1), (2), and (8) are accessible to the reader.
Chapter I presents in a concentrated form some biographical and bibliographical facts about Bolzano. The manuscripts used are described and dated. Some topics lying outside of logic proper and dealt with in published or unpublished works of Bolzano are also touched upon. The most representative portraits of Bolzano - one of which has not been published before - are reproduced.
Chapter II describes the logical machinery to be used in the formalization and the comparative analysis of Bolzano's logic. This chapter may be read cursorily and used as future occasion may require. The system expounds the so-called elementary logic, i.e., classical predicate logic of first order with identity. In view of its simplicity and non-controversial character, elementary logic seems at present to be an expedient object for comparison in research in the history of logic. To be sure, we know nothing for certain about the future of logic; maybe the logics of tomorrow will differ greatly from those of today. However, it is likely that our elementary logic will be translatable into or representable within these conceivable

new systems. But there is always the possibility that new systems of logic will promote a deeper understanding of certain features of the objects analysed.
Chapters III-VII expound those aspects of Bolzano's theories of logic, semantics, and mathematical philosophy which seem to me fundamental. Certain sections of these chapters are subdivided into two parts; part A describes Bolzano's ideas and part B offers commentaries on A. In order to shorten the exposition, definitions and arguments are sometimes formalized even in A, and sometimes auxiliary notions not found in Bolzano are introduced with explicit caution. In doing so I have always attempted to stay within or very close to Bolzano's sphere of ideas. This does not mean, of course, that I have always followed the order in which Bolzano presents his definitions and theories in the *Wissenschaftslehre* or elsewhere. Nor have I stinted myself at times in giving very free paraphrases of Bolzano's mode of expression. In proving theorems I try to reproduce Bolzano's line of thought without copying his manner of speaking. In part B, Bolzano's logic is compared with modern theories. I attempt to show how certain gaps could be filled in and how Bolzano's theories could be elaborated and made more precise. Moreover, under B some forerunners of Bolzano and selected parts of the modern literature concerning him are discussed.
In the annotated bibliography, part A embraces the literature on Bolzano and his own works. References to part A are given by names followed by numerals within parenthesis. References to part B of the bibliography are effected by placing a "B" after the parenthesized numerals. In references to manuscripts, folio numbers are qualified by "r" and "v", meaning, as usual, recto and verso respectively." (*Introduction*).

19. ———. 1966. "Bolzano's Notion of Proposition." In *Ost und West in der Geschichte des Denkens und der kulturellen Beziehungen. Festschrift für Eduard Winter zum 70. Geburtstag. Mit einem Geleitwort von A.P. Juskevic*, edited by Steinitz, Wolfgang, 519-526. Berlin: Akademie-Verlag.

20. ———. 1967. "What Is a Proposition?" *Logique et Analyse* no. 39/40:293-306
"Certain theories of modern logic have the purpose of defining interesting classes of linguistic expressions, such as the set of sentences of a language, or relations between expressions, such as derivability among formulas. Other theories aim at describing semantic relations between linguistic expressions and nonlinguistic objects, such as the relation of being the meaning of an expression. Yet a third kind of theories may give a direct analysis of non-linguistic objects which could stand in semantic relations to linguistic expressions.
This paper first propounds and discusses certain constructions of the second kind and then attempts an explication of the third kind of the notion of non-linguistic proposition. However, only a limited class of propositions (called "elementary propositions") will be explained, viz., propositions corresponding to the sentences of a language of elementary logic. Admittedly, this explication will have merely a remote connection with the problems of ordinary language. On the other hand, a tradition of logical semantics has accumulated since the 19th century dealing with technical and more or less formalized languages, and it may be worth while to attempt a solution of some problems encountered in such studies.
In writing this paper I have profited from comments and criticism of Professor A. Wedberg, University of Stockholm." (p. 293)

21. ———. 1972. "Bolzano's Theory of an Ideal Language." In *Contemporary Philosophy in Scandinavia*, edited by Paul, Anthony, Olson, Raymond and Wright, Georg Henrik von, 405-415. Baltimore: John Hopkins Press
"In his logical inquiries Bolzano employed a partly formalized language embracing an ordinary language extended by constants, variables, and certain technical expressions. In the second volume of the Wissenschaftslehre he investigated the relations of this semiformalized philosophical language to colloquial language (WL, sections 127-46, 169 84). He believed that all sentences of colloquial language were 'reducible' to sentences of certain canonical forms expressed in the philosophical

language. These canonical sentences were said to mirror their corresponding propositions in the sharpest way.
Had Bolzano's theory of reduction been completely developed it might have resulted in the construction of an ideal language for philosophical analysis. In this ideal language, however, sentences of canonical form would not play quite the same role as the atomic sentence forms on the basis of which more complex forms are built up in modern quantification theory. It seems, on the contrary, that Bolzano intended even the most complicated sentences to have canonical forms or to be reducible to sentences having such form.
This paper attempts a reconstruction of an extensional Bolzanian ideal language on the level of elementary logic. After some preliminary explanations of fundamental notions in Bolzano's logic, the main points of his theory of reduction of sentences are described. Two principles that determine the construction of an elementary Bolzanian ideal language emerge from the exposition. We then move toward building such a language and begin by modifying the standard representation of elementary logic, replacing the universal and existential quantifiers by Hilbert's e-operator. By further modifications of both the syntax and the underlying semantics, a logical language satisfying the two principles is obtained." (pp. 405-406)

22. ———. 1977. "Bolzano's Contribution to Logic and Philosophy of Mathematics." In *Logic Colloquium '76*, edited by Gandy, Robin O. and Hyland, John Martin, 147-171. Amsterdam: North-Holland
"The *Wissenschaftslehre* was intended merely as a prelude to Bolzano's work on mathematics. His main ambition was to recreate the whole body of contemporary mathematics in accordance with the vision of an abstract hierarchy of true propositions. For Bolzano this task implied the creation of entirely new foundations for certain branches of mathematics, as may be seen from his highly interesting efforts directed toward basing geometry on topological concepts.
In carrying out this program, most of the means of expression of modern quantification theory were in essence available to Bolzano.
He came very close to modern notions of satisfaction, logical truth, consistency and logical consequence. On the other hand, the formal deductive machinery of quantification theory is practically non-existent in Bolzano's works. This syntactic machinery appears only in Frege, who created the first strictly logistical system at the end of the 19th century. Bolzano's lack of interest in developing particular logical calculi most probably stems from his aspects of logic and mathematics and of science in general.
The notion of calculus in the modern logistical sense was first clearly considered by Leibniz. His basic dream was of an effectively decidable, interpreted calculus embracing all "eternal" truth. Bolzano was justifiably critical of this overambitious program and presented instead his own theory of the *Abfolge* structure of nonlinguistic propositions, thereby taking his stand away from that line of development in logic which leads to modern syntactic concept formation. A reason for Bolzano's general lack of interest in questions of logical syntax was no doubt his profoundly intensional, non-linguistic approach to logic.
Bolzano's central thesis, that there are abstract objects which differ from both mental occurrences and all kinds of linguistic expressions, has been advocated by later philosophers of the German-speaking countries, inter alia by Lotze, Brentano in his earlier period, Meinong and Frege. Lotze and Frege never refer to Bolzano's work, though, and the others protested their independence of Bolzano. Husserl admits that he received vital influences from Bolzano, but his notions of "ideal" objects derive from Lotze's and not from Bolzano's logic.
Among the great Western philosophers Bolzano is perhaps the least influential. In epistemology, logic and mathematics his most fervent disciples were not able to propagate his ideas with sufficient vigor.
His keen criticism of German idealistic philosophy and his important discoveries in logic: semantics and mathematical philosophy silently died away.

A contributing cause of Bolzano's lack of influence on the development of the philosophical disciplines was, of course, the fact that most of his works were, for political reasons, published anonymously in editions not easily accessible. Furthermore, an immense number of unpublished manuscripts in a partly almost indecipherable handwriting is to be found in archives in Prague and Vienna. Several unfruitful attempts have been made in the last 150 years to bring out more or less complete editions of Bolzano's works. It is to be hoped that the latest venture launched in Stuttgart, West Germany, will prove more successful." (pp. 170-171)

23. ———. 1982. "A Requirement for the Logical Basis of Scientific Theories Implied by Bolzano's Logic of Variation." *Acta Historiae Rerum Naturalium Necnon Technicarum* no. 12:415-425
Bernardo Bolzano (1781-1848) - Bicentenary. Impact of Bolzano's epoch on development of Science - Conference papers, Prague 7-13 September 1981.

24. ———. 1986. "A Logic of Terms with an Existence Operator." In *Logic and Abstraction. Essays Dedicated to Per Lindström on his Fifthieth Birthday*, edited by Furberg, Mats, Wetterström, Thomas and Aberg, Claes, 71-94. Göteborg: Acta Universitatis Gothoburgensis
"1. Introduction
In this paper a language L^* of elementary logic satisfying the following two conditions will be constructed:
(C 1) each expression in L^* consists of an n-ary function symbol **f** applied to n arguments (n > 0);
(C 2) if in L^* **f** is a predicative function symbol, $\mathbf{t_i}$ a term, and J an interpretation for a particular i (1 < i < n) under which $\mathbf{t_i}$ is empty, then $\mathbf{f}(t_1 \cdot \cdot \cdot t_n$ is false under J.
Here **f** is a predicative function symbol of L^* if **f** under a suitable mapping, corresponds to a predicate of one of the standard versions of elementary logic.
The language L^* differs from standard representations of elementary logic in that it replaces the universal and existential quantifiers with Hilbert's ε-operator and modifies both the syntax and the underlying semantics accordingly. An elementary logic with a non-trivial existence predicate is in itself interesting, and when combined with a logic of terms can be utilized for research in the history of logic. (Cf. Berg [1972]."
References
J. Berg, Bolzano's theory of an ideal language. R. E. Olson & A. M. Paul (Eds.): *Contemporary philosophy in Scandinavia* pp. 405-415 (Baltimore).

25. ———. 1987. "Bolzano and Situation Semantics: Variations on a Theme of Variation." *Philosophia Naturalis* no. 24:373-377
"The distinction between logical and non-logical notions plays a significant role in Bolzano's theory and he is fully aware of its importance even though he has to admit that various scholars may differ in their opinion on what a *logical idea* is (WL § 148.3). Tarski has tried to clarify the distinction between logical and non-logical notions (Adam Tarski, *What are logical notions?* History and Philosophy of Logic, 7, 1986, pp. 143-154). A notion of Euclidean geometry, e.g., is invariant under all similarity transformations, and a topological notion is invariant under all continuous transformations. Analogously, a *logical notion* may be conceived of as a concept which is invariant under all bijective mappings of the domain of individuals onto itself. (In this sense even the classical reduction problem of critical realism can be solved: A physical notion is a concept invariant under a Galilei or a Lorentz transformation.)
It is possible to vary not only the non-logical ideas-as-such contained in propositions but even some or all logical ideas. Actually, such a variation is implied by the algebraic approach to logic. The propositions of Bolzano correspond to the values of the propositional variables of modern logic. (p. 374)
(...)

Bolzano seems to have intended, however, variation exclusively over non-logical ideas-as-such. That he did not allow a variation of the copula is abundantly clear from his way of introducing the notion of variation in his *Einleitung zur Grossenlehre* (Bolzano (2A7), p. 62). Here he presupposes that only the subject and predicate ideas of a proposition or parts thereof be varied. Furthermore, his proofs of certain variation-logical theorems show that he would not allow a variation of the logical constants of negation, truth, and the copula in the form of an inclusion between ideas-as-such (WL §§ 154.19, 155.21)." (p. 375)

26. ———. 1987. "Is Russell's Antinomy Derivable in Bolzano's Logic?" *Philosophia Naturalis*:406-413
"In his encyclopedic work *Wissenschaftslehre* Bernard Bolzano expounded a theory of logical truth which constitutes an outstanding achievement in the history of Western thought. This informal theory is essentially based on a substitutional truth-value semantics without certain existence presuppositions and contains a general proof theory. In his substitutional semantics Bolzano introduced notions such as universal validity, consistency, consequence, analyticity, and probability by means of the technique of variation of concepts. In his proof theory he treated the notion of entailment, which is a generalization of a special case of the relation of logical consequence, and studied proof trees generated by the relation of entailment which exhibit the objective connection between all true propositions. In view of this wealth of important notions it seems worthwhile to investigate the possibility of a consistent reconstruction of Bolzano's logic. In particular, it must be examined how his theory fares with a fundamental set-theoretic antinomy such as that of Russell." (p. 406)
(...)
"There is, therefore, no such thing as an idea of all ideas which are not objects of themselves, and Bolzano could scarcely be blamed for having no idea of something which does not exist in any sense at all. Under a reasonable interpretation of Bolzano's theory of ideas-as-such there is no trouble-maker around who could generate an antinomy analogous to that of Russell." (p. 411)

27. ———. 1987. "Bolzano on Induction." *Philosophia Naturalis* no. 24:442-446
"Bolzano combined the fundamental notions of his theory of probability and his proof theory to achieve a logical analysis of the principles of induction. The relation between the conclusion and the premisses of an inference of incomplete induction or analogy is an interior probability relation in Bolzano's sense. The principles of induction endow the relation between the premisses and the conclusion with the character of a relation between ground and consequence. From Bolzano's subsumption of the rule of incomplete induction under the syllogistic rule of Barbara, it follows that this relation is a special case of Bolzanian derivability." (p. 442)

28. ———. 1992. *Ontology Without Ultrafilters and Possible Worlds: An Examination of Bolzano's Ontology*. Sankt Augustin: Academia Verlag
Contents: Vorwort der Herausgeber [Edgar Morscher] 7; Einleitung von Edgar Morscher 13;
Jan Berg: Ontology Without Ultrafilters and Possible Worlds 29
Introduction 31; § 1. Collections, sets, and sums 34; § 2. Numbers, infinite sets, and infinitesimals 39; § 3. Ontology without ultrafilters 48; § 4. Ideas, properties, and intuitions 52; § 5. Propositions, sentences, and judgements 64; § 6. Validity, derivability, and entailment 79; § 7. Substances, adherences, and causes 88; § 8. Ontology without possible worlds 91;
List of special symbols 95; References 97-100.
"The first basic notion of Bolzano’s ontological system is the part relation. Its domain, i.e., the set of all objects bearing it to something, embraces concrete substances, abstract objects, and collections. The converse domain of the part relation, i.e., the set of all objects to which it is borne, contains collections only.

Some collections are concrete entities existing in space and time, the rest are abstract sums or other sets. Concrete sums are composed of substances and adherences, i.e., forces. Forces applied to certain substances give rise to subjective ideas or judgements. Further results of such applications are the concrete sentence occurrences. A subjective idea is a part of a judgement which is not itself a judgement. The set of judgements is ordered by a special causal relation.
Bolzano's abstract world is constituted of sets, abstract sums, certain attributes (i.e., properties or relations), ideas-as-such, and objects constructed on the basis of these entities. Thus, sentence shapes are a kind of properties, and certain complexes of ideas-as-such constitute propositions. The notion of an idea-as-such can be constructed from expressions of a language by means of axioms for the relation of being an object of something. Analogously, properties can be generated by axioms for the relation of something being applied to an object. The converse of this relation, i.e., the relation of an entity having a property, and the relation of being an object of an idea-as-such are fundamental ontological constants of Bolzano's." (p. 31)
(...)
"The question whether a rational reconstruction of Bolzano's ontology is possible will be sustained like a pedal point throughout the present study. In many respects, indeed, his ontological system is a model of thrift, comprehensiveness, and deductive cogency. He shows us how to grasp a self-contained, abstract "third" world (in Popper's sense) embracing the realms of classical logical truth and additive probability spaces without indulging in possible worlds, states of affairs, facts, and all that. Admittedly, from a modem point of view certain aspects of his ontology may look like Dr. Johnson's dog walking on its hind legs: it is not always done quite well, but you are surprised to find it done at all. To rational bipeds of our time it should be more instructive, though, to watch this performance rather than amazing at metaphysical cephalopods wallowing in clouds of ontological splendors, or gazing at recondite cogitators crawling on all fours through a self-induced verbal fog." (p. 33)

29. ———. 1992. "The Connection Between Bolzano's Logic of Variation and His Theory of Pprobability." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 107-120. Firenze: Leo S. Olschki
"In his monumental four-volume work *Wissenschaftslehre* (1837) - in the sequel denoted by 'WL' - Bolzano introduced several new concepts for the analysis of the structure of scientific theories. In particular, he tried to lay down a logically satisfactory foundation of mathematics and the theory of probability. During the search for such a foundation he became aware of the distinction between the actual thoughts of human beings and their linguistic expressions on the one hand, and abstract propositions (*Sätze an sich*) and their components which exist independent of these thoughts and expressions on the other hand.
Bolzano described the relations of propositions to other relevant notions such as those of sentence, truth, existence, and analyticity. Furthermore, he studied relations among propositions and defined highly interesting notions of validity, consistency, derivability, and probability, based on the method of «replacing» certain components in proposition. A proposition in Bolzano's sense is a structure of ideas-as-such (*Vorstellungen an sich*). According to Bolzano, each complex idea-as-such can be analyzed into a sequence of simple ideas which include certain logical constants such as those expressed by the words 'not', 'and', 'some', 'all', 'to have', or 'ought' (WL §§ 61, 78.1, 116.3). The manner in which a complex idea-as-such is built up from simple ones may be expressed in a language by a chain of definitions." (p. 1907)

30. ———. 1994. "The Ontological Foundations of Bolzano's Philosophy of Mathematics." In *Logic and Philosophy of Science in Uppsala*, edited by Prawitz, Dag and Westerstähl, Dag, 265-271. Dordrecht: Kluwer
"The basic notion of Bolzano's ontological system is the part relation.

Its domain embraces concrete substances, abstract objects, and collections; the converse domain contains collections only.
Some collections are concrete entities existing in space and time, the rest are abstract sets.
Bolzano's notion of a set implies that a set cannot be a member of itself. Hence, there is no danger of an antinomy similar to that of Russell arising in Bolzano's ontological system of sets.
Bolzano's abstract world is constituted of sets, certain attributes (i.e., properties and relations), ideas-as-such, and objects constructed on the basis of these entities.
Thus, certain complexes of ideas-as-such constitute propositions. The notion of an idea-as-such can be constructed from expressions of a language by means of axioms for the relation of being an object of something. Analogously, properties can be generated by axioms for the relation of something being applied to an object. The converse of this relation and the relation of being an object of an idea-as-such are fundamental ontological constants of Bolzano's." (p. 265)

31. ———. 1997. "Bolzano, the Prescient Encyclopedist." *Grazer Philosophische Studien* no. 53:13-32
Abstract: "In his *Wissenschaftslehre* Bernard Bolzano tried to lay down a logically satisfactory foundation of mathematics and theory of probability. Thereby he became aware of the distinction between the actual thoughts and judgments of human beings, their linguistic expressions and the abstract propositions *(Sätze an sich)* and their components (*Vorstellungen an sich).* This ontological distinction is fundamental in Bolzano's thinking paired with a universal world view in the sense that philosophy, mathematics, physics and metaphysics should be build upon the same logical foundations. Bolzano's enterprise is sketched in the light of examples from his logical semantics, proof theory, number theory, theory of truth and his variation logic."

32. ———. 2000. "From Bolzano's Point of View." *The Monist.An International Quarterly Journal of General Philosophical Inquiry* no. 83 (1):47-67
"I am going to present logic, logical semantics, ontology, proof theory, the foundations of mathematics, and certain aspects of the philosophy of nature from Bolzano's point of view.
In his monumental four-volume work *Wissenschaftslehre* (1837) Bolzano introduced several new concepts for the logical analysis of the structure of scientific theories. In particular, he tried to lay down a logically satisfactory foundation of mathematics and the theory of probability.
During the search for such a foundation he became aware of the distinction between the actual thoughts and judgements of human beings, their linguistic expressions, and the abstract propositions (*Sätze an sich*) and their components which exist beyond space and time. This ontological distinction is fundamental in Bolzano's philosophy. In his terminology, real things have actuality whereas abstract objects have logical existence bare of actuality.
Bolzano worked extensively with the relation of being an object of an idea-as-such (a *Vorstellung an sich*). The object of an idea-as-such can be either an abstract object or a concrete object existing in space and time.
The relation of being an object of an idea-as-such corresponds in modem semantics to the relation of being an element of the extension of a concept." (47)
(...)
"Bolzano is indubitably one of the greatest philosophers of the German language. His world view was a universal one in the sense that philosophy, mathematics, physics, and metaphysics should build upon the same logical foundations. In fact, he already recognized many of the essential things to come in logic and the foundations of mathematics." (p. 67)

33. ———. 2003. "Bolzano's Heuristics." In *Bernard Bolzanos Leistungen in Logik, Mathematik und Physik*, edited by Morscher, Edgar, 35-56. Sankt Augustin: Academia Verlag

"In the fourth part of the *Wissenschaftslehre* [WL], contained in the third volume of the original 1837 edition, Bolzano treats *heuristics* or the "art of discovery", i.e., the "rules to be observed in the search for new truths" (§ 9. Note 3; cf. also § 15.2). The first main section of Bolzano's heuristics embraces the *general rules* of this discipline (§§ 325 -348).
Logic in Bolzano's sense is a theory of science the objects of which are the different sciences and their linguistic representations (§ 15). According to Bolzano a science is a set of true propositions (*Sätze an sich*) worthy of representation in a textbook. Logic or the *theory of science* is a set of rules which are necessary and sufficient for a representation to satisfy certain criteria concerning scientific textbooks (§ 1). In view of this very broad conception of logic it is fairly obvious that heuristics is an integrant part thereof." (p. 35)
This paper was already presented in 1991 at the International Bolzano Symposium in Salzburg, but has never been published since.

34. ———. 2003. "The Importance of Being Bolzano." In *Bernard Bolzanos Leistungen in Logik, Mathematik und Physik*, edited by Morscher, Edgar, 153-166. Sankt Augustin: Academia Verlag
"1. Logical consequence
Ever since Aristotle philosophers have occupied themselves with the question whether a given statement follows from another statement. The first published precision of this notion in modern times was undertaken by the Polish logician Alfred Tarski in 1936. Accordingly, a closed formula F is a logical consequence of a set of formulas F if and only if F is true under every interpretation of the nonlogical constants under which all elements of F are true. Logical constants are inter alia connectives of sentential logic (expressed by words like "not", "and", "or", "if - then") and quantifiers of predicate logic (such as "for all" and "there is"); hence, the interpretation of these constants is determined.
But who conceived this notion of logical consequence (mutatis mutandis) already a hundred years earlier?'
Right: The Bohemian philosopher, ontologist, logician, mathematician and theologian Bernard Bolzano!
Upon substitution of abstract nonlinguistic propositions for closed formulas and variants of propositions for interpretations, we get precisely a special case of Tarski's notion of logical consequence. (A variant of a proposition P is a proposition identical with P up to at least one nonlogical component.) Incidentally, at the university of Warsaw Tarski was a student of Lukasiewicz's who lectured inter alia on Bolzano's logic.
Just like Bolzano Tarski admitted being unable to exactly distinguish between logical and nonlogical constants. Not until thirty years later did he formulate a necessary condition for the property of being a logical constant. Furthermore, if all constants of the formal language in question were regarded as logical, the notion of material implication, would emerge. Even this weakest of all notions of consequence was introduced by Bolzano and is playing an important role in some of his deduction rules.
Tarski presupposed a fixed domain as a realm of reference for the interpretations. Even Bolzano did not conceive of a combined quantification over domains and components of propositions. (By introducing a predicate for domains and letting the quantifiers refer to this predicate, however, one can represent all theorems of the model theory developed later on.)
Nowadays we know, of course, that Tarski's notion of logical consequence is unsuitable if the set-theoretic language is enlarged by a generalized existence quantifier expressing that there is an absolutely infinite class C (in the sense that C does not include exactly K elements for any cardinal number K). This esoteric fact of modern set theory cannot, however, diminish our appreciation of Bolzano's achievement.
2. Analytic propositions

A fundamental distinction in Kant's Critique of Pure Reason is that between analytic and synthetic judgements. In modern logical semantics analyticity is often considered a relation between a sentence S, a set of definitions, and a language L. For instance, one can say that S in L is analytic with respect to D if S is a logical consequence of D in L which embraces S and the elements of D.
But who formulated an analogous explication of analyticity within the system of abstract propositions already in the 1830s?
Right: Bernard Bolzano!" (pp. 153-154)
(...)
"6. Situation semantics
In modern so-called situation semantics, established at the beginning of the 1980s by the American logician and linguist Jon Barwise, a notion of consequence is introduced which is stronger than that of Tarski. In situation semantics certain set-theoretic structures are considered models and a situation is a partial submodel thereof. The primitive notion is the confirmation of a sentence in a model by a situation. For example, a sentence of the form of "A or not A" is a logical consequence of any sentence in the sense of Tarski but not a strong consequence of it.
But who discovered this notion of strong consequence even a hundred years earlier?
Right: Bernard Bolzano!
In his logic Bolzano considered not only the variants with respect to the sequence of all nonlogical components of propositions but also the variants with respect to all subsequences. By that counterparts of main laws of situation semantics turn into theorems of Bolzano's logic." (p. 156)
(...)
17. Estimation
Thus some outstanding achievements of Bolzano's on the fields of logic, semantics, and mathematics have been delineated. The fact that the connection of most of these achievements with modern research remained unknown until the 1960s is due to the circumstance that the study of Bolzano's work took a new turn then and that eventually editions of the often hardly legible manuscripts of the literary remains could be published in the Collected Works of Bernard Bolzano.
Moreover, particularly in Bolzano's logical semantics there are many original ideas which have no precise affinity with modern theories. In addition to that he accomplished extensive investigations into concepts of epistemology, philosophy of nature, physics, metaphysics, ethics, and theology." (p. 165)

35. Berka, Karel. 1982. "Bolzano’s Philosophy of Science." In *Bernard Bolzano, 1781-1848 Bicentenary. Impact of Bolzano's Epoch on the Development of Science*, 427-442. Prague: Institute of Czechoslovak and General History CSAS.

36. ———. 1983. "The Ideal of Mathematization in B. Bolzano." In *Nature Mathematized. Historical and Philosophical Case Studies in Classical Modern Natural Philosophy. Vol. 1*, edited by Shea, William R., 291-298. Reidel: Kluwer
"In my contribution I would like to draw attention to the views on the ideal of mathematization held by B. BoJzano, a later follower of Leibnizian rationalism. This analysis will show the evolution of conceptions elaborated in the epoch of *mathesis universalis* on this topic in a period basically influenced by the philosophy of Kant and other representatives of German classical philosophy." (p. 291)
(...)
"The discussions concerning the acceptability of the fifth postulate of Euclid's *Elements* and the various attempts to prove it, seem to him to be clear evidence that the problem in question does not lie in the demonstration of the certainty of this postulate, but in finding the objective ground of its validity. In his work *Die drey Probleme der Rectification, der Complanation und der Cubirung* (1811), he claims that we cannot accept as a basic truth any proposition which admits a further ground of its truth. BoJzano does not doubt that this postulate is true, requiring only to have its validity grounded in an objective way, independently of our subjective feeling of certainty." (p. 291)

(...)
"Bolzano's conception, which extends and modifies the Leibnizian project of mathematization is explicitly proclaimed in part II, "On the mathematical method", of his *Beyträge zu einer begründeteren Darstellung der Mathematik* (1810) and further elaborated in other mathematical works, especially in his *Einleitung zur Grossenlehre*, and in the *Wissenschaftslehre*, where the logical aspects of mathematics and its methodology are taken into consideration." (p. 292)

37. ———. 1988. "Natural Deduction in Bolzano's *Wissenschaftslehre*." In *Intensional Logic, History of Philosophy and Methodology. To Imre Ruzsa on the Occasion of his 65th Birthday*, edited by Bodnàr, István M., Maté, András and László, Pólos, 203-212. Budapest: Department of Symbolic Logic, Eotvos University.

38. ———. 1998. "Bernard Bolzano. A Historian of Logic." *History of Science and Technology* no. 31:121-130
Abstract: "Bolzano´s *Theory of Science* (*Wissenschaftslehre*) contains a great amount of very valuable information concerning the development of logic from its beginnings in Aristotle till the post-Kantian period. In a critical exposition Bolzano presents views of his predecessors and compares them with his own standpoint. The paper presents a selective survey of various conceptions developed by Aristotle, G. W. Leibniz and his followers G. Ploucquet, J. H. Lambert and S. Maimon together with their Bolzanian interpretation. The historical analyses in his principal logical work are, thus, at the same time a witness of his own opinions toward different topics in logic."

39. Betti, Arianna. 1998. "*De Veritate*: Another Chapter. The Bolzano-Lesniewski Connection." In *The Lvov-Warsaw School and Contemporary Philosophy*, edited by Kijania-Placek, Katarzyna and Wolenski, Jan, 115-137. Dordrecht: Kluwer
"In 'De Veritate: Austro-Polish contributions to the theory of truth from Brentano to Tarski' Jan Wolenski and Peter M. Simons related an intriguing story of the "Austro-Polish obsession with truth". Wolenski and Simons mention the Bohemian philosopher Bernard Bolzano several times, with particular reference to absoluteness and sempiternity of truth in Twardowski and Lesniewski.
(...)
In the following I wish to point out three issues. First, in the so-called prelogistic writings the early Lesniewski defines truth of sentences in such a way that truth conditions are the same - *mutatis mutandis* - as Bolzano's.
Secondly, from this point of view the links between the early and the late Lesniewski, in this case between some parts of his early writings and some aspects of Ontology, are closer than they are commonly believed to be. Thirdly, in this perspective it can be shown that some of Bolzano's views come near to Lesniewski's Ontology. In discussing Bolzano's views I shall mostly follow Casari's reading of Bolzano's *Wissenschaftslehre*." (p. 115)

40. ———. 2006. "Sempiternal Truth. The Bolzano-Twardowski-Lesniewski Axis." In *The Lvov-Warsaw School: The New Generation*, edited by Jadacki, Jacek Jusliuz and Pasniczek, Jacek, 371-399. Amsterdam: Rodopi
"Twardowski [*] had revived Bernard Bolzano's ideas on the subject [eternity and sempiternity of truth], and, mainly thanks to him, these became known in the Lvov-Warsaw School (see, for instance, Jadacki 1993, p. 191). There is no doubt that Lesniewski knew Twardowski's ideas and it seems evident that the latter influenced him: Lesniewski's results are mostly compatible with the "absolutistic" content of Twardowski's 1900 article. And, similarly, no doubts can be raised about the Bolzanian origin of the aspects of eternity and sempiternity of truth defended by Twardowski in *Relative Truths* (see, for instance, Wolenski and Simons 1988, p. 430, n. 24; and Simons 1992, Ch. 2, p. 15, n. 11; see also Smith 1988, p. 325): though his name is not quoted, traces of Bolzano's legacy can be found even in the examples given by Twardowski, some of which are the same as used by Bolzano in his *Wissenschaftslehre*. Yet, since Bolzano, Twardowski and Lesniewski supported different theories of meaning with different ontological presuppositions,

"sempiternity of truth" actually stands for three different conceptions. This paper is a survey of these three conceptions. I suggested elsewhere a comparison between Bolzano and the early Lesniewski as to their theories of meaning and truth, claiming the possibility of a (direct or indirect) influence of Bolzano upon Lesniewski. The analysis presented here is also meant as a contribution to the picture sketched there." (p. 372, notes omitted)
[*] "On the So-Called Relative Truths" (1900) in J. Brandl and J. Wole?ski (eds.), *Kazimierz. Twardowski - Actions, Products and other Topics in Philosophy,* Amsterdam: Eodopi 1999, pp. 147-168. J. Brandl and J. Wole?ski (eds.),
References
Jadacki, J.J. (1993). Kazimierz Twardowski's Descriptive Semiotics. In: Coniglione et *al.,* eds. (1993), pp. 191-206.
Coniglione, F., R. Poli and J. Wolenski, eds. (1993). Po*lish Scientific Philosophy: The Lvov-Warsaw School.* Poznan Studies in the Philosophy of the Sciences and the Humanities, vol. 28. Amsterdam: Rodopi.
Simons, P.M. (1992). Ph*ilosophy and Logic in Central Europe from Bolzano to Tarski. D*ordrecht: Kluwer.
Smith, B. (1988). Ka*simir Twardowski: An Essay on the Borderlines of Ontology, Psychology and Logic.* In: Szaniawski, ed. (1988), pp. 313-375.
Szaniawski, K., ed. (1988). Th*e Vienna Circle and the Lvov-Warsaw School. T*he Hague: Nijhoff.
Wolenski, J. and P.M. Simons (1988). D*e veritate: Austro-Polish Contributions to the Theory of Truth from Brentano to Tarski. I*n: Szaniawski, ed. (1988), pp. 391-443.

41. ———. 2006. "The Strange Case of Savonarola and the Painted Fish. On the Bolzanization of Polish Thought." In *Actions, Products, and Things. Brentano and Polish Philosophy*, edited by Chrudzimski, Arkadiusz, 55-81. Frankfurt: Ontos Verlag
"I have previously discussed in several papers specific Bolzanian elements present in the Polish tradition. This paper will not, for the most part, add anything in particular to that. The new - and rather blunt hypothesis to be put forward here is that, despite appearances, Twardowski also contributed *de facto* to slowing down the reception of Bolzano's most modem logical discoveries. For in Poland Bolzano was to remain one logician among many for rather long. It was chiefly thanks to two factors that Bolzano's star could, slowly, begin to rise in Poland, or, at least, that the fundamental achievements of his logic could be known. One factor is antipsychologistic (more precisely Platonistic) influence coming from Husserl and from Twardowski's student Lukasiewicz. The other factor is the change in the conception of logic which took Polish logic from, say, Sigwart, to Tarski through Lesniewski and Lukasiewicz," (p. 55)

42. ———. 2010. "Explanation in Metaphysics and Bolzano's Theory of Ground and Consequence." *Logique et Analyse* no. 56:281-316
"In "Troubles with Truth-making: Necessitation and Projection." Erkenntnis 64: 61-74 (2006a, and in "Truth-Making without Truth-Makers." Synthese 152: 21-46 (2006b), Benjamin Schnieder criticizes truthmaking as a relation between entities in the world and the truths those entities 'make true'. In (2006b), his criticism exploits a notion of conceptual explanation that is very similar to Bolzano's grounding. In the first part of this paper, I offer an analysis of Bolzano's grounding. I discuss some open problems and argue that Bolzano's grounding is not a systematization of the ordinary notion of 'because' as others have maintained, but of the technical notion of explanatory proof in the context of an axiomatic conception of (proper) science. On the basis of this analysis, in the second part, I offer a critical discussion of Schnieder 2006b's arguments against truthmaking. I conclude that the latter are not very effective from a methodological point of view and that Bolzano's original position fares better in this respect; still, truthmaker theorists will be able to defend truthmaking only at a high price."

43. ———. 2012. "Bolzano's Universe: Metaphysics, Logic, and Truth." In *Categories of Being. Essays on Metaphysics and Logic*, edited by Haaparanta, Leila and Koskinen, Heikki J., 167-208. New York: Oxford University Press
"Thanks to a handful of publications from the last decade, however, Bolzanian metaphysics has begun to receive more attention than ever before.(1)
It is not difficult to show why Bolzanian metaphysics matters. Bolzano's logic builds on firm ontological and mereological foundations. Logic as a science has a realm of its own, that of the *an sich*, in the strong sense that logic is the science of a special kind of object, namely, propositions-in-themselves and ideas, and their qualities. Furthermore, the edifice of logic rests on a mereological conjecture regarding the basic form of propositions and is constructed by exploiting mereological relations between propositions and ideas, plus a device of semantic ascent, involving very special ideas with very special qualities, called symbolic ideas.
Th e first and main aim of this essay is to present an overview of Bolzano's universe from the point of view of his metaphysics and its relationship to logic, relying fundamentally on his major work, the *Wissenschaftslehre.* This I shall do in sections II–VI. Although these sections are chiefly intended as an exposition of the state of the art on the matter, I shall make no secret of preferring a reading of Bolzano as a "Platonistic nominalist," as Textor puts it—as a Platonist about propositions and a nominalist about properties. (2) My second aim, in sections VII–IX, shall be to answer the open question of whether in Bolzano there is any "ontology of truth," as one may call it, though with some hesitation." (pp. 167-168)
(1) Among others, Künne 1998; Schnieder 2002; Textor 2004.
(2) Textor 2004 , 10. That Bolzano is a Platonist about propositions is the predominant view, which I follow here. Among those who disagree, cf. Cantù 2006 , 10.
References
Cantù, Paola. 2006. Bolzano et les propositions en soi: une théorie objective des vérités. In *Propositions et états de choses* , ed. J. Benoist. Paris: Vrin.
Künne, Wolfgang. 1998. Substanzen und Adhärenzen—Zur Ontologie in Bolzanos Athanasia. *Philosophiegeschichte und logische Analyse* 1: 233–50.
Schnieder, Benjamin. 2002. *Substanz und Adhärenz: Bolzanos Ontologie des Wirklichen*. Sankt Augustin: Academia.
Textor, Mark. 2004. Bolzanos Ontologie. In *Die Bedeutung Bernard Bolzanos für die Gegenwart*, ed. K. Strasser. Prague: Filosofia.

44. Beyer, Christian. 2004. "Bolzano and Husserl on Singular Existential Statements." In *Phenomenology & Analysis: Essays on Central European Philosophy*, edited by Chrudzimski, Arkadiusz and Huemer, Wolfgang, 69-88. Frankfurt: Ontos Verlag
"Which form does the propositional content take that is judged when a given speaker sincerely utters a sentence in order to assert a singular existential statement? Two thought-provoking answers to this question have been proposed by Bernard Bolzano and, when commenting upon Bolzano's proposal, by Edmund Husserl. In Section 1 of this paper the author clarifies what he means by "singular existential statements". In Section 2 Bolzano's proposed analysis is sketched. In Section 3 the author exposes the earlier Husserl's conception of "logical reflection" and draws upon it to explain why Husserl, around 1900, subscribed to Bolzano's proposal. Following this, he reconstructs and considers in detail the later Husserl's discussion of that proposal and Husserl's own mature theory of singular existential statements as manifested in a 1917/18 lecture series, both of which shed light upon a conception that is of central importance for Husserlian phenomenology: the conception of "noematic sense" (Section 4)." (p. 71)

45. Bodnar, Joanne. 1976. *Bolzano and Husserl: Logic and Phenomenology*
Unpublished Ph.D. thesis, State University of New York at Buffalo, available at ProQuest Dissertation Express.
Contents: Introduction 1; I. Bolzano's Anti-Psychologism 5; II. Bolzano's Theory of Meaning 41; III. Bolzano's Basic Logical Relations 51; IV. Truth to Bolzano 63; V.

Husserl's Anti-Psychologism 76; VI. Husserl's Theory of Meaning 99; VII. Basic Logical Relations in Husserl 113; VIII. Truth to Husserl 128; IX. Recapitulation 144; X. Conclusion 151; Bibliography 164-168.

"Bernard Bolzano and Edmund Husserl both present some form of ontological framework for logic rather than a linguistic framework. Their works predate the pragmatic and semantic theories of Tarski and Carnap. Bolzano's *Wissenschaftslehre* appeared in 1837. (2)And Husserl's major logical thinking was formulated before 1935. (3) But neither of them seem receptive to a semantic foundation for logic, because of their rationalist-platonist leanings. Both strongly oppose the view that logic is taken from psychological experience by generalization. They have a viewpoint which is perhaps closer to the classical outlook than to either of the others, since they consider the foundation of logic to be the acceptance of meanings as entities — entities which are in some important ways related to actual and possible being and its structure.

Bolzano and Husserl each make crucial modifications on traditional platonism as a philosophy of logic. A basic thesis which they both do accept is that the logical entities such as the proposition with its elements and its relations are ideal unities, which are independent of their being thought. But the logical entities are not platonic forms in which spatio-temporal existences "participate" — nor are they determined by spatio-temporal existence in any way. They are independent of the subject or knower as well as of the facts of material existence.(4)

Thus the logical entities are what they are whether they ever come to expression or not. They have a character similar to that of numbers or other "abstract" mathematical objects, but it cannot be said that they arise in experience as abstractions from the empirical world. Although there is disagreement about the question of abstraction in Bolzano — with some Bolzano commentators such as Rolf George seeing little difference between Bolzano and Carnap — this tendency to read Bolzano's work as if he were a pragmatist obscures the originality of Bolzano.(5)

His differences from semantic and empiricist thinking are well worth investigating. Husserl's approach too deserves consideration for its uniqueness. Crediting Bolzano with giving a starting point in philosophy of logic, Husserl "discovered" Bolzano and brought his work out of obscurity. He makes use of the work of Bolzano however, only to transform it thoroughly. If certain common themes are selected for exposition, the positions of Bolzano and Husserl are both seen to be modifications of the classical platonism. This provides a basis for a comparison of Bolzano and Husserl." (pp. 3-5).

(2) Bolzano's work under consideration is *Theory of Science*, ed. and trans. by R. George (Berkeley, 1972), hereafter cited simply as Bolzano. English paginations are used, but section numbers apply to all German editions as well. The Theory of Science is a condensation of Wissenschaftslehre, vols. 1-4 (Sulzbach, 1837). R. George follows in large measure the F. Kambartel edition of Bolzano's *Wissenschaftslehre* vols. 1-2, entitled *Grundlegung der Loqik* (Hamburg, 1963) in which Bolzano's original has been condensed, with the omitted passages summarized by the editor.

(3) Works by Husserl principally under consideration are *Logische Untersuchungen* (1900 and 1913), trans. by J. N. Findlay (New York, 1976); and *Formale und Transzendentale Loqik* (1929), trans. by D. Cairns (Hague, 1969).

(4) See: U. Neemann, *Bernard Bolzanos Lehre von Anschauung und Begriff in ihrer Bedeutung für erkenntnistheoretische und pädaqogische Probleme* (Paderborn, 1972) pp. 81 and 144. for discussions of how Bolzano's logical entities differ from Plato's forms and from Kant's subjective categories.

(5) See: R. George, "Editor's Introduction" in Bolzano's *Theory of Science*, (Berkeley, 1972) p. xxx. Also note J. Berg, *Bolzano's Logic* (Stockholm, 1962) pp. 49-50, where he expresses the view that Bolzano takes logical entities as abstractions.

46. Cantù, Paola. 2011. "Bolzano Versus Kant: Mathematics as a Scientia Universalis." In *Mind, Values, and Metaphysics: Philosophical Essays in Honor of Kevin Mulligan. Vol. 1*, edited by Reboul, Anne. Dordrecht: Springer
Abstract: "The chapter will discuss some changes in Bolzano's definition of mathematics attested in several quotations from the *Beyträge, Wissenschaftslehre* and *Größenlehre*: Is mathematics a theory of forms or a theory of quantities? Several issues that are maintained throughout Bolzano's works will be distinguished from others that were accepted in the *Beyträge* and abandoned in the *Größenlehre*. Changes will be interpreted not only as a consequence of the new logical theory of truth introduced in the *Wissenschaftslehre* but also as a consequence of the overcome of Kant's terminology, and of the radicalization of Bolzano's anti-Kantianism.
It will be argued that Bolzano's evolution can be understood as a coherent move, if one compares the criticism on the notion of quantity expressed in the *Beyträge* with a different and larger notion of quantity that Bolzano developed already in 1816. This discussion is based on the discovery that two unknown texts mentioned by Bolzano can be identified with works by von Spaun and Vieth. Bolzano's evolution will be interpreted as a radicalization of the criticism of the Kantian definition of mathematics and as an effect of Bolzano's unaltered interest in the Leibnizian notion of *mathesis universalis*. As a conclusion, it will be argued that Bolzano never abandoned his original idea of considering mathematics as a *scientia universalis*, i.e. as the science of quantities in general, and it will be suggested that the question of ideal elements in mathematics, which has been interpreted as a main reason for the development of a new logical theory, can also be considered as a main reason for developing a different definition of quantity."
References
Vieth G.U.A. (1805) *Anfangsgründe der Mathematik. Lehrbuch der angewandten Elementarmathema*tik. Barth, Leipzig-
von Spaun F.A.R. (1805) *Versuch das Studium der Mathematik durch Erläuterung eniger Grundbegriffe und durch zweckmässigere Methoden zu erleichtern*. Göbhardt, Bamberg-

47. Casari, Ettore. 1989. "Remarks on Bolzano's Modalities." In *Atti del Convegno Internazionale di Storia della Logica: le teorie della modalità*, edited by Corsi, Giovanna, Mangione, Corrado and Mugnai, Massimo, 319-322. Bologna: CLUEB
"1. *The Roots of Bolzano's Interest in Modalities*.
From an autobiographical note quoted by Winter ([1], p.32), we learn that as Bolzano was 17 years old and began to read the *Kritik der reinen Vernunft*, he was soon strongly attracted by the distinctions of Judgments into a priori and a posteriori and into analytic and syntetlc as well as by the distinction of representations into Intuitions and concepts, whereas he was very hurted by the immediate use, without any previous explanation, of the concepts of experience and of necessity. A significant part of Bolzano's work may be seen as an attempt to clarify the preceding notions and to substantiate their distinctions. In particular, his theory of (absolute) modalities is his answer to the question about necessity, an answer which he derives from his answer to the question about experience. Necessity is indeed first reduced to necessary truth; this latter is identified with true proposition in Itself which doesn't depend upon experience; depending upon experience is identified with containing intuitions; intuitions are a logically well defined kind of representations In themselves (Ideas).
Bolzano's theory of modalities has been scarcely considered up to now; the most careful analysis has been done by E. Morscher ([2], pp.87-92). In the following we will embed Bolzano's theory of the absolute modalities, as presented mainly in the *Wissenschaftslehre* §182, into the general framework of his logic we have reconstructed elsewhere ([3], [4]) and which will be only sketched here. We will not consider his theory of relative modalities." (p. 319)
(...)

"*Last Remarks*. As alluded to in §1, Bolzano's primary interest was in the notion of necessary truth. The whole of his modal theory, although interesting in many respects, is far from being satisfactorily refined. So, for instance, we remark that according to Bolzano, every standard proposition whose subject is unobjectual [*gegenstandIos*], that is, referring to no object, is false, it follows that all such propositions are possible (although their being true may be Impossible). That truth is a quality and that there is an Idea which refers to it, are, of course, rather disquieting assumptions. From their discussion in [4], it follows that a very important question is whether the idea [*p*], which refers to *p*, has also *p* as its part. In the present context, the question presents itself, in particular, with respect to the problem about the conceptuality of a proposition having the idea [*p*] as its subject, under the hypothesis of the conceptuality of the proposition *p* and vice versa; a problem which immediately arises, when considering, for instance, iterated modalities." (p. 323)

Bibliographical Note

[1] E. Winter, *Die geistige Entwicklung Bolzanos*, in E. Winter, P. Funk, J. Berg, *Bernard Bolzano, Ein Denker und Erzieher Im Österreichischen Vormärz*, Sitz.-Ber. d. Öst. Ak.d. Wiss., Phil.-Hist.Kl., Bd. 252, Abhdl. 5, Hf. 8, Wien 1967, pp. 29-74.
[2] E. Morscher, *Philosophische Logik bei Bernard Bolzano*, in Bolzano-Symposion: "Bolzano als Logiker", Sitz.-Ber. d. Öst. Ak.d. Wiss., Phil.-Hist. Kl., Bd.293, Abhdl. 5, Hf. 12, Wien 1974, pp. 77-105.
[3] E. Casari, *Bemerkungen über die Bolzanosche Wissenschaftslehr*e, in *Logik und Grundlagenforschung*, H.Scholz-Kolloquium, Aschendorff, Münster i. W., 1985, pp. 53-66.
[4] E. Casari, *An Interpretation of some ontological and semantical notions In Bolzano's logic,* to appear in the proc. of the meet. (1987) of the Florence Center for Hist, and Phil. of Sc.: *Wissenschaftslehre 1B37-1987.*

48. ———. 1992. "An Interpretation of Some Ontological and Semantical Notions in Bolzano's Logic." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 55-105. Firenze: Leo S. Olschki

"In the following, the attempt is done to clarify some significant features of Bolzano's logical system with particular attention to its development in the *Wissenschaftslehre* (WL). This system is viewed as a theory trying to identify certain quite general properties, relations and operations of *things [Dinge],* in the most general and unbiased sense of this word. For sake of simplicity and determinateness of the formulations, the current logical symbolism is *used.* The point of view of the theory is *elementary,* that is to say, we always work with *particular notions* about the things without allowing us any consideration of *arbitrary notions* about the things. 'x', 'y', 'z', ... are used as variables for things." (p. 55)

49. ———. 2006. "Some Remarks on Bolzano's Notion of a Quality." In *Logic and Philosophy in Italy. Some Trends and Perspectives. Essays in honor of Corrado Mangione*, edited by Ballo, Edoardo and Franchella, Miriam, 185-201. Milano: Polimetrica.

50. ———. 2016. *Bolzano's Logical System*. Oxford: Oxford University Press

"As already mentioned, many specialist studies have analysed many of the questions that arise from the first three parts of Bolzano's work, as well as providing comprehensive expositions of them, often very successfully. Yet, it seems to us that there remains room for a more systematic reconsideration of Bolzano's logical thought.

This book is concerned precisely with this aim. In undertaking this task, the book is intended as an exploration, not so much of the more specifically discursive aspects of Bolzano's logical thought—already amply studied—as much as one aimed at identifying the singularly coherent and systematic nature of the logic presented in the *Wissenschaftslehre*.

In order to render as visible as possible the systematic nature of that logic, I have decided to present it within a formal system. Despite being surprising even to me, it has become clear that in pursuing this aim, it is sufficient to adopt the approach of the predicate calculus with identity and choice operator, that is, enlisting the wellknown Hilbert's epsilon calculus. As this book reveals, the formalization of Bolzano's logic in this calculus emerges quite effortlessly." (Preface, p. VIII)

51. ———. 2017. "Husserl and Bolzano." In *Essays on Husserl's Logic and Philosophy of Mathematics*, edited by Centrone, Stefania, 75-91. Springer
Abstract: "The paper examines the all too often neglected role of the Czech philosopher and mathematician Bernard Bolzano for Husserl's work, from ca. 1893–1894 onwards. Husserl himself finds it important to stress in an appendix to chapter 10 of the *Prolegomena to Pure Logic* that his investigations are not "in any sense mere commentaries upon, or critically improved expositions of, Bolzano's thought patterns", but that they "have been crucially stimulated by Bolzano... ". The paper examines early Bolzano's ideas on the ground-consequence relation, Bolzano's logical universe as presented in his masterpiece, the monumental *Wissenschaftslehre*, the role of Hermann Lotze in making Husserl receptive for Bolzano and, finally, a lecture course on logic held by Husserl at the University of Halle in 1896 [*], working out just what Husserl is taking, and not taking, from Bolzano."
[*] E. Husserl, [LV'96] *Logik: Vorlesung 1896,* ed. by E. Schuhmann. *Husserliana* Materialienbände I (Kluwer, Dordrecht, 2001)

52. Cellucci, Carlo. 1992. "Bolzano and Multiple-Conclusion Logic." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 179-189. Firenze: Leo S. Olschki
"The aim of this paper is to assess Bolzano's logical work in the light of contemporary logical developments. This has been done before by others, most recently by van Benthem, (1) but everybody has his own approach and my approach -- whatever its value -- will be somewhat different from the current one. Make no mistake, I am not going to discuss once again to what extent Bolzano anticipated modern logic. On the contrary I will try to show how far he was from moden logic. In order to do so I will compare Bolzano with the tradition of multiple-conclusion logic." (p. 179)
(1) J. van Benthem, *The Variety of Consequence, According to Bolzano*, Studia Logica 44, 1985, pp. 389-403.

53. Centrone, Stefania. 2010. "Functions in Frege, Bolzano and Husserl." *History and Philosophy of Logic* no. 31:315-336
Abstract: "This explorative article is organized around a set of questions concerning the concept of a function. First, a summary of certain general facts about functions that are a common coin in contemporary logic is given. Then Frege's attempt at clarifying the nature of functions in his famous paper *Function and Concept* and in his *Grundgesetze* is discussed along with some questions which Freges' approach gave rise to in the literature. Finally, some characteristic uses of functional notions to be found in the work of Bernard Bolzano and in Edmund Husserl's early work are presented and elucidated."
"4. Bernard Bolzano
In this section, I want to show that the set–theoretical notion of a function is implicitly at work in Bolzano's logic of variation. Bolzano's own use of the term 'function' is not pertinent here, for he employs this term only in the context of 'x is a function of y1, y2,..., yn' where the correlated entities are what he calls *Größen* (magnitudes).(24) Thus, Bolzano's usage of the term (unlike Frege's) is restricted to the field of mathematics. The entities his logic of variation is concerned with are not magnitudes, but propositions and their non-propositional parts." (p. 325)
(24) Bolzano 1830–1835, [J. Berg, ed., *Einleitung zur Grössenlehre. Erste Begriffe der allgemeinen Grössenlehre*, BGA Series 2A, vol. 7, 1975], p. 229.

54. ———. 2016. "Early Bolzano on *Ground-Consequence* Proofs." *The Bulletin of Symbolic Logic* no. 2:215-237
Abstract: "In his early *Contributions to a Better-Grounded Presentation of Mathematics* (1810) Bernard Bolzano tries to characterize rigorous proofs *(strenge Beweise*). Rigorous is, prima facie, any proof that indicates the grounds for its conclusion. Bolzano lists a number of methodological constraints all rigorous proofs should comply with, and tests them systematically against a specific collection of elementary inference schemata that, according to him, are evidently of ground-consequence-kind. This paper intends to give a detailed and critical account of the fragmentary logic of the *Contributions*, and to point out as well some difficulties Bolzano's attempt runs into, notably as to his methodological ban on 'kind crossing'."

55. Chattopadhyaya, Debi Prasad. 1979. "Bolzano and Frege: A Note on Ontology." In *Logic, Ontology and Action*, edited by Banerjee, K.K., 214-242. Atlantic Highlands: Humanities Press.

56. Chihara, Charles. 1999. "Frege's and Bolzano's Rationalist Conceptions of Arithmetic." *Revue d'Histoire des Sciences* no. 52:343-361
Abstract: "In this article, I compare Gottlob Frege's and Bernard Bolzano's rationalist conceptions of arithmetic. Each philosopher worked out a complicated system of propositions, all of which were set forth as true. The axioms, or basic truths, make up the foundations of the subject of arithmetic. Each member of the system which is not an axiom is related (objectively) to the axioms at the base. Even though this relation to the base may not yet be scientifically proven, the propositions of the system include all of the truths of the science of arithmetic. I conclude the article by analyzing the respective views of Frege and Bolzano in the light of Gödel's first incompleteness theorem."

57. Chisholm, Roderick M. 1986. "On the Positive and Negative State of Things." In *Non-Existence and Predication*, edited by Haller, Rudolf, 97-106. Amsterdam: Rodopi
Abstract: "Following Balzano, I suggest that there are two types of entity: those that are states of other things and those that are not. The second type includes, not only substances, in the traditional sense, but also such abstract objects as numbers, attributes and propositions. It is argued that the theory of states, when combined with an intentional account of negative attributes, will yield a theory of negative entities and of events."

58. ———. 1986. "The Self in Austrian Philosophy." In *Von Bolzano zu Wittgenstein. Zur Tradition der österreichischen Philosophie = From Bolzano to Wittgenstein. The Tradition of Austrian Philosophy*, edited by Nyíri, János Kristóf 71-74. Wien: Hölder-Pichler-Tempsky
Reprinted in: R. M. Chisholm, *On Metaphysics*, Minneapolis: University of Minnesota Press, 1989, pp. 156-161.
"Bolzano's definition of substance provides us with a kind of key to the conceptions of the self in Austrian philosophy. His definition is as clear as anyone could possibly wish. He says that there are two kinds of things: (I) those things that are states or conditions of other things ("Beschaffenheiten von anderen Dingen"); and (II) those things that are not states or conditions of other things: "the latter are what I call *substances.*"(1) Examples of things that are states or conditions of other things are "the color, smell and weight of a body," the beliefs that a particular person has, the sensations that he has, and the actions that he performs. Examples of substances-of things that are not states or conditions of other things -are physical bodies and selves.
Bolzano says, in Leibnizian fashion, that, if there are things that are states or conditions of other things, then there are things that are not states or conditions of other things.(2) If we use the term "substance" in the way he suggests, then we need not ask whether a given philosopher believes in substances; we need ask only what the things are that function for him as substances." (p. 156 of the reprint)

(1) Bolzano, *Athanasia oder Gründe für die Unsterblichkeit der Seele* (Sulzbach: J. G. v. Seidleschen Buchhandlung, 1838), [second enlarged edition; first edition 1827] p. 283.
(2) Bolzano (1827), p. 22. He holds that *Beschaffenheiten* may themselves have *Beschaffenheiten* and that such things as numbers also have *Beschaffenheiten* (p. 22), and he seems to hold that God has a *Beschaffenheit* (p. 22).

59. ———. 1989. "Bolzano on the Simplicity of the Soul." In *Traditionen und Perspektiven der analytischen Philosophie*, edited by Gombocz, Wolfgang, Rute, Heiner and Sauer, Werner. Vienna: Hölder-Pichler-Tempsky.

60. ———. 1991. "Bernard Bolzano's Philosophy of Mind." *Philosophical Topics* no. 19:207-216
"The views of Bernard Bolzano (1781-1842) concerning the nature of psychological properties and the nature of what it is that has those properties are of first importance to philosophy. I shall discuss some of them here in the hope that what I say may lead to a more systematic study and evaluation.
Bolzano's best known works are the *Theory of Science* [Wissenschafteslehre], first published in 1837, and *The Paradoxes of the Infinite* [Paradoxien des Unendlichen], first published in 1851. The present topic is discussed in detail in *Athanasia: Or Grounds for the Immortality of the Soul*, published in 1838.(2) This work has not been translated into English." (p. 205, note 1 omitted)
(...)
"Bolzano, then, is concerned with presenting considerations which, he thinks, indicate that only simple substances can think. In order to avoid a fundamental misunderstanding, we must be clear about one fundamental point. In setting out on his investigations, Bolzano assumes that it is not known that thinking things are identical with physical bodies. Hence he does not presuppose the thesis according to which we are identical with our bodies or with some proper part of our bodies. Others, of course, may presuppose the contrary of this thesis. But any *criticism* of Bolzano that is based upon the contrary thesis and that does not include a positive defence of this contrary thesis would be question begging." (p. 207)
(2) *Athanasia; oder Griinde fur die Unsterblichkeit der Seele* (Sulzbach: J. G. v. Seidleschen Buchhandlung, 1838). [Second enlarged editon; first edition 1827.]

61. Claas, Jan. 2021. "Leibniz and Bolzano on Conceptual Containment." *European Journal of Philosophy*:1-19
Abstract: "Philosophers often rely on the notion of conceptual containment and apply mereological terminology when they talk about the parts or constituents of a complex concept. In this paper, I explore two historical approaches to this general notion. In particular, I reconstruct objections Bernard Bolzano puts forward against a criterion that played a prominent role in the history of philosophy and that was endorsed, among others, by Leibniz. According to this criterion, a concept that represents objects contains all and only the concepts that represent properties the objects must have in order to be represented by the former concept.
Bolzano offers several counterexamples and arguments against the criterion. I argue that while some of them presuppose a strongly mereological understanding of containment, which Leibniz is not committed to, one of them also succeeds without relying on demanding mereological principles."

62. Coffa, J. Alberto. 1982. "Kant, Bolzano and the Emergence of Logicism." *Journal of Philosophy* no. 74:679-689
"Bolzano was the first to recognize the fallacy behind the principle of synthetic judgments. The crucial step in Kant's inference for the need to appeal to intuition in synthetic judgments was the premise that from concepts alone only analytic knowledge can be derived. Astonishingly, there isn't a single argument in the Critique for this claim; all Kant says about it is that "it is evident" (A47, B64).(6) What is evident, instead, is that Kant had confused true in virtue of concepts with true in virtue of definitions, or, in his own language, he had erroneously identified judgments whose predicate is not contained in their subject-concept with judgments

that extend our knowledge (*Erweiterungsurteile*). Against this, Bolzano was the first to make a point that even Frege would miss: that Kant's analytic judgments, far from exhausting the grounding power of the conceptual resources of our language, mobilize only a very modest fraction of them, the logical concepts. Bolzano's characterization of analyticity is well known, and it has often been noted that it anticipates not Frege's proof-theoretic treatment but the more modern semantic approach by means of interpretations. What is less well known is the reasoning that led Bolzano to this proposal. After reviewing a number of attempts to explain the point of Kant's notion of analyticity, Bolzano comments that "none of these explanations singles out what makes these [analytic] propositions important. I believe that this consists in the fact that their truth or falsity does not depend upon their constituent representations but remains unaltered, whatever changes one may make in some of these representations . . . This is the ground of my preceding definition."(7) Thus, the reason why Bolzano came to his celebrated insight on the semantic characterization of logical truth is that he saw that Kant's analytic judgments, far from being those grounded on the information implicit in the constituent concepts, were grounded on only a few of those concepts, thus concluding that a proper definition of analyticity should emphasize the extent to which all other concepts are to be ignored." (p. 684)
(6) For a very modest effort toward an argument, see Ak 20, 340.
(7) *Wissenschaftslehre* (Hamburg: Meiner, 1929), vol. II, sec. 148, p. 88.

63. ———. 1991. *The Semantic Tradition from Kant to Carnap*. Cambridge: Cambridge University Press
Second Chapter: *Bolzano and the Birth of Semantics* pp. 22-40.
"While the idealists were removing every trace of objectivity from Kant's semantics, there was in a corner of the Austro-Hungarian empire, ignored by the leaders of German philosophy, a Czech priest by the name of Bernard Bolzano, who was engaged in the most far-reaching and successful effort to date to take semantics out of the swamp into which it had been sinking since the days of Descartes. Bolzano was the first to recognize that transcendental philosophy and its idealistic sequel were a *reductio ad absurdum* of the semantics of modern philosophy. He was also the first to see that the proper prolegomena to any future metaphysics was a study not of transcendental considerations but of what we say and its laws and that consequently the *prima philosophia* was not metaphysics or ontology but semantics. The development of these ideas in his monumental *Wissenschaftslehre* and in a variety of other writings established Bolzano as the founder of the semantic tradition.
Bolzano's philosophy was the kind that takes from and then gives life to science. His approach to semantics was developed in dialectical interplay with his decision to solve certain problems concerning the nature of mathematical knowledge. Kant had not even seen these problems; Bolzano solved them. And his solutions were made possible by, and were the source of, a new approach to the content and character of a priori knowledge. We shall illustrate the point by focusing on one of Bolzano's favorite mathematical topics, the calculus." (p. 23)

64. Cohen, Jonathan L. 1982. "Bolzano's Theory of Induction." In *Impact of Bolzano's Epoch on Development of Science - Conference Papers Prague 1981*, 443-457. Prague: Ustav ceskoslovenských a svetových dejin CSAV
Also published in: Merrilee H. Salmon (ed.), *The Philosophy of Logical Mechanism*, Dordrecht: Springer 2011, pp. 29-40.
Abstract: "Bolzano's Wissenschaftslehre was published in 1837, although most of it seems to have been written during the decade 1820–1830. John Stuart Mill's System of Logic was published in 1843, but had been in gestation or preparation since 1825. Neither author seems to have exercised any influence on the other, and in their views about the fundamental nature of logical and mathematical reasoning they notoriously represented very different trends. Bolzano sought to direct philosophers' attention away from mental processes towards relationships between ideas in themselves and between propositions in themselves, while Mill's logic

insisted on a study of the mental process which takes place whenever we reason, of the conditions on which this process depends, and of the steps of which it consists. But in their views about the methodology of natural science the divergences are much more finegrained. Both assign a central role to the search for causes and both discuss the same basic procedures for the discovery of these. It is just that Bolzano shows a greater sensitivity than Mill does to the inherent difficulties of the enterprise."

65. Corcoran, John. 1975. "Meanings of Implication." *Diálogos* no. 9:59-76
Reprinted in: R. I. G. Hughes (ed.), *A Philosophical Companion to First-order Logic,* Indianapolis: Hackett 1993, pp. 85-100.
"In philosophical and mathematical discourse as well as in ordinary scholarly contexts the term 'implies' is used in several clear senses, many of which have already been noticed and explicated. The first five sections of this article codify and interrelate the most widely recognized meanings. Section 6 discusses a further significant and common use. Section 7 discusses and interrelates Tarski's notion of logical consequence, the "model-theoretic" notion of logical consequence, and Bolzano's two grounding relations. The eighth section employs the use-mention distinction to separate the three common grammatical categories of 'implies'. Section 8 also shows that criteria based on use-mention are not reliable indications of intended usage of 'implies'. The ninth and last section relates the above to the counterfactual and gives reasons for not expecting to find 'implies' used to express counterfactuals. A summary is provided."
"Summary and Conclusion: In the first five sections we have distinguished twelve uses of the term 'implies'. At the outset we distinguished: implies1 (truth-functional), implies2 (logical consequence) and implies3 (logical deducibility). Next we distinguished three elliptical or enthymematic varieties of implication: C-implies1, C-implies2 and C-implies3. In none of these six senses did "A implies B" presuppose the truth of A. Then we discussed the cases wherein "A implies B" is used to mean "The-fact-that-A implies B," which does presuppose the truth of A. We paraphrased the latter as "A is true and A implies 13" where 'implies' indicates any of the previous six senses of the term. Thus, at that point, twelve senses of implies were distinguished, six which do not presuppose the truth of the implying sentence and six which do. Of the six which do, three are enthymematic.
In addition, the three original senses were carefully distinguished and interrelated, and possible causes of confusion were identified.
Then, building on some off-hand observations of Russell, we related the truth-functional use of 'implies' to two further notions which have been used as explications of traditional logical consequence. We also brought in Bolzano's relative implication and his two grounding relations.
We argued briefly that counterfactuals are not normally expressed using 'implies' and that the distinction between use and mention cannot be used as a test for distinguishing different meanings of 'implies'.
Use of 'implies' as a transitive verb taking a human subject has been ignored."

66. de Jong, Willem R. 2001. "Bernard Bolzano, Analyticity and the Aristotelian Model of Science." *Kant-Studien* no. 91:328-349
"In this article I intend to make clear that Bolzano's perception and use of the distinction in question [analytic-synthetic] should also be understood in the framework of this model of science. The effect of doing so is to render more comprehensible Bolzano's highly personal and, in its application, upon first acquaintance rather strange characterization of the analytic-synthetic distinction. This characterization can then also be placed more easily in its historical context. [Joëlle] Proust aside, most interpreters have looked somewhat askance at Bolzano's notion of analyticity. And most of them seem not to be able to go on and do much with this apparently anomalous element in Bolzano's thinking. (4)
In § 2 Bolzano is presented as an adherent of the Aristotelian model of science. Section 3 discusses briefly Kant's view of the analytic-synthetic distinction; Bolzano studied it thoroughly. In § 4 his criticism of Kant's notion of analyticity is

considered, while in § 5 and § 6 Bolzano's own characterization of this distinction is discussed. Section 7 connects Bolzano's notion of analyticity with his view of derivability or (logical) inference. In the following two sections this theme is further elaborated and developed in the light of the Aristotelian model of science and the notion of scientific demonstration implicit in it. Finally, § 10 presents some conclusions." (pp. 328-329)
(4) Cf. Y. Bar-Hillel, "Bolzano's Definition of Analytic Propositions." *Theoria* 16 (1950), pp. 91-117; p. 100. W. and M. Kneale, *The Development of Logic*, Oxford 1962, p. 367. J. Berg, "Introduction." In: B. Bolzano, *Theory of Science* (ed. by J. Berg; transl. by B. Terrell), Dordrecht 1973, pp. 12-44; p. 18. Coffa, *The Semantic Tradition*, Cambridge, 1991, p. 34.

67. ———. 2010. "The Analytic-Synthetic Distinction and the Classical Model of Science: Kant, Bolzano and Frege " *Synthese* no. 174:237-261
Abstract: "This paper concentrates on some aspects of the history of the analytic-synthetic distinction from Kant to Bolzano and Frege. This history evinces considerable continuity but also some important discontinuities. The analytic-synthetic distinction has to be seen in the first place in relation to a science, i.e. an ordered system of cognition. Looking especially to the place and role of logic it will be argued that Kant, Bolzano and Frege each developed the analytic-synthetic distinction within the same conception of scientific rationality, that is, within the Classical Model of Science: scientific knowledge as cognitio ex principiis. But as we will see, the way the distinction between analytic and synthetic judgments or propositions functions within this model turns out to differ considerably between them."

68. Detlefsen, Michael. 2010. "Rigor, Re-proof and Bolzano's Critical Program." In *Construction. Festschrift for Gerhard Heinzmann*, edited by Bour, Pierre Edouard, Rebuschi, Manuel and Rollet, Laurent, 171-184. London: King's College Publications
"Introduction
The so-called critical movement in nineteenth and twentieth century foundational thinking(1) was described by the American mathematician George Miller (1863–1951) as one in which "[o]ur geometric intuitions are forced into the background" [27, p. 530] as, more and more, "logical deductions from definitions" (*loc. cit.*) take their place.
The main sources of this movement, as both Miller and others described them, were the widely advertised problems concerning geometrical intuition as a guide to our thinking about continuity and differentiability. As mathematicians became increasingly sensitive to the press of these problems, they also "naturally became . . .more exacting in regard to rigor" (*loc. cit*.), and this renewed emphasis on rigor became the central element of nineteenth and early twentieth century attempts to "arithmetize" mathematics.
How the notion of rigor mentioned was conceived and what its principal benefits were taken to be are prime concerns for me here. A better understanding of these matters should contribute to a better understanding of rigor and its motives and benefits overall. Therewith, I believe, should also come a fuller appreciation of the attention given to rigor by nineteenth century foundational thinkers. These at any rate are my chief goals here." (p. 171)
(1) "Critical" was the term that was used by Felix Klein (cf. [20]) and various other writers (cf. e.g. [22]), F. Engel ([8]), J. Merz (cf. [25] and [26]), C. Keyser (cf. [17], [18] and [19]) and G. Kneebone (cf. [24]) to describe the proposals in the nineteenth century that called for the reformation of proof practices in mathematics, particularly analysis.

69. Drozdek, Adam. 1997. "Logic and Ontology in the Thought of Bolzano." *Logic and Logical Philosophy* no. 5:3-18
"Logic and theology were two domains of great importance to Bolzano. His attempt to reconcile the demands of these two domains led Bolzano to very strong logical

realism, or, objectivism, whereby theology could be put on a firm ground. The paper analyzes the problem of objective concepts, propositions, and truths, with an attempt to give an interpretation of these entities, to account for their puzzling ontological status in Bolzano's system.
Bolzano is one of the forerunners of modern logic; however, his logical, and also mathematical, discussions were conducted in the context of very serious concern about the ontological status of the logical constructs. In the context of logic, he discusses the problem of propositions (*Sätze*) and their special category, namely truths; and ideas (*Vorstellungen*), and their special categories, namely intuitions (*Anschauungen*); and concepts. What is interesting in Bolzano's analyses is the considerable effort he devotes to distinguishing subjective propositions and ideas from objective propositions and ideas, the latter also called propositions and ideas in themselves. What is particularly puzzling in Bolzano's philosophy is the ontological status of the latter. According to Bolzano, objective propositions and ideas do not exist, they are not real, and yet they make logic possible." (pp. 3-4)

70. Dubucs, Jacques, and Lapointe, Sandra. 2006. "On Bolzano's Alleged Explicativism." *Synthese.An International Journal for Epistemology, Methodology and Philosophy of Science* no. 150:229-246
Abstract: "Bolzano was the first to establish an explicit distinction between the deductive methods that allow us to recognise the certainty of a given truth and those that provide its objective ground. His conception of the relation between what we, in this paper, call "subjective consequence", i.e., the relation from *epistemic reason* to consequence and "objective consequence", i.e., grounding (*Abfolge*) however allows for an interpretation according to which Bolzano advocates an "explicativist" conception of proof: proofs par excellence are those that reflect the objective order of grounding. In this paper, we expose the problems involved by such a conception and argue in favour of a more rigorous demarcation between the ontological and the epistemological concern in the elaboration of a theory of demonstration."

71. Duhn, Anita von. 2001. "Theoretical Laws and Normative Rules: Kant and Bolzano's Views on Logic." In *Kant und die Berliner Aufklarung. Akten des 9. Internationalen Kant-Kongresses. Band V: Sektionen XV-XVIII*, edited by Gerhardt, Volker, Horstmann, Rolf-Peter and Schumacher, Ralph, 3-12. Berlin: Walter de Gruyter
"Does logic instruct us how to think correctly? If so, what place does methodology have in logic? Is logic an instrument which provides rules for correct thinking or a system of proof for scientific theories, or is the doctrine of method merely an appendix to a doctrine of elements? The question whether logic is an *organon* is related to the question whether logical laws are theoretical truths or normative laws. Kant and Bolzano agree that logical laws basically provide us with truths, but that they can be apprehended as telling us how to think. (1) So a theoretical judgment that something is the case precedes the normative judgment that we may or should do something about it. Does it follow that Kant and Bolzano also agree on the question of whether logic is an *organon* which instructs us how to think? I will show that despite their divergent positions on logic, both authors claim that we apply normative rules because they are true." (p. 3)
(1) Kant and Bolzano agree with Husserl and Frege, who thought that a normative act, such as demanding or permitting, presupposes a theoretical act, such as judging or believing and that every law that states what *is* can be apprehended that one *ought* to think in accordance with it. Cf. Frege (1893) *Grundgesetze der Arithmetik,* intro. XV; Husserl (1900) *Prolegomena,* §§ 3, 13-14. I discuss this issue in *"Is logic a theoretical or practical discipline? Kant and/or Bolzano",* to appear in the *Archiv für Geschichte der Philosophie.* [vol. 84, no. 3 (2002) pp. 319-333]

72. ———. 2003. "Bolzano's Account of Justification." In *The Vienna Circle and Logical Empiricism: Re-evalutation and Future Perspectives*, edited by Stadler, Friedrich, 21-33. Dordrecht: Kluwer

"Bolzano investigated the following problem. How can we determine whether or not a certain truth is basic without recourse to subjective criteria based on intuition or immediate perceptual knowledge? For him, the criterion of self-evidence is not a means for justifying propositions because it does not provide us with a scientific proof presenting the objective reasons for a proposition, reasons that hold independently of our knowledge.(1) Bolzano intended to provide a workable alternative to the criterion of intuitive self-evidence, and claims that we have to search for proof even of self-evident propositions – at least until it becomes clear that and why no proof could be required.(2)
I reconstruct Bolzano's account of justification, which is designed to replace the criterion of self-evidence and provide a scientific basis for the demonstrative sciences. I then argue that although Bolzano succeeded in devising a procedure for grounding truths, his theory fails on the account that it implicitly reintroduces an epistemological problem." (p. 21)
(1) 1804, § 3 (*Betrachtungen über einige Gegenstände der Elementargeometrie* (1804) in Bolzano 's early mathematical works, Czechoslovak Studies in the History of Science, Prague, 1981. Partial English translation by S. Russ in W. Ewald, *From Kant to Hilbert,* vol. 1, OUP, 1996); *Beyträge II* (1810), §§2, 11, 12, 21 (*Beyträge zu einer begründeteren Darstellung der Mathematik* (1810) in *Bolzano's early mathematical works*, op.cit. (*Beyträge*). English translation by S. Russ in W. Ewald, *op.cit*.); 1817 (*Purely analytic proof ...*), § 1; English translation by S. Russ in W.Ewald, *op.cit*; WL IV, §525; (*Wissenschaftslehre* (1837), 4 vols, Aalen, Scientia Verlag,
1981 (WL)) and the *Anti-Euklid*, a manuscript in Bolzano's Nachlass edited by Karel Vecerka, Sbornik, Prague, 1967, pp. 204-215, who dates the text around 1840. Jan Sebestik, however, situates the text closer to 1816.
(2) 1804, §3.

73. Dummett, Michael. 1997. "Comments on Wolfgang Künne's Paper." *Grazer Philosophische Studien* no. 53:241-248
Comments on: W. Künne, *Propositions in Bolzano and Frege* (1997).
"Entertaining and judging (§§ 1,3)
I feel some doubt about Wolfgang Künne's definition (E1)
x is a subjective idea
iff
x is not a judgement *&*
possibly for some *y* (*y* is a judgement and *x* is part of *y*).
A judgement must be some particular person' s judgement, and occur at a particular time. Künne might now be judging that Bolzano misunderstood Kant; but suppose he is not. Are we to say that Künne now has a subjective idea of misunderstanding on the strength of the fact that he might be making that judgement?
The awkwardness arises from the difficulty of fitting (merely) entertaining or grasping a proposition into Künne's Figure 1. A proposition is indeed always something that it is possible to judge; but "*X* entertains the proposition *P*" cannot be defined as "Possibly *X* judges that *P*". Entertaining a proposition has to be acknowledged as a type of mental act in its own right, and as one more generic than judging: one that, like judging, has a proposition as its object (content, matter). Failure to acknowledge this leads to the complications of Figure 5." (p. 241)

74. Etchemendy, John. 1990. *The Concept of Logical Consequence*. Cambridge: Harvard University Press
Chapter 3: *Tarski on Logical Truth*, pp. 27-50.
"Though my concern in this book is not historical, a few preliminary words should be said about the complicated heritage of the model-theoretic definitions of the logical properties. As I mentioned, these definitions are generally credited to Tarski's 1936 article, and for the purposes of this book, there is no need to question this attribution.
What is clearly right about it is that Tarski's article contains the only serious attempt to state, in its most general form, the analysis underlying the standard definitions,

and to put forward a detailed philosophical justification for that analysis. It is, so to speak, the philosophical locus of the model-theoretic definitions.
From a historical point of view, though, attributing the definitions to Tarski alone oversimplifies the situation a great deal.(4) For one thing, most of the main features of the analysis were anticipated, in various different ways, by earlier authors, including Bolzano (1837), Padoa (1901), Bernays (1922), Hilbert and Ackermann (1928), and Gödel (1929). Of all of these, Bolzano's discussion is by far the most extensive; in Chapter 3, I will briefly describe his account and motivate certain features of Tarski's analysis by comparing it with Bolzano's." (p. 7)
(4) For a more detailed discussion of the historical relationship between Tarski's analysis and the model-theoretic definitions, see Etchemendy (1988).
"I approach Tarski's account of logical truth and logical consequence indirectly, by considering first a simpler account developed by Bolzano nearly a century earlier. (1) The two accounts are remarkably similar; indeed, Tarski initially entertains what is, for all intents, precisely the same definition as Bolzano's, but modifies it for reasons I will eventually explain. But in spite of the striking similarity in the two accounts, Tarski was unaware of Bolzano's work until several years after the initial publication of his article. The key difference between the two accounts is simply that Bolzano employs substitution where Tarski uses the more technical, and for the purposes more adequate, notion of *satisfaction*." (p. 27).
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Bolzano, Bernard. 1837. *Wissenschaftslehre*. Sulzbach.
Etchemendy, John. 1983. "The Doctrine of Logic as Form." *Linguistics and Philosophy* 6: 319-334.
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Gödel, Kurt. 1929. *Uber die Vollstandigkeit des Logikkalkuls*. Diss., University of Vienna. Reprinted, with translation, in Godel, *Collected Works*. Volume I. Ed. Solomon Feferman, et al. Oxford: Oxford University Press, 1986.
Hilbert, David, and Wilhelm Ackermann. 1928. *Grundzüge der theoretischen Logik*. Berlin: Springer. Second edition (1938) translated as *Principles of Mathematical Logic*. New York: Chelsea, 1950.
Padoa, Alessandro. 1901. "Essai d'une theorie algebrique des nombres entiers, precedéédune introduction logique a une theorie deductive quelconque." *Bibliotheque du Congres International de Philosophie*, Paris, 1900. Paris: Armand Colin. Translated as "Logical Introduction to Any Deductive Theory." In Jean van Heijenoort, ed., *From Frege to Godel*. Cambridge, Mass.: Harvard University Press.

75. Fine, Kit. 2022. "Some Remarks on Bolzano on Ground." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin. New York: Oxford University Press
"Bolzano takes the notion of grounding to be *factive*, so that grounds and consequences are always true propositions. But he also acknowledges that we sometimes invoke a non-factive notion of grounding.
Kit Fine discusses how Bolzano proposes to understand such a notion and critically assesses Bolzano's proposal from the perspective of the recent debate about grounding and its logic." (p. 37)

76. Føllesdal, Dagfin. 1981. "Comments on Quine." In *Philosophy and Grasmmar: Papers on the Occasion of the Quincentennial of Uppsala University*, edited by Kanger, Stig and Öhman, Sven, 29-35. Dordrecht: Reidel
Comments on W.V.O. Quine, *Grammar, Truth, and Logic*, same volume, pp. 17-28.
"I shall now comment on some points in the paper which, it seems to me,would be well worth discussing by our group. I will concentrate on the following three points:
1. Logical particles.
2. Syntactic ambiguities.

3. Demonstratives." (p. 29)
(...)
"One hundred and fifty years ago, Bolzan02 was the first to have the idea of demarcating logic the way Quine does with the help of a set of logical particles which are held constant, while the other non-logical expressions are freely substituted for one another. However, Bolzano's idea received little attention until it was rediscovered afresh in the mid-thirties by Quine and Ajdukiewicz(3) independently of one another. All the basic ingredients are there in Bolzano: the steps that Bolzano goes through are the same as Quine's and in the same order:
1. Specify a vocabulary of logical particles.
2. Define what it means for two expressions to have the same logical form:
Two expressions have the same logical form if they can be obtained from one another by the substitution of non-logical expressions for non-logical expressions.
3. Define logical truth:
A sentence is logically true if and only if all sentences with the same logical form are true." (pp. 29-30, a note omitted)
(3) Quine, W. V., 'Truth by Convention', in O. H. Lee (ed.), *Philosophical Essays for A. N. Whitehead* (Longrnans, New York, 1936). Reprinted in W. V. Quine, *The Ways of Paradox* (Random House, New York, 1966), and in various other places, including Herbert Feigl and Wilfrid Sellars (eds.), *Reading in Philosophical Analysis* Appleton-Century-Crofts, New York, 1949). Ajdukiewicz, Kazimierz, 'Sprache und Sinn', *Erkenntnis* 4 (1934), 100-138.

77. ———. 1997. "Bolzano's Legacy." *Grazer Philosophische Studien* no. 53:1-11
Original German published as: *Bolzanos bleibende Leistungen* in: Arkadiusz Chrudzimski and Wolfgang Huemer (eds.), *Phenomenology and Analysis. Essays on Central European Philosophy,* Frankfurt: Ontos Verlag, 2004, pp. 57-68.
Abstract: "Bemard Bolzano (1781-1848) was an original and independent thinker, who left a lasting legacy in several areas of philosophy. Four such areas are singled for special attention: political philosophy, ethics and theology, logics and semanties, and mathematics. In all these areas he was far ahead of his time. He had pioneering ideas in political philosophy and in ethics and philosophy of religion, and he argued for them in a brilliantly clear way. In logic and semantics he anticipated Frege, Camap and Quine on important points, and he had intriguing, yet to be explored, ideas on intuition and other fundamental philosophical notions. In the foundations of mathematical analysis and the theory of infinite sets he anticipated Weierstrass and Cantor."

78. ———. 2001. "Bolzano, Frege and Husserl on Reference and Object." In *Future Pasts: The Analytic Tradition in Twentieth Century Philosophy*, edited by Floyd, Juliet and Shieh, Sanford, 67-80. Oxford: Oxford University Press
"Bolzano was a main influence on the development of Husserl's phenomenology. Husserl gives generous credit to Bolzano in several of his works and refers to him frequently. Husserl first came across Bolzano when, barely twenty, he read *Paradoxien des Unendlichen*(2) during his studies with Weierstrass in Berlin. And he renewed this acquaintance with *Paradoxien des Unendlichen* in 1884-1885 when he followed Brentano's lectures in Vienna on "Die elementare Logik und die in ihr notigen Reformen."
But it was only later, in the mid-1890s, that Husserl started serious study of Bolzano's *Theory of Science*,(3) which he earlier had regarded as "strange" ("fremdartig"). Husserl had then decided to give up work on the second volume of the psychologistic *Philosophy of Arithmetic* (1891) and had started working on what was to become his first phenomenological work, the *Logical Investigations* (1900-1901). Husserl states that he came to appreciate Bolzano, and in particular his theory of propositions (*Satze an sich*) and representations (*Vorstellungen an sich*), through studying Lotze's interpretation of Plato's theory of ideas. Husserl interpreted Bolzano in a platonistic manner, which Husserl claimed—I think unjustly—was foreign to Bolzano ([*Husserliana*] XXII, *Aufsätze und Rezensionen* (1890-1910)] p. 130)." (pp. 67-68)

(2) Bernard Bolzano, *Paradoxien des Unendlichen*, ed. F. Prihonsky (Berlin: Mayer and Miiller, 1889; originally published 1851).
(3) Bernard Bolzano, *Theory of Science,* abridged, ed. and trans. Rolf George (Berkeley: University of California Press, 1972, originally published 1837).

79. Fossati, Lorenzo. 2019. "Neither Aristotle nor Kant. Bernard Bolzano on Categories." In *Categories: Histories and Perspectives 2*, edited by D'Anna, Giuseppe and Fossati, Lorenzo, 77-94. Hildesheim: Georg Olms
"The second Book of the *Wissenschaftslehre*, the *Elementarlehre* (Theoryof Elements) is divided into four Parts; the first one is dedicated to the ideas in themselves (it is in the first volume of *Wissenschaftslehre*, the second volume take into account propositions in themselves, true propositions and inferences). This first Part, which includes §§ 46-120, is divided into four Chapters and introduces the notion of objective representation (often indicated as "idea"), its internal attributes and the distinction between the representations on the basis of their interrelation and of their relation to other objects (WL I: 214--571).
Each paragraph is followed by some notes where Bolzano appeals to ancients and modems to point out his own theses. At the end of the Chapter on ideas in themselves he adds a further Appendix (§§ 115-120) entitled "Previous Treatments of the Subject Matter of this Part," which helps better point out the big picture.
In particular, two paragraphs are devoted to categories-§ 118 to the categories of the "ancients" and § 119 of the "modems." He thus underlines his willingness to investigate any aspect and to involve all different kinds of interlocutors, but first and foremost his constant necessity to confront Aristotle and Kant." (pp. 77-78, notes omitted)

80. Fraisopi, Fausto. 2014. "The Quasi-Ontology of "An-Sich". Bernard Bolzano's *Theory of Science* between Leibnitian *Ars Combinatoria* and the Husserlian Idea of *mathesis universalis*." *Avello Publishing Journal* no. 4:1-25
Abstract: "Starting from the critical position that Husserl assumes against Bolzano and hisidea of *mathesis universalis*, this paper focuses and emphasizes Bolzano's project for a *mathesis* and the differences between this project and Leibniz's. Putting into an historical perspective these three forms of *mathesis*, by Leibniz, Bolzano, and Husserl, we / I open in so doing a theoretical perspective concerning the nonontological dimension of idealities they form and articulate *mathesis* as such. The an-ontological Combinatorics of propositions and of ideas in themselves, suggests, Bolzano maintains, the possibility of a treatment of Combinatorics independently from these ontological and metaphysical presuppositions that formed and structured the Leibnitian *ars combinatoria*. In this sense, the philosophical position of a "semantic Platonism," assumed by Bolzano, opens the perspective of a non-metaphysical but modular *mathesis* that we can articulate and widen beyond an ontological commitment."

81. Franks, Curtis. 2014. "Logical Completeness, Form, and Content: An Archaeology." In *Interpreting Gödel: Critical Essays*, edited by Kennedy, Juliette, 78-106. Cambridge: Cambridge University Press
§ 2: *Bolzano's question*, pp. 81-92.
"Bernard Bolzano engaged in the profound study of two distinct notions of logical consequence over several decades in the early nineteenth century.
The work most remembered and highly regarded by modern logicians, because of its striking resemblance to twentieth century set-theoretical definitions of consequence, concerns the Ableitbarkeit ("derivability") relation.
In his 1837 masterpiece, Wissenschaftslehre, Bolzano in fact defines a network of concepts – validity, compatibility, equivalence, and derivability – in terms of one another in a way very similar to contemporary presentations. Here is his definition of the last of these: [Wissenschatslehre, § 155, text omitted]." (p. 81)
"Bolzano's two theories of logical consequence are themselves not precise enough for their correspondence with one another to be subject to proof. All the same, the question is at the center of Bolzano's thought.

The procedural *Ableitbarkeit* relation provides a calculus of inference.
The ontological *Abfolge* relation is a feature of the world absolutely independent of our ability to reason about it. By establishing that these notions correspond, we would ensure that the logical structure of the world is accessible, that some line of thought could trace the dependencies of truths, that the reasons behind the complex facts of reality are discoverable and comprehensible." (p. 92)

82. George, Rolf. 1961. *The Problems of the Infinite and the Continuum in Some Major Philosophical Systems of the Enlightenment*, Michigan State University
Unpublished Ph.D thesis, available at Michigan State University, Digital repository.
Contents: Introduction 1; Chapter I: Leibniz 19; Chapter II: Berkeley 63; Chapter III: Bayle 111; Chapter IV: Kant 133; Chapter V: Bolzano 192; Conclusion 215; Bibliography I-V.
"The philosophers discussed in this dissertation are Leibniz, Berkeley, Bayle, Kant, and Bolzano. Its aim is to show that certain difficulties connected with infinite and continuous sets were recognized by these philosophers, and that their systems were, at least in part, designed in such a way that these difficulties did not arise in them.
(...)
Bolzano was the first to realize that the so-called Paradox of Galileo is no paradox at all, but simply describes a common property of all infinite sets.
As concerns the constitution of continua the problem was that neither the assumption that a continuum ultimately consists of unextended parts, nor that it consists of extended parts seemed defensible. Against the former case it was argued that unextended parts, no matter how many, cannot make a finite extension, against the'latter that extended parts are not ultimate, but are further divisible. Bayle held that none of the logical alternatives are defensible, so that no one need bother to change whatever opinion he happens to have on the subject.
(...)
Bolzano declared that in a continuum every point has a neighbor within any distance, no matter how small. This definition, although ultimately unsatisfactory, proved to be of great help in discovering various important properties of continuous sets." (Fom the Abstract)

83. ———. 1972. "Enthymematic Consequence." *American Philosophical Quarterly* no. 9:113-116
"Enthymemes were traditionally defined as incomplete or incompletely stated syllogisms.
Arguments of this sort, though formally invalid, must be allowed to have some merit, and although the restriction to syllogisms is undesirable, the definition at least has the advantage of precision.
(...)
I shall argue in this paper that, while it is true that enthymematic arguments can be augmented so that valid arguments result, it is not wise to define enthymemes in these terms. I shall instead give a definition of enthymematic consequence which is similar to Tarski's definition of logical consequence; one can even arrange matters so that the latter becomes a limiting case of the former.
The definition can then be used to generate additional premisses which will convert enthymematic arguments into logically valid ones. It will thus automatically provide the desired restriction upon missing premisses.
I shall then show that the definition gives the same results as the traditional account within the domain of syllogisms, and that outside this domain it singles out a class of invalid but plausible arguments which seem to answer to many logicians' intuition of what an enthymeme is, if we can take the examples in their textbooks as a clue." (p. 113)
(...)
"It remains to give a logician his due who more than a hundred years ago propounded a theory of logical consequence which in one definition accounted for both logical and enthymematic validity: Bernard Balzano. He defined consequence thus: "I say that propositions *M, N, O* ... follow from Propositions *A, B, C, D*, ...

with respect to the variable parts *i, j*, ... if every class of representations whose substitution for *i, j*, ... makes all of *A, B, C, D*, ... true also makes all of *M, N, O*, ... true."(6) It has been pointed out that Balzano anticipated Tarski by almost exactly a hundred years in his definition of logical consequence.(7) Indeed,
if the variable parts *i, j,* ... are taken to consist of all and only the extralogical terms of *A, B, C, D*, ... , *M, N, O*, ... the definition is close to that of Tarski (though Bolzano demands that the premisses be consistent). Cases where *i, j,* ... include more or fewer than the extralogical terms were generally regarded as somewhat quaint. In particular, it has not been seen that cases of Bolzano-entailment where the class of "variable" terms is smaller than the class of extralogical terms are just those argument forms which we are wont to call enthymemes." (p. 116)
(6) Bernard Bolzano, *Wissenschaftslehre*, Vol. 2 (Sulzbach, 1837), p. I 14.
(7) E.g., by Heinrich Scholz.

84. ———. 1983. "Bolzano's Consequence, Relevance and Enthymeme." *Journal of Philosophical Logic* no. 12:299-318
"Historians of logic tend to view their task as the application of modern insights and symbolic techniques to old texts. Perhaps they do this on the assumption that what is good in these works must be an adumbration of what was recently done and is now well known. This holds, at any rate, for most discussions of Bolzano's theory of logical consequence.
In the present paper I shall reverse this procedure and comment on some problems and beliefs of contemporary logic from what I take to be Bolzano's point of view. This will have the advantage of bringing out more forcefully than a straight exegesis what his view was and will also, I hope, put in doubt certain contemporary dogmas.
I begin by applying his definition of consequence to propositional logic. Bolzano did not entertain this branch of logic, and to this extent my account is ahistorical. That it is, nonetheless, a straight extension of his theory is shown by the fact that all 23 theorems about consequence which he proves in his *Theory of Science* hold in this application I then consider how C. I. Lewis's so-called "independent proof" for *A* & -A |= B fares in this system (it fails). After some comments on the proof, I show that in Bolzano-consequence premisses and conclusion share a subsentence (a necessary condition of relevance). There follows a discussion of enthymemes and a general procedure for generating the so-called "nutting premiss". At the end I sketch a taxonomy of consequence relations and briefly remark on earlier interpretations of Bolzano's work. In using the first person plural (from now on) I mean to speak for those who think Bolzano's approach sound, a group that includes at least Bolzano and myself." (p. 299, notes omittred)

85. ———. 1983. "A Postscript on Fallacies." *Journal of Philosophical Logic* no. 12:319-325
"Bolzano is justly esteemed for his opposition to psychologism in logic. It is most fitting, therefore, that his definition of consequence has enabled us to strike a blow at the residual psychologism that is found in the customary treatment on enthymemes.(1) We shall now do the same for the so-called formal fallacies." (p. 319)
(1) See section (9) of the preceding essay. [*Bolzano's Consequence, Relevance and Enthymeme*]

86. ———. 1986. "Bolzano's Concept of Consequence." *Journal of Philosophy* no. 83:558-564
Reprinted in: Dale Jacquette (ed.), *Philosophy of Logic: An Anthology*, Malden: Blackwell, 2002, pp. 205-209.
"Plainly, to identify a speech as an argument and to understand its premises and conclusion is not the same as knowing what argument is intended. What is missing? Bernard Bolzano defines the concept of consequence thus:
Propositions *M, N, O*, . . . follow from propositions *A, B, C, D*, . with respect to variable parts *i, j,* . . . if every class of ideas whose substitution for *i, j,* . . . makes

each of *A, B, C, D,*. . . true also makes all of *M, N, O*, . . . true.(1)
The *i, j* . . . are constants tagged for substitution; I shall call them *variands*." (p. 558)
(...)
"The conception of consequence here adumbrated has two features that should recommend it to logicians who are concerned not with the development of formal systems, but with the analysis of infor- mally stated arguments and the identification of fallacies. The first of these is that arguments of invalid form are invalid. In the classical view, this is not the case, as Gerald Massey has pointed out with clarity and vigor.(3)
(...)
In Bolzano's view, the evaluation of any argument must begin with the identification of variands. If their variation generates an invalid form, the argument is invalid; if not, not. It is of course possible to make mistakes in this, just as sentences can be misunderstood. It is a cultural, and perhaps even a human, failing that we do not usually indicate the variands explicitly. But these are problems of communication. Plainly, it is often possible, and sometimes important, to identify formal fallacies. It therefore seems that in this respect Bolzano's account of consequence is superior to the classical. A second positive feature of Bolzano's conception is that it gives a promising account of enthymemes. Although he concentrates on arguments in which all indexical elements are variands (this being the proper province of logic, cf. WL § 223), his definition does not exclude cases in which only some of them are. We readily identify 'Socrates' as the variand in 'Socrates was a man, therefore Socrates was mortal'. That is, we understand this argument as implicitly claiming that every substitution on 'Socrates' that makes the premise true also makes the conclusion true. If we had to construct a device for computing the "missing premise" (which we intuitively take to be 'All men are mortal'), we would have it state that fact. It would, that is, form the universal closure on the variand, over the conditional consisting of premise and conclusion, and voila, the missing premise results. This procedure works for all syllogistic enthymemes, and is only slightly more complex when no singular terms are involved. No principle of charity or other proviso is needed. I venture the guess that some such computation is going on even in our own minds when, with a speed that must compel wonder, we determine what all the world takes to be the missing premise in such a case."
(1) *Wissenschaftslehre* (Sulzbach, 1837), § 155, no. 2, vol. ii, pp. 199 ff. Translated as *Theory of Science*, R. George, ed. (Oxford: Blackwell, 1972), p. 209. Henceforth WL.
(3) "The Fallacy behind Fallacies," in P. A. French, T. E. Vehling, Jr., and H. K. Wettstein, eds., *The Foundations of Analytic Philosophy* (Minneapolis: Minnesota UP, 1981), pp. 499 ff.

87. ———. 1987. "Bolzano on Time." *Philosophia Naturalis* no. 24:452-468
"In the first volume of the *Wissenschaftslehre* Bolzano claims that "by the word 'time' we mean nothing but that particular determination in a real thing which is the condition for correctly attributing to it a given property."(1) He says that from this *all* properties of time can be deduced. This is supported by just one example, namely, that several contrary properties can be attributed to the same substance only on condition that times differ. This follows directly, since sentences with contrary predicates can be true only if their subjects differ. Hence one and the same substance can have contrary attributes only on the assumption that its time determinations are not the same.
In Chapter 412 he maintains that a theory can have the status of a science even if its extent is very small. Consequently, he says, "the theory of time (the properties of time, not of the art of measuring it) deserves to be treated as a special science (i.e. the pure theory of time) although this science can consist of only a very few propositions."(2) Kant, he objects, should not have denied it the name of science for no other reason than its small extent.

In the following chapter Bolzano adds that a theory need not be denied the status of a science even if everyone already knows its propositions. Again the theory of time serves as an example. He maintains that all theorems of the pure theory of time are obvious to everyone *(sind jedem von selbst schon bekannt)* (3) , but that it should be considered to be a science nonetheless.
These are sweeping claims. Given the voluminous publications, the many controversies and the continuing interest in the subject of time they seem strange, even absurd. I begin by discussing these assertions, then add some reflexions on Bolzano on *time perception,* and end with a brief account of his criticism of Kant's views." (p. 452)
(1) *Wissenschaftslehre* I, 365. Citations follow the first edition.
(2) IV, 52.
(3) IV, 53.

88. ———. 1992. "Concepts of Consequence." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 3-26. Firenze: Leo S. Olschki
"It has been held since antiquity that in all deductive argumentation there is a formal element or aspect. I wish to distinguish, and contrast, two ways of characterizing this. One of them I call «logic of schemata», or the «Received View», and the other, which was first articulated by Bolzano, «logic of variation». I shall investigate how these concepts of consequence succeed in addressing five concerns, not all of them logical issues, as we now understand logic, but connected with argumentative practice and certain epistemic matters.
(1) For the sake of completeness I mention first that a definition of consequence should fix a relation that satisfies certain formal requirements, i.e. a cut rule, thinning, and the like. There is a conventionally accepted set of these, described, e.g. by Gentzen. If a consequence relation shows deviations from this, it must be a reasoned difference that should be argued for. Also, a consequence relation (specifically logical, rather than enthymematic consequence) should be defined in such a way that first order predicate logic is strongly complete, that is, that if A is a consequence of a set of sentences X, then A should be deducible from X in a finite sequence of steps.
(2) A defensible definition of consequence should have the form, broadly, «If an argument satisfies this definition, it is valid, otherwise not». Contemporary definitions fail, as a rule, to satisfy the «otherwise not» clause. It is, however, argumentative practice to convict arguments of being formally fallacious. This can only be based on the assumption that if we have *fully understood* an argument, we can judge it to be valid or invalid - setting aside such esoterica as undecidable cases. I think it desirable that a definition of consequence allow an account of invalidity as well as validity.
(3) I shall consider a definition of logical consequence to be superior if is it broad enough to explain why we concede merit to some formally invalid arguments enthymemes), but withhold approbation from others (gross non-sequiturs), that is, if it treats *logical consequence* as a special, though perhaps the most important and interesting, case.
(4) Arguments as presented in both informal and formal contexts can be ambiguous, even if they are constructed of unambiguous sentences, and even if they are couched in a language that stipulates a rigid distinction between logical and extralogical constants. I call an argument *naked* if all that is presented are premisses, conclusion, and an inference indicator, like «therefore». I shall maintain that when we understand an argument, we understand more than the sentences of which it is composed, and more than the unspecified claim that the conclusion *somehow* follows from the premisses. That is, we grasp more than the naked argument. If we fail in this, we may misconstrue arguments, which amounts to saying that naked arguments can be ambiguous. I suggest that an acceptable theory of consequence should allow us to bring into focus the problem of argument ambiguity.

(5) It is desirable that a concept of consequence, if it does not itself define a «relevant» relation, can at least be augmented so that it does. (A consequence relation is here called relevant if it stipulates or implies that premisses and conclusion share some element)." (pp. 3-4)

89. ———. 1997. "Psychologism in Logic: Bacon to Bolzano." *Philosophy and Rethoric* no. 30 (3):213-242
Reprinted in: Dale Jacquette (ed.), *Philosophy, Psychology, and Psychologism. Critical and Historical Readings on the Psychological Turn in Philosophy*, Dordrecht: Kluwer, 2003, pp. 21-49.
"The first logician to conceive of logic as a matter wholly apart from psychology was Bolzano. He did, however not neglect the old concerns. Of the four volumes of his *Wissenschaftslehre* only the first two (and not all of them) deal with the objective world of propositions in themselves. The third is epistemology, dealing with the manifestation of propositions in the mind: a judgment, in contrast to a proposition, which is abstract and mind independent, now is a mental episode whose "matter" is a proposition in itself. In this part of the work he discusses all those issues that tended to be mixed into the discussion of logic itself: clarity and obscurity of representations, knowledge and error, as well as the "art of discovery" which now has its proper place as a part of epistemology. The last volume, finally, is given over to the presentation of a science in the form of a treatise of the subject. This is the old "methodology", the theory of combining discovered truths into the system of a science." (p. 39 of the reprint)
(...)
"I hope to have clarified in this paper at least some of the strands of psychologism that ran through the history of logic between Bacon and Bolzano. Much had to be left out. My thesis — if I may be said to have argued one — has been that there were different kinds of intrusion of psychology into logic, some due to a conception of logic that included much of what is now assigned to other fields, others due to cultural and ideological persuasions, and still others to the obsession that logic is the science of thinking." (p. 44 of the reprint)

90. ———. 1997. "Bolzano's Programme and Abstract Objects." *Grazer Philosophische Studien* no. 53:167-180
Abstract: "Most of the Bolzano literature is exegetical, neglecting, unfortunately, the great potential of his logic as the beginning of a *Programme*. Specifically, his unorthodox construal of the consequence relation as triadic, and his account of logical form are promising beginnings which even as they stand shed light on question of relevance, the ancient problems of enthymemes and others. Instead of developing these suggestions, Bolzano scholars have been occupied with elucidating the ontology of sentences in themselves, and related topics. I argue, and believe to be in agreement with Bolzano, that the nature of sentences is fully explained by the relations that hold between them, just as money has no nature or essence beyond the transactions it makes possible. It follows that the development of his logic would contribute at least as much to the understanding of sentences than any exegesis."

91. ———. 2003. "Bolzano and the Problem of Psychologism." In *Husserl's Logical Investigations Reconsidered*, edited by Fisette, Denis, 95-108. Dordrecht: Kluwer
"As we saw, the view that subjective ideas are parts of judgments was not new, but Bolzano's theory of objective contents allowed him to avoid a certain confusion. It was generally acknowledged that ideas pass through the mind when one thinks, i.e. judges. At the same time they were thought to be sensations, or the copies of sensations, that is, visual or auditory sense data.
They were often described in terms not consistent with their roles as terms of judgments, i.e. as extended, round, moving, receding, as semblances of their objects, etc. (cf. Exner, supra) [*]. But mental occurrences of this sort cannot be terms of judgments. Hume, for example, claimed reasoning to be the operation of

our thoughts and ideas, but it is not very plausible to think of it as an operation on something that can be blue, round, divided, or point-like.
Bolzano was not a victim of that confusion. For him a subjective idea is part of a mental proposition or at least could be such a part, and must have the character that goes with this role. He concentrates on the logical functions of ideas, thus avoiding certain classical mistakes. In particular the view that knowledge consists in the similarity or resemblance between our ideas and their objects is exposed as fallacious. Terms of propositions refer to their objects, they need not resemble them. The truth of a proposition, and hence our knowledge of an object, does not depend upon the similarity between idea and object. Rather, "a proposition is true if we connect with the idea of an object the idea of an attribute which this object actually has" (WL §42). This rejection of the resemblance theory is not based on the classical argument that we can never know whether our ideas resemble their objects since we can never compare the two, the object being altogether inaccessible. Rather, the critical point is that it is of no consequence whether an idea resembles its object." (pp. 105-106)
(...)
"Bolzano had a very generous conception of the scope of logic, which for him included a logic of discovery, epistemology and a lot of communication theory. He insisted that logic in this broad sense needed to make use of psychological theory. However, the Theory of Elements in the first two volumes of WL on which "logic as a science must be built" (Husserl) is a historical first in avoiding all connection with psychological doctrine." (p. 108)
[*] Bernard Bolzano, *Letter to Franz Exner,* 18th December 1834, in: *On the Mathematcal Method and Correspondence with Exner*, Amsterrdfam: Rodopi 2004, pp.157-174.

92. ———. 2004. "Intuitions—the Theories of Kant and Bolzano." In *Semantik und Ontologie. Beiträge zur philosophischen Forschung*, edited by Siebel, Mark and Textor, Mark, 319-354. Frankfurt: Ontos Verlag
"Bolzano credits Kant with impressing on the philosophical public the distinction between intuition (*Anschauung*) and concept (*Begriff*). But making the distinction is one thing, explaining it is another. Bolzano is not happy with Kant's account (*WL* I, § 77),(1) but his critique does not connect well with Kant's theory. The gulf between them, in both substance and terminology, is too deep. Despite the divergence between the two philosophers on almost any topic, Bolzano paid more attention by far to Kant and Kantian logicians than any other tradition or school, for good reasons."
(...)
"Mathematical propositions are purely conceptual, and so intuitions will play no role in their proof or analysis. They can be established a priori because they are purely conceptual. Bolzano's theory of intuition supports this profoundly important tenet of his thought. His redefinition of "Anschauung" was thus not merely an exercise in persuasive definition, and the appropriation of a popular and important expression for different purposes.
According to Bolzano (and in truth, I might add) there are no such things as Kantian intuitions. Bolzano's construal of the word, whatever its shortcomings, certainly removes the temptation to seek geometrical and arithmetic truth in intuitions, yet preserves the root connotation that *Anschauungen* are those thought episodes that represent our direct empirical awareness." (p. 35)
(1) Bolzano 1837. The *Wissenschaftslehre* is cited as *WL* plus number of volume.

93. Gieske, Carsten Uwe. 1997. "Bolzano's Notion of Testifying." *Grazer Philosophische Studien* no. 53:249-266
Abstract: "The notion of testifying (or testimony) is the central notion of Bolzano's theory of communication. In his Wissenschaftslehre (Theory of Science) Bolzano gives an analysis of this notion. It shows surprising parallels to Paul Grice's attempt to define "A meantNN something by x". I will begin with an explanation of some parts of the analysis and continue with an investigation of the relationship between

Bolzano's analysis and that of Grice. In conclusion I would like to present some evidence supporting the hypothesis that several of the virtues of Grice's theory had already been developed by Bolzano, whose approach even has the advantage of a better definition than Grice's, as Bolzano's analysis provides a better basis for defining a notion of successful communication of information."

94. Grossmann, Reinhardt. 1961. "Frege's Ontology." *Philosophical Review* no. 70:23-40
Reprinted in: E. D. Klemke, *Essays on Frege*, Urbana: University of Illinois Press 1968, pp. 79-98.
On Bolzano see pp. 23-27.
"I begin by describing some features of Bolzano's *Wissenschaftslehre*, for much of what I shall have to say about Frege can best be understood against the background of Bolzano's view.(4) According to Bolzano, all things are of one of three kinds: First, there are different kinds of mental states (*subjective Vorstellungen*), namely, (a) individual ideas (*subjective Einzelvorstellunge*n), (b) general ideas (*subjective Allgemeinvorstellungen*), and (c) thoughts (*gedachte Saetze*). Things of these three kinds are supposed to exist in individual minds; in this respect they are "subjective" rather than "objective."(5) Second, there are so-called objects$_1$ (*Gegenstaend*e), namely, (a) individual things and (b) properties (*Beschaffenheiten and Relationen*). These things are not in any individual mind, but exist independently of minds and are therefore "objective" rather than "subjective."(6)
Third, there are senses (*objective Vorstellungen*), namely, (a) individual concepts (*objective Einzelvorstellungen*), (b) general concepts (*objective Allgemeinvorstellungen*), and (c) propositions (*Saetze an sich*). These things differ from mental states in that they are as "objective" as objects$_1$. But they also differ from the latter. One important difference is that they are more closely connected with mental states than are objects$_1$. (7)" (pp. 23-24)

(4) Bolzano, *Wissenschaftslehre* (new ed., 4 vols.; Leipzig, 1929). Compare also Y. Bar-Hillel, "Bolzano's Definition of Analytic Propositions," *Methodos*, II (1950), 32-55; and H. R. Smart, "Bolzano's Logic," *Philosophical Review*, LIII (I944), 513-533.
(5) *Wissenschaftslehre*, I, 77, 99, 219.
(6) *Ibid*., pp. 219-222, 331, 378-387.
(7) *Ibid*., pp. 2I6-2I8.

95. Hafner, Johannes. 2000. "Bolzano's Criticism of Indirect Proofs." *Revue d'Histoire des Sciences* no. 52:385-399
Abstract: "The bearing of *Ableitbarkeit* and the compatibility requirement on the possibility of indirect proofs in Bolzano's logic has frequently been misconstrued. Without additional assumptions concerning the logical structure of indirect proofs and the relationship between proofs and *Ableitbarkeit* the compatibility requirement does not in general preclude indirect proofs. Bolzano's own objections to them are raised in the context of *Abfolge*, not *Ableitbarkeit*. Closer inspection shows that there are in fact two distinct criticisms in play. Identifying and analyzing them clarifies what exactly Bolzano views as the problem of indirect proofs."

96. Hale, Bob, and Wright, Crispin. 2015. "Bolzano’s Definition of Analytic Propositions." *Grazer Philosophische Studien*:325-364
Abstract: "We begin by drawing attention to some drawbacks of what we shall call the Frege-Quine definition of analytic truth. With this we contrast the definition of analytic propositions given by Bolzano in his *Wissenschaftslehre*.
If Bolzano’s definition is viewed, as Bolzano himself almost certainly did not view it, as attempting to capture the notion of analyticity as truth-in-virtue-of-meaning which occupied centre stage during the first half of the last century and which, Quine’s influential assault on it notwithstanding, continues to attract philosophical attention, it runs into some very serious problems. We argue that Bolzano’s central idea can, nevertheless, be used as the basis of a new definition which avoids these problems and possesses definite advantages over the Frege-Quine approach. Our

itle notwithstanding, we make no claim to contribute to the exegesis of Bolzano's thought and works, which we must leave to those more expert in these matters than we are. Naturally, we have done our best not to misrepresent Bolzano's views, and believe we have avoided doing so. But it bears emphasis that it is no part of our intention to suggest that the modifications to his definition which we propose would have had any appeal for him, or that he had, or would have had, any sympathy with the project which motivates them."

97. Haller, Rudolf. 1992. "Bolzano and Austrian Philosophy." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 191-206. Firenze: Leo S. Olschki
"It would be fruitful to compare in detail some of the formulations in Twardowski, Husserl, Meinong, Mier, and Kerry, with the original work of Bolzano, a task which cannot be done here. That we cannot rely in all cases on a clear-cut causal relation from reading Bolzano to the adoption of his arguments may not wonder us. To speak about an entire tradition is always a tricky thing, since traditions are not easily to be identified. But if we may use the expression `tradition' then part of a philosophical tradition is that its main tenets recur in different writings and the same or at least similar methods are applied. The fact, however, that even the philosophers of the Vienna Circle claimed to be part of this tradition has been overlooked for a long time. After all, *logical empiricism* was only one of the labels they accepted. Neurath's preferred name «rational empiricism» is somewhat nearer to what was the significant principle of Austrian philosophy. It was the attempt to base the system of science on an ontology of objects. For both fields the tradition starting with Bolzano provided a good basis to build up a philosophical program. To investigate how many of the philosophers of this tradition came to similar conceptions under an influence of Bolzanoan ideas *without a wider knowledge of his work* and to *explain,* how at the same time we find a strong impact of this conception in different philosophers will remain a task for further research." (pp. 205-206).

98. Jaray, Kimberly. 2006. "Reinach and Bolzano: Towards A Theory of Pure Logic." *Symposium.Journal of the Canadian Society for Continental Philosophy* no. 10:473-502
"The work of Adolf Reinach (1883-1917) on states of affairs, judgment, and speech acts bears striking similarities to Bernard Bolzano's work in the area of general logic. It is my belief that these similarities suggest that Reinach used Bolzano's logical work to assist with his own. Three considerations support this view. First, Bolzano's work in *Die Wissenschaftslehre* (Theory of Science) was considered by Husserl to be the necessary foundation for any work in logic. Second, Bolzano's logic was a suitable alternative to Immanuel Kant's in that he formulated his essential relations as inexistent yet real, not Platonic or belonging to a transcendental realm. Third, Reinach did not openly criticize Bolzano in the manner he did the Austrians of the Brentano school, suggesting that Bolzano's logic was more complementary with his own. Due to his untimely death in 1917, Reinach's work on states of affairs and logic remains incomplete, some of it even lost or destroyed. I shall here offer a few brief remarks about Husserl as he was Reinach's mentor and friend, but an in depth discussion of the differences between Reinach and Husserl will not be offered in this paper. Secondary literature tells us that Reinach admired Husserl's *Logical Investigations*, in which phenomenology was said to concern itself with "primarily the discovery of the terra firma of pure logic, of the *Sachen* (things) in the sense of objective entities in general and of general essences in particular," and further "this phenomenology must bring to pure expression, must describe in terms of their essential concepts and their governing formulae of essence, the essences which directly make themselves known in intuition, and the connections which have their roots purely in such essences." These acts of discovering and describing essences or things themselves became the foundation of Reinach's realist ontology: things themselves surround us in the world and our access to them does not require a transcendental turn. It was precisely this

realist foundation that allowed Reinach to develop and extend his phenomenological work to logic, legal philosophy, and speech acts as well. This conception of the nature and goal of phenomenology allowed Reinach and other phenomenologists a manner in which to analyze experience with its essential connections without either falling prey to psychologism or resorting to Platonism: phenomenology for them was truly a realist alternative." (p. 473)

99. Kasabova, Anita. 2002. "Is Logic a Theoretical or Practical Discipline? Kant and / or Bolzano." *Archiv für Geschichte der Philosophie* no. 84:319-333
"Does logic describe something or not? If not, is it a normative or practical discipline? Is there a radical division between the practical or normative level and the theoretical or descriptive level? A discipline is theoretical, we may say, if its main propositions contain descriptive expressions, such as "is" or "have", but no normative expressions, such as "ought", "ought not" or "may". A discipline is normative if its main propositions are of the form "it ought to be". Theoretical propositions express what is, whereas practical propositions express what should be. So a theoretical discipline is descriptive and a normative discipline is prescriptive, but what does a theoretical discipline describe?
According to one view, logic is only theoretical and only describes how things are. Logic as a purely theoretical discipline can then be said to be about mental or linguistic activities, or about non-temporal entities and their non-natural connections, such as entailment or derivability. The practical alternative of this purely theoretical view is that logic is only a practical discipline. Its propositions tell us how we may, should or should not judge and reason. Logic as a normative discipline states norms for human activities. According to another view, logic is primarily a theoretical discipline and its counterpart says that logic is primarily a practical discipline. Yet another view of logic says that it can be conceived as both theoretical and practical." (p. 319).
"Which view of logic does Bolzano take? Whereas Husserl insists on delineating a separate pure logic, Bolzano's Theory of Science combines theoretical and practical logic. Unlike Husserl and contrary to Kant, Bolzano claims that logic as a theory of science, must have both a theoretical and a practical character. Bolzano's wide understanding of logic as a Wissenschaftslehre or doctrine of how to present sciences (WL I, § 1) extends to epistemology and methodology, including didactic and methodological rules for classifying and teaching the sciences. These latter are collections of truths (WL I, § 1) and it is the practical task of a theory of science or logic to direct our acquaintance with these collections of true propositions. Bolzano even claims that logic in this wide sense is essentially a normative discipline, which depends on psychology (WL I, § 11) (21) and that logic proper (22) is a methodology containing laws that regulate our acquisition of knowledge (WL I, § 15.2) (23)." (p. 326).
(21) Cf. also Heinrich Fels, "Die Philosophie Bolzanos", Philosophisches Jahrbuch der Görres-Gesellschaft, vol. 40, pp. 319-448, 1927, pp.319-448).
(22) Bolzano calls the 4th part of the *Theory of Science* "Eigentliche Wissenschaftslehre".
(23) Cf. Heinrich Scholz, *Die Wissenschaftslehre Bolzanos*, Verlag Oeffentliches Leben, Berlin.1937, p.421.

100. ———. 2004. "Colour Sensations and Colour Qualities: Bolzano Between Modern and Contemporary Views." *British Journal for the History of Philosophy* no. 12:247-276
"What are colour sensations? Sensations are the basic constituents of our perceptual states. They are primitive mental events and are usually distinguished from the conceptual component of more complex mental states, such as beliefs or judgements. For instance, we may see a certain colour or hear a sound without understanding what it is, but we do not remember a colour or sound, nor believe that there is a colour such as tawny, or want to hear a certain sound, without having some idea of what it is." (p. 247)
(...)

"How does Bolzano distinguish between colour sensations and colour qualities? He explains the fact that we have colour sensations by assuming that these latter are caused by real properties of objects and, in the *Wissenschaftslehre* and the *Athanasia*, he claims that colours are dispositional properties or secondary qualities. His causal thesis on colour perception is that colours are properties or attributes of things and we assume that these properties are the cause of our colour sensations and the reason for our judgements that we are seeing coloured things.(12)
His claim that colours are dispositional qualities underlies his examination of physical experiments on colours, which I reconstruct in the next but one section. I then bring the implications of his view into the contemporary discussion of whether colours are dispositional or physical qualities of objects." (p 249)
(12) *Aetiologie*, in *Mathematische und Philosophische Schriften 1810–1816,* BBGA, 2, Nachlass A, vol. 5. §§ 14–15.

101. ———. 2006. "Bolzano's Semiotic Method of Explication." *History of Philosophy Quarterly* no. 23:21-39
"This paper is programmatic: it presents a so-far undiscussed part of Bolzano's *Theory of Science*, namely the *Semiotics*.(1) Bolzano's account of explication is reconstructed to show his contribution to the contemporary discussion." (p. 21)
(...)
"In the second section of the semiotics dealing with the use of signs in treatises and manuals, Bolzano introduces the notion of *Verständigung*.
In German, a *Verständigung* means to inform someone of something, to communicate with someone and to make oneself (or something) understood.(7) Bolzano's English and French translators use the word *explication* for translating *Verständigung*, for this notion concerns the interpretative relation between linguistic and mental events: the relation between signs and intentions and the way in which we understand words. A *Verständigung* is more than the mere grasping or understanding of a word, however, for this word designates the linguistic act of making something explicit in such a way that it is understood by others and thus this concept plays an important role in communication." (p. 21-22)
(1) Bolzano, Bernard (1837), *Wissenschaftslehre (Theory of Science)*, Sulzbach, Seidel, [WL] IV, §§ 637-677; (1833-1841) *Von der mathematischen Lehrart (On the mathematical method)*, in Bernard Bolzano *Gesamtausgabe*, Stuttgart: Fromann-Holzboog, 1969-, Nachlass II, A, 7, [ML] § 9.
(7) The noun *Verständigung* is the nominalization of the verb *verständigen*, which means "to inform" ("den Leser zu verstandigen") or "to communicate."
The second use occurs especially with constructions using the genitive, e.g., "den Gastfreund der Ursache ihres Kummers zu verständigen" or "der jungen Fiirstin meine Liebe zu verständigen." Sich verstandigen means "to make oneself understood" and, more specifically, "to correct mistakes or misunderstandings" (Missverständnisse). See H. Paul, *Deutsches Worterbuch* (Halle: Niemeyer, 1896, 1935), pp. 608-609, as well as contemporary dictionaries of the German language, such as the *Wahrig* (1966), Bertelsmann, (2002).

102. Kasabova, Anna. 2012. "Bolzano's Semantic Relation of Grounding: A Case Study." In *Inference, Consequence, and Meaning: Perspectives on Inferentialism*, edited by Gurova, Lilia, 85-103. Newcastle upon Tyne: Cambridge Scholars Publishing
Abstract: "I reconstruct Bolzano's account of the grounding relation (*Abfolge*) which, I argue, is a precursor of inferentialism as a basis for semantics and I apply the grounding relation to a particular case: episodic memory. I argue that the basis of episodic memory is not the empirical relation of causality but the semantic relation of grounding which explains why we remember some things rather than others."

103. Kasabova, Anita. 2013. "Dubislav and Bolzano." In *The Berlin Group and the Philosophy of Logical Empiricism*, edited by Milkov, Nikolay and Peckhaus, Volker, 205-228. Dordrecht: Springer

"Brief Introduction
Walter Dubislav (1895–1937) was an active member of the Berlin Group of logical empiricismin the early 1930s. A philosopher, mathematician and logician, he shared the thematic focus of the Berlin Group on the natural sciences, mathematics and logic. He shared the methodological demand of the Berlin Group that philosophical method of inquiry should follow the rigor and precision of formal sciences in exposition and logical reasoning (Rescher 2006, 283). A rigorous methodology for philosophy was also required by Bernard Bolzano (1781–1848), the Prague mathematician, logician and philosopher. Was it Bolzano's efforts to separate logic from psychology in the *Theory of Science* (Bolzano 1837) or his reconstruction of mathematics in the *Contributions to a Better Founded Exposition of Mathematics* (1810) which attracted Walter Dubislav's attention?
Dubislav was not interested in Bolzano's early attempts to develop a mathematical method for expounding objective dependence relations which hold between judgments as grounds and consequences (Bolzano 1810, II, § 2). His research is focused on the later Bolzano (1837). In a series of papers published between 1929 and 1931, he deals with Bolzano's Kant-criticism and Bolzano's contribution to modern logic. More specifically, he examines what he calls Bolzano's propositional functions (*Aussage- oder Satzfunktion*), his notion of analyticity and analytic statements, as well as his notions of probability (*Wahrscheinlichkeit*) and derivability (*Ableitbarkeit*)." (p. 205)

104. ———. 2013. "Bolzano on Kant's Definition of Analyticity – Does it Fall Short of Logical Precision?" *Philosophical Alternatives* no. 6:13-34
Abstract: "My commentary is Kant-friendly and I begin by re-situating the Siebel-Bolzano-Kant discussion on analytic judgments in regard to their history, namely, to Aristotle's predication. I focus on Siebel-Bolzano's objections that Kant's analytic judgments (i) have a definiens permitting too broad an interpretation, and (ii) that the definiens is too narrow. I re-examine Kant's use of 'covertly' and 'identity of concepts' and argue pace Mark Siebel that Kant's analytic judgments make explicit the shared content of subject and predicate. I then re-examine Kant and Bolzano's notion of (essential) distinctive feature (*Merkmal*) discussed by Siebel in the context of the 'contained in'/'contained under' issue, and show that Kant's analytic judgments are nominal definitions."

105. Kluge, Eike Henner. 1980. "Bolzano and Frege: Some Conceptual Parallels." *Grazer Philosophische Studien* no. 10:21-41
"Recent Frege scholarship has evidenced a growing interest in the historical basis of Frege's thought. By and large, that interest has focussed on the figure of Leibniz, and although there is still some disagreement over the precise nature and extent of the latter's influence, the fact that it exists is apparently beyond dispute. However, there is another historical figure, of some importance in his own right, whose influence on Frege - or, to be more precise, the possibility of whose influence on Frege - has largely been ignored. I am referring to Bernard Bolzano (1781-1848). The purpose of this paper is to expose some interesting not to say profound similarities between certain fundamental doctrines of the two thinkers, and to suggest on that basis the likelihood of an influence of the former on the latter should be seriously considered." (p. 21)

106. Kneale, Wiiliam, and neale, Martha. 1962. *The Development of Logic*. Oxford Clrendon Press
Chapter V. *Logic after the Renaissance.* § 5. *Bolzano and Mill*, pp. 358-371.
"According to Bolzano a science in the objective sense of that word is a sum of objective truths. If it is set forth in a treatise, the truths of which it consists must, of course, be known to some man, but truths are not in general to be identified with truths known to men. On the contrary it is reasonable to suppose that the great majority of them are known only to God. For an objective truth is a true proposition-in-itself (*Satz-an-sich*), that is to say, a true propositional content, something thinkable or expressible but not necessarily thought or expressed.(6)

Often the word 'judgement' is used in this sense, but it is not suitable as a technical term because it is sometimes used also for the act as opposed to the content of judging; and apart from that it would be misleading if applied to a content which was not believed but merely considered as an hypothesis. 'Judgement' is in fact just one of many words that we can use to refer to propositional contents in special contexts. Others are 'premiss' and 'conclusion', which logicians introduced as descriptions for sentences occurring in certain positions in arguments but used later as though they were designations for propositional contents." (p. 360)
(6) *Wissenschaftslehre* § 12.

107. Konzelmann, Ziv Anita. 2009. "Naturalized Rationality. A Glance at Bolzano's Philosophy of Mind." *Baltic International Yearbook ofCognition, Logic and Communication* no. 4:1-21
Abstract: "Bernard Bolzano's philosophy of mind is closely related to his metaphysical conceptions of substance, adherence and force. Questions as to how the mind is working are treated in terms of efficient (causal) faculties producing simple and complex representations, conclusive and non-conclusive judgments, and meta-representational attitudes such as believing and knowing. My paper outlines the proximity of Bolzano's account of "mental forces" to contemporary accounts of faculty psychology such as Modularity Theory and Simple Heuristics. While the modularist notions of domain specificity and encapsulated mental faculties align with Bolzano's allotment of domain specific tasks to correspondingly specified psychological forces (e.g. judging to "judgmental force", inferring to "inferential force" etc.), the emphasis of Simple Heuristics on accurate "fast and frugal" processes aligns with Bolzano's views regarding cognitive resources and the importance of epistemic economy. The paper attempts to show how Bolzano's metaphysics of mind supposes a conception of bound rationality that determines his epistemology. Combining the rationalist concern for epistemic agent responsibility in the pursuit of knowledge with a strong confidence in the reliability of causal processes to generate the right beliefs, his epistemology shows close affinities with contemporary Virtue Epistemology. According to Virtue Epistemology, knowledge requires that true beliefs be generated by reliable processes typical of a virtuous character. The thesis that Bolzano anticipates virtue epistemological considerations is corroborated by his discussion of heuristic principles that set the norms for the acquisition of knowledge. The paper explores possible relations between such principles and the presumed low-level heuristics of cognitive processes."

108. ———. 2011. "Bolzanian Knowing: Infallibility, Virtue and Foundational Truth." *Synthese* no. 183:27-45
Abstract: "The paper discusses Bernard Bolzano’s epistemological approach to believing and knowing with regard to the epistemic requirements of an axiomatic model of science. It relates Bolzano’s notions of believing, knowing and evaluation to notions of infallibility, immediacy and foundational truth. If axiomatic systems require their foundational truths to be infallibly known, this knowledge involves both evaluation of the infallibility of the asserted truth and evaluation of its being foundational.
The twofold attempt to examine one’s assertions and to do so by searching for the objective grounds of the truths asserted lies at the heart of Bolzano’s notion of knowledge. However, the explanatory task of searching for grounds requires methods that cannot warrant infallibility. Hence, its constitutive role in a conception of knowledge seems to imply the fallibility of such knowledge. I argue that the explanatory task contained in Bolzanian knowing involves a high degree of epistemic virtues, and that it is only through some salient virtue that the credit of infallibility can distinguish Bolzanian knowing from a high degree of Bolzanian believing."

109. Koren, Ladislav. 2014. "Quantificational Accounts of Logical Consequence I: From Aristotle to Bolzano." *Organon F* no. 21:22-44

Abstract: "So-called quantificational accounts explicate logical consequence or validity as truth-preservation in all cases, cases being construed as admissible substitutional variants or as admissible interpretations with respect to non-logical terms. In the present study, which is the first from three successive studies devoted to quantification accounts, I focus on the beginning of systematic theorizing of consequence in Aristotle‘s work, which contains the rudiments of both modal and formal accounts of consequence.
I argue, *inter alia*, that there is no evidence for the claim that Aristotle propounded a quantificational account, and that for a full-fledged quantificational approach in a modern style we need to turn to Bolzano’s substitutional approach, whose motivation, structure and problems are explained in the second part of this study."
"Bolzano might have been the first to elaborate rigorously on this very idea in his account of logical validity and deducibility. The following passage deserves a full quote:
Among the definitions of [the concept of deducibility] … one of the best is that of Aristotle: ‘a syllogism is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so.’ Since there can be no doubt that Aristotle assumed that the relation of deducibility can hold between false propositions, the ‘follows of necessity’ can hardly be interpreted in any other way than this: that the conclusion becomes true whenever the premises are true. Now it is obvious that we cannot say of one and the same class of propositions that one of them becomes true whenever the others are true, unless we envisage some of their parts as variable.
For propositions none of whose parts change are not sometimes true and sometimes false; they are always one or the other. Hence when it was said of certain propositions that one of them becomes true as soon as the others do, the actual reference was not to these propositions themselves, but to a relation which holds between the infinitely many propositions which can be generated from them, if certain of their ideas are replaced by arbitrarily chosen other ideas. (Bolzano 1972, § 155, 219-220)" (p. 33)
References
Bolzano B. (1837/1972): *Theory of Science*. Translated and edited by R. George. Oxford: Basil Blackwell. Translation of selected parts of *Wissenschaftslehre. Versuch einer ausführlichen und grösstentheils neuen Darstellung der Logik mit steter Rücksicht auf deren bisherige Bearbeiter*. 4 Vols. Sulzbach: J. E. v. Seidel.

110. Krämer, Stephan. 2011. "Bolzano on the Intransparency of Content." *Grazer Philosophische Studien* no. 82:189-208
Summary: "Content, according to Bolzano, is intransparent: our knowledge of certain essential features of the contents of our contentful mental acts (such as their identity and composition) is often severely limited. In this paper, I identify various intransparency theses Bolzano is committed to, and present and evaluate the defence he offers for his view. I argue that while his intransparency theses may be correct, his defence is unsuccessful. Moreover, I argue that improving on his defence would require substantial modifications to his general epistemology of content."

111. Krause, Andrej. 2006. "Are Bolzano's Substances Simple?" *American Catholic Philosophical Quarterly* no. 80:543-562
Abstract: "This article analyzes one aspect of Bolzano's metaphysics. It discusses the question of whether, according to Bolzano, substances are simple or not. In the opinion of some commentators, he accepts composed substances, that is, substances having substances as proper parts. However, it is easily possible to misinterpret his position. This paper first tries to reconstruct Bolzano's definitions of the concept of substance and suggests that he should be able to agree with the following final definition: x is a substance if and only ifx is real and not a property. After this, it is shown that, according to Bolzano, every substance is simple in a fourfold sense: No substance has (1) adherences as parts, (2) substances as proper parts, (3) spatially extended parts, and (4) temporal parts."

112. Kriener, Jönne. 2017. "Bolzano." In *The History of Philosophical and Formal Logic: From Aristotle to Tarski*, edited by Malpass, Alex and Antonutti Marfori, Marianna, 121-142. New York: Bloomsbury Academic
"This chapter presents core elements of the logic developed by the Austrian mathematician and philosopher Bernard Bolzano during the fi rst decades of the nineteenth century. * For Bolzano, logic deals with scientific reasoning quite generally. A science for him is an ordered body of true propositions. Accordingly, I will begin by explaining Bolzano's notion of proposition.
When we engage in science, our reasoning crucially involves the derivation of some propositions from others. Bolzano's most advanced innovation in logic is his theory of deducibility (*Ableitbarkeit*). Famously, it anticipates some aspects of the modern concept of logical consequence.
Finally we deal with a more demanding, and less well understood, way in which Bolzano took scientific truths to be ordered: his notion of grounding (*Abfolge*). Grounding is central to Bolzano's thinking about science, and thus an important part of Bolzano's logic." (p. 121)

113. Künne, Wolfgang. 1997. "Propositions in Bolzano and Frege." *Grazer Philosophische Studien* no. 53:203-240
Reprinted in W. Künne, *Versuche über Bolzano / Essays on Bolzano*, pp. 157-195 and in Michael Beaney and Erich H. Reck (eds.), *Gottlob Frege. Critical Assessments of Leading Philosophers. Vol. I: Frege's Philosophy in Context*, New York: Routledge, 2005, pp. 124-153.
Abstract: "In the Preface to his book *Frege and Other Philosophers* [New York: Oxford University Press, 1996] Michael Dummett says: "The only nineteenth-century philosopher of whom it would be reasonable to guess, just from the content of his writings and those of Frege, that he had influenced Frege, is Bernhard Bolzano, who died in the year Frege was born; but there is no evidence whatever that Frege ever read Bolzano".(1) Apart from one grave mistake this seems to me to be exactly right. Did you notice the "grave" mistake? Bolzano's first name is spelled with an "h" and thereby deprived of its Italian flavour.(2)
To be sure, there were two mathematically minded philosophers and one philosophically minded mathematician who emphatically appealed to Bolzano in the course of their discussions with Frege. So he was made aware of the fact that Bolzano's work was potentially relevant for his own concerns. But Husserl, Kerry and Korselt were critical of Frege, and Frege in turn was very critical of them. Perhaps that's why he never bothered to read an author they praised, — who knows... (3)
There are many respects in which a comparison between Bolzano and Frege could be philosophically fruitful. But what is most striking for everyone who reads both Frege's *Logische Untersuchungen* and Bolzano's *Wissenschaftslehre* is the close similarity between what Frege calls *Gedanken* and what Bolzano calls *Sätze an sich*. In the literature this resemblance is frequently mentioned, but I have never seen a detailed investigation into this topic.(4) In this paper I shall recall some of the well-known respects, and point out some less well-known respects, in which F(rege)-Propositions and B(olzano)-Propositions (as I shall call them) resemble each other. But I am at least as keen to underline some philosophically important differences beneath those similarities."
(1) Dummett, vii. The same claim is to be found in Dummett *Ursprünge der analytischen Philosophie*, Frankfurt /M., 1988, 34; *Origins of Analytical Philosophy*, Cambridge/MA, 1993, 24, and *Frege. Philosophy of Mathematics*, London, 1991, 47. I cannot take seriously E.-H. Kluge's contention that there was "a de facto, perhaps even unconscious influence that manifested itself in a similarity of conceptual approach and a parallelism of positions defended" (Kluge "Bolzano and Frege: Some Conceptual Parallels, in: *Grazer Philosophische Studien* 10 (1980), pp. 21-42, 21 ff.). Several extremely careless translations from the *Wissenschaftslehre* in Kluge's article seem to be symptomatic of a rather superficial

acquaintance with Bolzano's work. I also disagree with much of his interpretation of Frege.
(2) Bemard(o)'s father was born at the Lago di Como. By the way, the misspelling is endemic. In Vienna it marred even the attempt to name a street after Bolzano.
(3) Cp. Künne "Die Ernte wird erscheinen...' Die Geschichte der Bolzano-Rezeption (1849-1939)", pp. 9-82, esp. 31-50; revised version in this volume: 326-359.
(4) Of course, in Dummett *Ursprünge.../Origins*... ch. 4, it is also duly registered, but the focus is rather on Frege.

114. ———. 1998. "Bolzano, Bernard." In *Routledge Encylopedia of Philosophy*, edited by Craig, Edward, 824-828. New York: Routledge
Abstract: "Bernard Bolzano was a lone forerunner both of analytical philosophy and phenomenology. Born in Prague in the year when Kant's first *Critique* appeared, he became one of the most acute critics both of Kant and of German Idealism. He died in Prague in the same year in which Frege was born; Frege is philosophically closer to him than any other thinker of the nineteenth or twentieth century. Bolzano was the only outstanding proponent of utilitarianism among German-speaking philosophers, and was a creative mathematician whose name is duly remembered in the annals of this discipline. His *Wissenschaftslehre* (Theory of Science) of 1837 makes him the greatest logician in the period between Leibniz and Frege. The book was sadly neglected by Bolzano's contemporaries, but rediscovered by Brentano's pupils: Its ontology of propositions and ideas provided Husserl with much of his ammunition in his fight against psychologism and in support of phenomenology, and through Twardowski it also had an impact on the development of logical semantics in the Lwów-Warsaw School."

115. ———. 2001. "Constituents of Concepts: Bolzano vs. Frege." In *Building on Frege. New Essays on Sense, Content, and Concept*, edited by Newen, Albert, Nortmann, Ulrich and Stuhlmann-Laeisz, Rainer, 267-285. Stanford: CLSI Publications
Reprinted in: W. Künne, *Versuche über Bolzano / Essays on Bolzano*, Sankt Augustin: Academia Verlag 2008, pp. 211-232.
"In section 1 of this paper I shall point out that in one respect the grandfather of analytical philosophy was more conservative than its great-grandfather: Frege at least partially endorsed the Canon of Reciprocity which was a prominent ingredient of the post-Cartesian logical tradition, Bolzano rejected it completely. In section 2 I shall try to defend one part of this bipartite principle. In section 3 I shall try to show that this line of defence is open to Frege. This claim is based on a reconsideration of Frege's notion of the marks (*Merkmale*) of a concept, — a notion which is generally treated rather cavalierly in the literature on Frege. In section 4 I shall present a problem that Bolzano and Frege share because they both think of complex senses in part-whole terms. Finally, in part 5, I shall briefly celebrate what I deem to be Bolzano's victorious attack on the other part of the Canon of Reciprocity (CR)." (p. 211)
(...)
Here is Kant's formulation of CR: (4)
(CR) Content and extension of a concept stand in an inverse relation. The more objects fall under a concept, the fewer conceptual components are contained within the concept, and vice versa.
Bolzano attacks CR in § 120 of his monumental *Wissenschaftslehre* (1837; henceforth '*WL*' for short). (5)
(4) 'Inhalt und Umfang eines Begriffs stehen gegen cinander in umgekehrtem Verhältnisse. Je mehr namlich ein Begriffunter sich enthalt, desto weniger enthalt er in sich und umgekehrt' (Kant (10), 148). Bolzano's contention in WL I 294, 570, repeated by many authors, that (CR) is to be found already in the Logic of Port Royal (Arnauld/Nicole) is not tenable (Schmauks 14f.). An early (if not the earliest) formulation of (CR) is given in Wolff (l), 138.
(5) Bolzano quotes (CR) in WL I 292.

[Another definition of CR: "Every concept, as partial concept, is contained in the representation of things; *as ground of cognition, i.e., as mark*, these things are contained under it. In the former respect every concept has a content, in the other an extension.
The content and extension of a concept stand in inverse relation to one another. The more a concept contains under itself, namely, the less it contains in itself, and conversely.
Note. The universality or universal validity of a concept does not rest on the fact that the concept is a partial concept, but rather on the fact that it is a ground of cognition." (I. Kant, *The Jäsche Logic*, § 7, *Content and extension of concepts*, in: *Lecture on Logic*, Cambridge: Cambridge University Press, 1992, p. 96]

116. ———. 2003. "Bernard Bolzano's *Wissenschaftslehre* and Polish Analytical Philosophy Between 1894 and 1935." In *Philosophy and Logic in Search of the Polish Tradition: Essays in Honour of Jan Wolenski on the Occasion of His 60th Birthday*, edited by Kijania-Placek, Katarzyna, 179-192. Dordrecht: Kluwer
"In this paper I want to examine some of the many Polish contributions to a critical discussion of Bolzano 's masterpiece.
Twardowski praised Bolzano for clearly distinguishing, under the headings [1] *subjektive Vorstellung*, [2] *Vorstellung an sich or objektive Vorstellung*, and [3] *Gegenstand*, what ought to be distinguished, namely [1] the mental act of representing an object, [2] the content of this act, and [3] its object. Twardowski's book [Twardowski 1892] voiced a fundamental disagreement with Bolzano, which, some would say, was to become rather fruitful, and it is marred by a fundamental misunderstanding.
The disagreement concerns the question whether all representings are objectual (*gegenständlich*) or whether some representings lack an object. For Bolzano this was a matter of course: The act of representing I give voice to when uttering the definite description 'the present King of Poland' has no object.
Twardowski disagreed: my representation does have an object, but it is a non-existent one,(4) This move paved the way for Meinong (as weIl as for Routley and Parsons)(5). Meinong's *Theory of Objects* is based upon the 'principle of the independence of being from being-so (*Prinzip der Unabhängigkeit des Soseins vom Sein*)' : an object can be thus-and-so even if it has no being (i.e, even if it neither 'exists' nor 'subsists'). Bolzano was strongly opposed to this: 'as the old canon has it (*wie schon der alte Kanon besagt*) - *nonentis nullae sunt offectiones*.(6) In 1894 another pupil of Brentano's, Edmund Husserl, who had already come across Bolzano as a mathematician, forcefully defended the claim that some representings have no object whatsoever against Twardowski's criticism.(7) (In some respects this controversy foreshadows that between Meinong and post-'On Denoting-Russell.)" (p. 179-180)
(4) Twardowski (1982), p. 24.
(5) Meinong 'Über Gegenstandstheorie'. On Meinong's reading of Bolzano cp. Künne (1997), §11.
(6) Bolzano, *Athanasia*; pp. 292 f. As to the Canon cp. Descartes, *Principia* I § 52.
(7) Husserl (1894), p. 303.
References
Bolzano, Bemard: 1837, Dr. B. Bolzanos *Wissenschaftslehre*, Seidel, Sulzbach; reprint, edited by Wolfgang Schultz, Leipzig, 1929/1931.
Bolzano, Bernard: 1838, Dr. Bolzano's *Athanasia* [. . .]. Zweite verbesserte Ausgabe, Seidel, Sulzbach.
Husserl, Edmund: 1894, 'Immanenter und wirklicher Gegenstand', Husserliana XXII, Nijhoff, TheHague,1979, pp. 303-338.
Künne, Wolfgang: 1997, ' "Die Ernte wird erscheinen", Die Geschichte der Bolzano-Rezeption', in: H. Ganthaler and O. Neumaier (eds.), *Bolzano und die österreichische Geistesgeschichte*, Academia, Sankt Augustin, pp. 9-82.
Meinong, Alexius: 1904, 'Über Gegenstandstheorie' , reprinted in *Gesamtausgabe*, Vol. VII, Graz 1968, pp. 483-535.

Twardowski, Kazimierz: 1982, *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen*, 1894, Philosophia, München.

117. ———. 2003. "Are Questions Propositions?" *Revue Internationale de Philosophie* no. 57:157-168
Reprinted in: W. Künne, *Versuche über Bolzano / Essays on Bolzano*, Sankt Augustin, Academia Verlag, 2008, pp. 197-210.
"In the Prolegomena to his *Logische Untersuchungen* (LU) Edmund Husserl praised the first two volumes of Bernard Bolzano's *Wissenschaftslehre* (WL) as 'far surpassing everything else world literature has to offer as systematic exposition of logic'. Eleven years later the key is a bit lower: These volumes, he now says, occupy 'the highest rank in the logical world literature of the 19th century'.(2)
To the best of my knowledge, the most extensive and most thorough discussion of a single contention in Bolzano's philosophy of logic that can be found in any of Husserl's books and articles published during his lifetime is contained in the last chapter of his LU.(3) The topic of this discussion is a courageous if not outrageous Bolzanian contention which, at least on the face of it, flatly contradicts what most philosophers since Aristotle took for granted. *Questions*, Bolzano claims, *are a special kind of propositions and hence truth-evaluable*. Let me call this Bolzano's *Tenet*.
In my little exercise I shall reconstruct and evaluate both Bolzano's *Tenet* and Husserl's criticism thereof. I shall argue that the latter is largely correct, but that in the end Husserl and Bolzano are both wrong. Somebody else got it right: a philosopher and mathematician for whom one would also claim a very high rank indeed in the logical world literature of the 19th, and of any, century. But this is to anticipate.
What exactly is it that Bolzano maintains when he says that questions are a kind of propositions? By 'proposition (Satz an sich)' he means something that is neither mental nor linguistic. Propositions are thinkables and sayables which can be singled out by that-clauses. Such thinkables and sayables are truth-evaluable, hence, assuming bivalence as Bolzano does, they are either true or false. If Bolzano's Tenet is to make any sense at all, by 'questions' he cannot mean anything mental or linguistic. Now the term 'question' is multiply ambiguous, and for our inquiry it is most important not to get entangled in this ambiguity. We must distinguish
Questions 1: mental acts of asking oneself a question,
Questions 2: illocutionary acts of asking a question,
Questions 3: interrogative sentences, and
Questions 4: askables.
Wonderings, i. e. sense-1-questions, are voiced by sense-2-questions. Husserl occasionally labels the former '*innerliche Fragen*' and the latter '*Anfragen*'. The second term (which in ordinary German has a far narrower application) is meant to register the fact that sense-2-questions are essentially addressed to someone. Sense-3-questions are linguistic vehicles of sense-2-questions; unsurprisingly Husserl calls them '*Fragesatze*'. Sense-4-questions, finally, are possible contents of sense-1- and of sense-2-questions, and sometimes they coincide with the conventional linguistic meaning of sense-3-questions. (They do so only if the latter are free of context-sensitive elements.) In Husserl's language, an askable is a '*Frageinhalt*', and he identifies it with the '*Bedeutung*' (meaning) des *Fragesatzes'*. (4) Askables are those thinkables and sayables which can be singled out by indirect sense-3-questions (for example, by the clauses in 'He asked whether the conference had started' or 'She asks when the conference will end'). So let us reformulate Bolzano's Tenet: *Askables are a proper sub-set of propositions*." (pp. 197-198).
(...)
"At the point we have now reached we can recognize that the following stance has a chance of being coherent: conceding that English yes/no interrogatives are not true or false (sc. in English) any more than any other interrogatives are, while maintaining that yes/no interrogatives, in contradistinction to search interrogatives, express propositions which are true or false (*simpliciter*). This is coherent if we take

yes/no interrogatives to be an exception to the right-to-left half of a bridge-principle that is unexceptionable as regards *declarative* sentences: Sentence S is true in language L at context c if and only if what is expressed by S in L at c is true. This move would mitigate the tension between Aristotle's and Bolzano's views about questions, which Husserl emphasized at the outset of his discussion of Bolzano's Tenet." (pp. 209-210).
(2) Husserl (3), I 225; letter to Friedjung, in Husserl (14), VII 97.
(3) In 1920 Husserl emphasized that he had refrained from modifying the text of the 1st edition only because in the meantime his views had changed too drastically (preface to the 2nd edition of Husserl(3), II/2 vii). I shall concentrate exclusively on his 1901 position, more precisely: on those aspects of that position which are relevant for an evaluation of Bolzano's thesis about questions. (Page references are always to the 2nd edition.)
(4) Husserl (3), II/2 211-212.

118. ———. 2006. "Analyticity and Logical Truth: from Bolzano to Quine." In *The Austrian Contribution to Analytic Philosophy*, edited by Textor, Mark, 184-249. New York: Routledge
Reprinted in: W. Künne, *Versuche über Bolzano / Essays on Bolzano*, Sankt Augustin: Academia Verlag 2008, pp. 233-303.
"Truth-value bearers and the concept of truth
For Bolzano analyticity, like truth and falsity, is a property of propositions (*Sätze an sich*). He takes the concept of a proposition to resist analysis or conceptual decomposition (*Erklärung*), but there are other ways of 'achieving an understanding (*Verständigung*)' of a concept.(3)
Consider a report of the following type: 'Johanna said that copper conducts electricity, Jeanne said the same thing, though in different words, and Joan believes what they said.' Here a that-clause is used to single out something that is [1] said by different speakers, [2] distinct from the linguistic vehicles used for saying it, and [3] believed by somebody. 'Now, this is the sort of thing I mean by *proposition*,' Bolzano would say, 'propositions are sayables and thinkables, possible contents of sayings and thinkings, that can be singled out by that-clauses.'(4)"
(3) 3 Cf. Bolzano, *Wissenschaftslehre* (henceforth: WL, quoted by volume and page number) IV 243–5, 488–90, 542–5, 547. The manuscript of *WL* was published only seven years after Bolzano had begun to search for a publisher (outside the borders of the Austrian Empire). The book was as unsuccessful as can be. It was only several decades after Bolzano's death that some philosophers in Vienna, Halle and Lemberg recognized some of the gold mines it contains. See Künne (2) and (5).
(4) Bolzano's views on propositions are examined, and compared with Frege's, in Künne (3).
References
Künne, W.:
(2) ' "*Die Ernte wird erscheinen*", *Die Geschichte der Bolzano-Rezeption*' [I], in H. Ganthaler and O. Neumaier (eds) *Bolzano und die österreichische Geistesgeschichte*, St Augustin: Academia Verlag, 1997: 9–82.
(3) 'Propositions in Bolzano and Frege', in (4): 203–40.
(4) with M. Siebel and M. Textor (eds) *Bolzano and Analytic Philosophy*, Grazer Philosophische Studien 53, 1997.
(5) '*Die Geschichte der philosophischen Bolzano-Rezeption*' [II], in H. Rumpler (ed.) *Bernard Bolzano und die Politik*, Wien: Böhlau, 2000: 311–52.

119. ———. 2007. "Some Varieties of Deception." In *Explaining the Mental. Naturalist and Non-Naturalist Approaches to Mental Acts and Processes*, edited by Penco, Carlo, Beaney, Michael and Vignolo, Massimiliano, 106-122. Cambridge: Cambridge Scholars Publishing
"Members of the family of concepts to which the title of this paper alludes play important roles in various areas of theoretical and practical philosophy. I want to throw some light on these concepts and their interrelations, and in doing so I also want to make Bernard Bolzano's analytical work in this area better available. The

great-grandfather of analytical philosophy, a contemporary of Hegel's, was a great mathematician, and he held the chair of Philosophy of Religion at Prague University until the Emperor sacked him. It was part of his job to deliver a sermon, a so-called *Erbauungsrede* or exhortation. on each and every Sunday and on church holidays. These sermons contain most of the material I shall exploit in this paper.(1) None of my definitions literally coincides with Bolzano's, but most of them are substantially due to him.
Bolzano never put his accounts of various kinds of *deceiving* and of various kinds of *trying to deceive* together. but if one attempts to arrange them systematically it runs out that for the most part they harmonize very well with each other. Whenever they don't I shall take the liberty of making adjustments that are meant lo enhance their plausibility." (p. 106)
(1) The pertinent sermons will be quoted as 'I', 'II', 'III ' and 'IV', followed by page number. I. deception & cheating, 13. 04.1817, in Bolzano. *ER4*, pp 306-313: II. self-deception, 15. 07.1810. in Bolzano. *ER4*, pp. 36-45: lll. Hypocrisy 16.02.1812. in Bolzano. *ER2*,. pp. 289-300. IV *Lying* 18. 03.1810, in Bolzano, *ER*2, pp. 73-81. Abbreviated references to Bolzano's works are spell out in the bibliography to this paper Quotations from Bolzano are always in italics.

120. ———. 2008. *Versuche über Bolzano / Essays on Bolzano*. Sank Augustin: Academia Verlag
Essays in English: Propositions in Bolzano and Frege 157; Are Questions Propositions? 197; Constituents of Concepts 211; Analyticity and Logical Truth: From Bolzano to Quine 233-304

121. ———. 2009. "Bolzano and (Early) Husserl on Intentionality." In *Acts of Knowledge: History, Philosophy and Logic, Essays Dedicated to Göran Sundholm*, edited by Primiero, Giuseppe and Rahman, Shahid, 95-140. London: College Publications.

122. ———. 2011. "On Liars, ‘Liars’ and Harmless Self-Reference." In *Mind, Values, and Metaphysics. Philosophical Essays in Honor of Kevin Mulligan. Volume 2*, edited by Reboul, Anne, 355-429. Dordrecht: Springer
Abstract: "The topics of this chapter are (1) the history of a mislabelled antinomy and of a pseudo-paradox and (2) some logico-semantical peculiarities of self-referential sentences that do not give rise to a paradox. My points of departure will be Bernard Bolzano’s discussions of a plain fallacy he called The Liar and of an antinomy that we unfortunately got used to calling The Liar. He found a pointer to the fallacy in Aristotle’s *Sophistical Refutations*. In a logic manual of the early renaissance, he came across a source of the antinomy in the form of a sentence that declares itself to be false. In Sect. 24.1, I shall praise Bolzano’s reaction to the fallacy and discuss his analysis of the concept of lying. I will present some ancient expositions of the antinomy and go on to criticize, along Moorean lines, Russell’s rather sloppy account. Finally, I will defend the author of the ‘Letter to Titus’ against the charge of being paradox-blind when he invoked a Cretan denigrator of all Cretans. (Some twentieth century logicians and analytic philosophers are the villains of this part of my chapter: I shall criticize their carelessness with respect to a well-entrenched concept, and I shall complain that they keep on alluding to ancient texts without bothering to read them closely.) In Sect. 24.2, I shall reconstruct Girolamo Savonarola’s excellent exposition of the antinomy [*], examine Bolzano’s criticism of the Florentine diagnosis and reject his own attempt to defuse the paradox. (I shall not try to improve on his attempt.) In this context, Bolzano makes a point concerning self-referential sentences that is not affected by the failure of his alleged dissolution of the antinomy. He rightly takes it to be a matter of course that there are ever so many harmlessly self-referential sentences. But he shows that some care is needed when one wants to formulate their negation. In Sect. 24.3, I will expound this point.
It turns out that similar problems arise when one uses harmlessly self-referential sentences in deductive arguments. Such sentences also enforce a revision of certain

intuitively plausible constraints on translation."
[*] Girolamo Savonarola's Compendium logicae (Bolzano, WL I 78–80; Savonarola, CL 151, lines 6–24).
References
Savonarola G. (1492) *Compendium logicae*. In: Savonarola G. (1982) Opere, vol 22, *Scritti filosofici*, 1 (Garfagnini G. Garin E. (eds)). Roma 3–208.

123. ———. 2015. "On Having a Property: Corrigenda in Bolzano's *Wissenschaftslehre*." *Grazer Philosophische Studien* no. 91:365-408.

124. ———. 2018. "Truth, Ascriptions of Truth, and Grounds of Truth Ascriptions: Reflections on Bolzano and Frege." In *Eva Picardi on Language, Analysis and History*, edited by Coliva, Annalisa, Leonardi, Paolo and Moruzzi, Sebastiano, 31-66. Cham (Switzerland): Palgrave Macmillan
"In Sect. 1 of this chapter, I shall discuss Bolzano's attempt to give a definition of the concept of truth, in Sect. 2 I shall ask whether Frege succeeds in showing that all such endeavours are doomed to failure. In this chapter I shall remain neutral as to the question of definability, but the key premise of his alleged proof of indefinability. The equivalence schema 'The thought that things are thus and so is true if, and only if, things are that way' captures an important feature of the concept of truth.
Frege went beyond this *true-iff* principle when he claimed that the two halves of such biconditionals do not only stand and fall together,—they even express one and the same thought. It is doubtful whether Frege has any good argument for this Identity Thesis. In Sect. 3 of this chapter I will give reasons for this doubt. In Sect. 4 I shall show that, and why, Bolzano rejects the Identity Thesis. Bolzano emphasizes an important feature of our concept of truth that is not captured by the equivalence schema. One can hint at this additional feature by saying, 'If the thought that things are thus and so is true, then it is true because of things' being that way, and not vice versa'. In Sect. 5 I shall locate this true because-of principle in the theory of grounding (*Abfolge*) that Bolzano outlined in the second volume of his monumental *Wissenschaftslehre* (henceforth: *WL*). In Sect. 6 I shall explore whether the Identity Thesis can be refuted by appealing to (the Bolzanian reformulation of) the true-because-of principle. On the following pages, I shall not try to argue for the true-because-of principle. Like Aristotle and Bolzano I shall accept it as a basic intuition concerning truth.(1) The brief Appendix points to a use of the notion of grounding that has been neglected in recent literature although Bolzano deemed it to be of great importance." (pp. 31-32)
(1) Any attempt at a proof of this principle from a definition of truth presupposes, of course, that pace Frege such a definition is to be had.

125. Lange, Marc. 2022. "Bolzano, the Parallelogram of Forces, and Scientific Explanation." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 394-417. New York: Oxford University Press
"Marc Lange turns to Bolzano's philosophy of physics and discusses his explanatory proof of the parallelogram law for the composition of forces. Lange argues that this proof is neither clearly causal nor clearly non-causal. In order to illuminate its explanatory potential, Lange compares it with Bolzano's explanation of the intermediate value theorem: Bolzano takes the latter to have a unified explanation covering all functions, and in a similar spirit he regards an explanation of the parallelogram law as unifying it with analogous laws regarding various other quantities that are potential causes." (p. 38)

126. Lapointe, Sandra. 2002. "Bolzano's Hidden Theory of Universal Quantification." In *The Logica Yearbook 2001*, edited by Childer, Timothy and Ondrej, Majer, 37-48. Prague: Filosofia. Publishing House of Prague Institut of Philosophy.

127. ———. 2004. "Why Frege Never Read Bolzano." In *The Logica Yearbook 2003*, edited by Behounek, Libor, 183-194. Prague: Filosofia. Publishing House of Prague

Institute of Philosophy.

128. ———. 2006. "Bolzano on Grounding or Why Is Logic Synthetic." In *The Logica Yearbook 2005*, 113-126. Prague: Filosofia.

129. ———. 2007. "Bolzano Semantics and His Critique of the Decompositional Conception of Analysis." In *The Analytic Turn*, edited by Beaney, Michael, 219-234. London: Routledge
"When asked to explain what conceptual analysis is, philosophers often resort to the idea of decomposition: to analyse an expression or a concept is to break it down into its (simpler) components. Although the notion of decomposition is a convenient figure of speech, without qualifications it can hardly be said to provide an informative description of what is involved in conceptual analysis. It could be argued, however, that this was not always the case. In Kant's theory, for instance, the conception of analysis is literally decompositional: notions such as *Zergliederung, Auflösung', `Inhalt'* and *enthalten sein'* are meant to provide a relatively straightforward description of the mereological conception of the formal features of and relations between concepts he had inherited from his predecessors, contrary to what influential interpretations such as Quine (1953: 21) suggest.(2) In what follows, I'll use the expression `decompositional conception of analysis' to refer to the conception of analysis that underlies Kantian semantics and, most notoriously, the Kantian definition of analyticity. My concern, though, is not primarily with Kant nor with analyticity but with Bernard Bolzano's conception of analysis. A superficial reading of Bolzano's *Theory of Science - Wissenschaftslehre* (Bolzano 1837; hereafter *WL)* - could lead one to think that Bolzano also subscribed to the decompositional conception of analysis. Yet, while Bolzano sanctions Kant's account in his earlier work (cf. Bolzano 1810: §5; 1812: §30) he came explicitly to reject it. Contrary to what is often assumed, Bolzano's understanding of what it means for a concept to be 'included' in another concept or for a given concept to have a particular content is radically different from Kant's and from that of Bolzano's other immediate predecessors. In fact, Bolzano anticipated some of the most important developments of twentieth-century semantics.(3)
I begin the paper with a brief sketch of the decompositional conception of analysis in section 1, and then in section 2 I present Bolzano's criticism of this conception. In section 3, I explain the main lines of Bolzano's reductive programme of analysis. Section 3, I hope, will go some way towards establishing the continued interest of Bolzano's semantic analyses. One of the main consequences of Bolzano's rejection of the decompositional conception of analysis is the need to find a new way to define semantic notions such as analyticity or validity. For that purpose, Bolzano developed a new and ingenious substitutional method. I sketch this method in section 4. I conclude by pointing out some important aspects of Bolzano's historical impact." (pp. 219-220)
(2) I deal in more length with this question in Lapointe *Qu'est-ce que l'analyse?*, Paris, Vrin, 2008.
(3) Superficial knowledge of medieval semantics suffices to convince that similarities are not scarce but this, unfortunately, remains to be studied.

130. ———. 2010. "Bolzano *a priori* Knowledge, and the Classical Model of Science." *Synthese* no. 174:263-281
Abstract: "This paper is aimed at understanding one central aspect of Bolzano’s views on deductive knowledge: what it means for a proposition and for a term to be known a priori. I argue that, for Bolzano, a priori knowledge is knowledge by virtue of meaning and that Bolzano has substantial views about meaning and what it is to know the latter. In particular, Bolzano believes that meaning is determined by implicit definition, i.e. the fundamental propositions in a deductive system. I go into some detail in presenting and discussing Bolzano’s views on grounding, a priori knowledge and implicit definition. I explain why other aspects of Bolzano’s theory and, in particular, his peculiar understanding of analyticity and the related notion of Ableitbarkeit might, as it has invariably in the past, mislead one to believe that

Bolzano lacks a significant account of a priori knowledge. Throughout the paper, I point out to the ways in which, in this respect, Bolzano's antagonistic relationship to Kant directly shaped his own views."

131. ———. 2011. *Bolzano's Theoretical Philosophy. An Introduction*. New York: Palgrave Macmillan

Contents: Michael Beaney: Foreword VIII; Acknowledgements XI; Introduction 1; 1. Kant and German Philosophy 11; 2. Decomposition 18; 3. Meaning and Analysis 29; 4. A Substitutional Theory 43; 5. Analyticity 59; 6. *Ableitbarkeit* and *Abfolge* 72; 7. Justification and Proof 91; 8. *A priori* Knowledge 102; 9. Things, Collections and Numbers 116; 10. Frege, Meaning and Communication 128; 11. Husserl, Logical Psychologism and the Theory of Knowledge 139; Notes 158; Bibliography 170; Index 180-183.

"Bernard Bolzano (1781-1848) occupies a unique place in the history of modern philosophy. Born in the year in which Kant's *Critique of Pure Reason* was published and dying in the year in which Frege was born, his philosophy - like his life - can be seen as offering a bridge between Kant's seminal work and the birth of analytic philosophy. In Bolzano's writings, one finds many of the characteristic themes of analytic philosophy anticipated. Like Frege and Russell after him, Bolzano was dissatisfied with Kant's account of mathematics and realised that a better conception of logic was required to do justice to mathematics. Bolzano's conception of logic was not Frege's or Russell's, but he did criticise traditional subject-predicate analysis, suggested that there was a fundamental form underlying all types of proposition and was insistent on the need to keep psychology out of logic. Like Frege, Bolzano construed existential statements as being concerned with the non-emptiness of appropriate 'ideas' *('Vorstellungen an sich'* in Bolzano's terms) or 'concepts' *(Begrime'* in Frege's terms), and his conception of 'propositions' *(Satze an sich')* is similar in many respects to Frege's conception of 'thoughts' *('Gedanken').* Like Frege, too, Bolzano emphasised that there is a class of entities, including both 'ideas'/'concepts' and 'propositions'/'thoughts', which are objective but not actual *('wirklich'),* in the sense of not existing in the spatio-temporal realm. Despite these similarities, however, Bolzano had no direct influence on any of the acknowledged founders of analytic philosophy. He had an influence on other German-speaking philosophers such as Franz Brentano, Benno Kerry, Edmund Husserl, Alwin Korselt and Kazimierz Twardowski, who themselves had an influence on the early analytic philosophers, both through correspondence and in their own publications (even if, often, mainly as a target of criticism). Through Twardowski, the founder of the Lvov-Warsaw school, he also had an influence on a whole generation of Polish logicians and philosophers, including Jan Lukasiewicz, Stanislaw Lesniewski and Alfred Tarski, who played an important role in the development of analytic philosophy. So a full account of the history of analytic philosophy must certainly pay attention to Bolzano's work. His significance, however, lies not just in these patterns of influence. The similarities and differences between his views and those of Frege, in particular, reveal much about the nature of analytic philosophy: the conceptions of analysis and logical form involved, for example, and key debates such as those about analyticity and other modal notions. These influences and connections are explored and elucidated by Sandra Lapointe in this book.

At the heart of Bolzano's logic - logic being understood in the traditional broad sense as including both methodology and theory of science (hence the title of Bolzano's major work, the *Wissenschaftslehre)* - lies his critique of Kant. As Lapointe explains in the first three chapters, Bolzano criticises Kant's theory of intuition and his decompositional conception of analysis. In doing so, Bolzano develops his own positive doctrines, concerning analyticity and logical consequence, in particular, based on a method of substitution, as Lapointe elaborates in Chapters 4-6. In the remaining chapters, further clarifying his semantic theory, she discusses his epistemological and ontological views and his connection with Frege and Husserl." (from the Foreword by Micharl Beaney).

132. ———. 2012. "Is Logic Formal? Bolzano, Kant and the Kantian Logicians." *Grazer Philosophische Studien* no. 85:11-32
Abstract: "In the wake of Kant, logicians seemed to have adhered to the idea that what is distinctive of logic is its “formality”. In the paper, I discuss the distinction Kant draws between formality and generality of logic and argue that he ultimately con ates the two notions. I argue further that Kant's views on the formality of logic rest on a series of non trivial assumptions concerning the nature of cognition. I document the way in which these assumptions were received in his successors. In the second part of the paper I focus on Bolzano's criticism of the Kantian position and his redefinition of the notion of form. I argue that while what contemporary, post-Tarskian philosophers generally understand as the formality of logic ought to be traced back to Bolzano there are also important differences between the two positions."

133. ———. 2012. "Bolzano and Kant: Introduction." *Grazer Philosophische Studien* no. 85:1-10.

134. ———. 2014. "Bolzano, Quine and Logical Truth." In *A Companion to W.V.O. Quine*, edited by Harman, Gilbert and Lepore, Ernie, 296-312. Malden: Wiley Blackwell
"In this paper, I compare Quine's discussion of logical truth to Bolzano's theory of "logical analyticity". It is by now a received view that Bolzano largely anticipated Quine's views on logical truth, a conclusion Quine himself was retroactively prompted to draw:
"[M]y much cited definition of logical truth was meant only as an improved exposition of a long-current idea. So I was not taken aback at Bar-Hillel's finding the idea in Bolzano [...]" (Quine 1960, 65; see also 1966b, 110)."
According to the standard interpretation, the similarity between Bolzano and Quine comes from the fact that they are both "demarcating logic [...] with the help of a set of logical particles which are held constant, while the other non-logical expressions are freely substituted for each other".(3) This interpretation assumes that Bolzano and Quine share at least some substantial views about what makes a term a "logical" term. I think that this interpretation is largely mistaken. My paper has four parts. In the first part, I give some background to Bolzano's theory, focusing on his views on syntax and form. In the second part, I show why it is mistaken to assume that Bolzano and Quine mean the same when they speak of logical concepts/words. In the third part of the paper I discuss Bolzano's views on logical truth and sentences that can be turned into logical truth by putting synonyms for synonyms. I conclude by asking whether Bolzano's position allows him to fulfil the epistemic requirement (and answer, with a twist, in the affirmative)." (p.297).
(3) “Comments on Quine” (Føllesdal 1980, p. 29, my emphasis).
References
Føllesdal, Dagfinn (1980). Comments on Quine. In S. Kanger and S. Öhman (eds.). *Philosophy and Gramma*r (29–35). Dordrecht: Reidel.
Quine, W.v.O. (1960). *Word and Object*. Cambridge, MA: MIT Press.
Quine, W.v.O. (1966b). Carnap and Logical Truth. In *The Ways of Paradox* (107–132). Cambridge, MA: Harvard University Press.

135. ———. 2014. "Bolzano and the Analytical Tradition." *Philosophy Compass* no. 9:96-111
Abstract: "In the course of the last few decades, Bolzano has emerged as an important player in accounts of the history of philosophy. This should be no surprise. Few authors stand at a more central junction in the development of modern thought. Bolzano's contributions to logic and the theory of knowledge alone straddle three of the most important philosophical traditions of the 19th and 20th centuries: the Kantian school, the early phenomenological movement and what has come to be known as analytical philosophy. This paper identifies three Bolzanian theoretical innovations that warrant his inclusion in the analytical tradition: the commitment to ‘logical realism’, the adoption of a substitutional procedure for the

purpose of defining logical properties and a new theory of a priori cognition that presents itself as an alternative to Kant's. All three innovations concur to deliver what counts as the most important development of logic and its philosophy between Aristotle and Frege. In the final part of the paper, I defend Bolzano against a common objection and explain that these theoretical innovations are also supported by views on syntax, which though marginal are both workable and philosophically interesting."

136. ———. 2014. "Bolzano's Logical Realism." In *The Metaphysics of Logic*, edited by Rush, Penelope, 189-208. Cambridge: Cambridge University Press
"Bolzano's *Theory of Science* (1837) presents the first explicit and methodical espousal of internal logical realism. It also contains a formidable number of theoretical innovations. They include (i) the first account of the distinction between "sense" (*Sinn, Bedeutung*) and "reference" (or "objectuality": *Gegenständlichkeit*), (ii) definitions of analyticity and consequence, i.e. "deducibility" (*Ableitbarkeit*) based on a new substitutional procedure that anticipates Quine's and Tarski's, respectively, and (iii) an account of mathematical knowledge that excludes, *contra* Kant, recourse to extraconceptual inferential steps and that is rooted in one of the earliest systematic reflections on the nature of deductive knowledge. (i)–(iii) all assume the existence of mind - and language-independent entities Bolzano calls "propositions and ideas in themselves" (*Sätze an sich*). Take (i) for instance. Appeal to propositions in themselves in this context serves Bolzano's antipsychologism in logic: according to Bolzano, the sense (*Sinn*) of a sentence – the proposition it expresses – is to be distinguished from the mental act in which it is grasped. Just like what is the case in Frege, a sentence has the semantic properties it has (e.g. truth) on Bolzano's account derivatively, by virtue of its relation to mind-independent entities: the primary bearers of semantic properties are the propositions that constitute their *Sinne*." (p. 195)

137. ———. 2017. "Bernard Bolzano." In *Sourcebook in the History of Philosophy of Language: Primary source texts from the Pre-Socratics to Mill*, edited by Cameron, Margare, Hill, Benjamin and Stainton, Robert J., 1029-1032. Dordrecht: Springer
"The views on language of Bernard Bolzano (1781–1848) overlap with two traditions. On the one hand, Bolzano tries to make sense of the idea that the signs we use designate ideas. On the other hand, Bolzano's theory is underpinned by a series of semantic and epistemological analyses that yield the first philosophical treatment of linguistic signs as endowed with both meaning and reference, understood in the contemporary sense. The resulting theory is an interesting combination of elements of post-Lockean epistemologies with a clear anticipation of post-Fregean semantics." (p. 1029)

138. ———. 2018. "Bolzano's Philosophy of Mind and Action." In *Philosophy of Mind in the Nineteenth Century*, edited by Lapointe, Sandra, 42-59. New York: Routledge
"In spite of the overwhelmingly sympathetic consensus on the significance of Bolzano's contribution to theoretical philosophy, little attention has to this date been paid to his views on mind."
(...)
"The present chapter is an attempt to go some way toward such an understanding. The first part of the chapter offers a brief comparison of Bolzanian and Brentanian views on representation and judgement. A brief survey of Brentano's main positions is informative as a theoretical point of comparison for Bolzano's own views. At the very least, it is helpful to have the Brentanian theory in mind when gauging the impact - however humble - Bolzano effectively had on the theories of mind of some of Brentano's students.(4) The comparison however does not fully do justice to Bolzano's views. This is mainly because Bolzano's approach to the philosophy of mind and action has more to share with contemporary theorists than with any of his predecessors or successors in the 19th century. This claim is likely to arouse perplexity. Bolzano puts forward his views on mind in *Athanasia* (1827), a treatise in which a hefty metaphysics of substance is put to work for the purpose of proving

the immortality of the soul, a context which *prima facie* is unlikely to afford much relevance. Those who have discussed Bolzano's views on mind, with few exceptions, have however consistently missed what is most remarkably interesting about them. First, the framework within which Bolzano develops his metaphysics of mind and agency is not dualistic and presents some anticipation of what will later be known as "neutral monism".(5) Second, the conceptual resources that are deployed to make sense of the way in which "body" and "soul" interact in living beings presuppose an understanding of organisms that goes against - or far beyond - much of what Bolzano's contemporaries and successors in the the 19th century wrote on the topic, especially the Idealists. More importantly, Bolzano puts forward an account of rational agency based on a theory of mind that anticipates crucial aspects of contemporary discussions on the role of intentions as "reasons" or "causes" for action. In the second and more substantive part of the paper, I focus on Bolzano's views on the ontology of mind and rational agency." (pp. 42-43)
(4) For a more detailed presentation of Brentano's views on mind, see Rollinger *infra*; see also Kriegel (forthcoming). [2017]
(5) For a discussion of Mach's view on neutral monism, for instance, see Banks, *infra*.
References
Erik C. Banks, *Ernst Mach' Contributions to the Philosophy of Mind*, same volume, pp. 77-95.
Robin D. Rollinger, *Brentano's Early Philosophy of Mind*, same volume, pp. 168-185.
Uriah Kriegel, "Brentano Concept of Mind" in *Innovations in the History of Analytical Philosophy*, Sandra Lapointe and Chris Pincock (eds.), Houndmills, Palgrave Macmillan, 2017.

139. ———. 2019. "Bolzano on Logic in Mathematics and Beyond." In *Logic from Kant to Russell: Laying the Foundations for Analytic Philosophy*, edited by Lapointe, Sandra, 101-122. New York: Routledge
"According to standard narratives, the origins of formal logic as we know it are to be found within the push toward logicism, axiomatisation and the foundations of set theory for which Frege's foundational project in mathematics often serves as muster. Frege, however, was by no means the first logician of the 19th century to seek to provide a new logical foundation to mathematical knowledge. At least one other author was driven by concerns, insights, ambitions and philosophical acumen that were as remarkable as Frege's. This author's efforts too resulted in a fullscale logical system whose conceptual resources, while they do not have the elegance and simplicity of Frege's "concept-script", are nonetheless as rich as those of first-order predicate calculus and powerful enough to generate Russell's paradox.(2) This author is Bernard Bolzano." (p. 101)
(2) Cf. Simons (1997) and Lapointe (2011, Chapter 3).
References
Lapointe, Sandra (2011) *Bolzano's Theoretical Philosophy*, Houndmills, Palgrave.
Simons, Peter (1997) "Bolzano on Collections," *Grazer Philosophische Studien* 53, 87–108.

140. ———. 2022. "Bolzano's Theory of Satz an sich." In *The Routledge Handbook of Propositions*, edited by Tillman, Chris and Murray, Adam Russell. New York: Routledge
To be published March 2022.

141. Lapointe, Sandra, and Armstrong, Chloe. 2014. "Bolzano, Kant, and Leibniz." In *New Anti-Kant*, edited by Lapointe, Sandra and Tolley, Clinton, 272-290. London: Palgrave Macmillan
"Both historically and philosophically Bolzano's contribution to philosophy is to be understood within the context of the reception of Kant's critical philosophy, or so we will argue. This claim is also likely to be controversial. Bolzano's contribution to philosophy, and in particular his contribution to the epistemology of logic and

mathematics, is more often than not positioned in stark opposition to Kant's, in the intellectual lineage of Leibniz. What we are proposing is deliberately meant to upset this picture. Bolzano's relationship to critical philosophy is far more complex than what is generally assumed. For one thing, Bolzano's relationship to Kantian philosophy is not exhausted by his relationship to Kant. Bolzano paid close attention to the logical theories of those who followed in Kant's stride, the "new logicians" (Bolzano's term), and he discussed their views in at least as much depth as he did Kant's. What's more, Bolzano sought to determine what is distinctive of the "new logic" and thus offered a philosophical reflexion that is still, even today, enlightening when it comes to understanding this aspect of the reception of Kant's first *Critique*." (pp. 273-274)
(...)
"Of course, there are connections between Bolzano and Leibniz. But Bolzano discusses Leibniz's work in fact comparatively rarely. We find over the some 2400 pages of the *Theory of Science* (1837) a mere 30 references to Leibniz, mostly to the *Nouveaux essais* (1704) – compare this with the some 150 references to Kant, and some 200 to Kiesewetter.[*] It is not only that the number of references is small, but also that many references are in footnotes, even in the sections entirely devoted to discussion of the views of other philosophers." (p. 275)
[*] Kiesewetter, Johann Gottfired Karl Christian. (1806). *Grundriss einer allgemeinen Logik nach Kantischen Grundsätzen*. Berlin: Lagarde.

142. Lapointe, Sandra, and Tolley, Clinton, eds. 2014. *New Anti-Kant*. London: Palgrave Macmillan
Contents: Michael Beaney: Series Editor's Foreword VI; Acknowledgements IX; Notes on Contributors X; PART I: 1. Sandra Lapointe and Clinton Tolley: Introduction 3; 2. Translators' Note 15; 3. František Přihonský: New Anti-Kant, or examination of the *Critique of Pure Reason* According to the Concepts Laid Down in Bolzano's *Theory of Science* (translated by Sandra Lapointe and Clinton Tolley) 18;
PART II: 4. Clinton Tolley: Bolzano and Kant on Space and Outer Intuition 157; 5. Nicholas F. Stang: Kant, Bolzano, and the Formality of Logic 192; 6. Timothy Rosenkoetter: Kant, Bolzano, and Moore on the Value of Good Willing 235; 7. Sandra Lapointe and Chloe Armstrong: Bolzano, Kant and Leibniz 272; Index 291-295.
"A unique philosophical dialogue
The present volume contains an altogether remarkable document in the history of nineteenth-century philosophy: a critical commentary on he most influential systematic work (the *Critique of Pure Reason*) of one major philosopher (Kant), written from the point of view of another major systematic philosopher (Bolzano), just decades after the former's publication. Bolzano, at the height of his powers, and with his mature philosophical views having fully taken shape with the publication of his Theory of Science (1837), undertakes the project of engaging, key point by key point, with Kant's masterwork. In collaboration with Bolzano, Frantisek Prihonsky (who would ultimately publish the final record of this work in 1850, shortly after Bolzano's death) both compiles a comprehensive and thorough summary of the main definitions, theses, and arguments in Kant's book, and then proceeds to bring to light the most important unclarities, confusions, and fallacies that he finds each step along the way. The result, *New Anti-Kant*, is not only an extremely useful and even-handed overview of the entire first *Critique* itself - including parts often neglected by even Kant's most sympathetic readers - but also a catalogue of philosophically insightful and textually well-grounded challenges to signature Kantian doctrines. This work helps us to see anew the overarching contours of Kant's philosophy, and brings a fresh focus onto deep points of tension within Kant's system - all the while serving to introduce us, through instructive contrast, to the powerful alternative perspective that Bolzano develops in his own systematic philosophy." (pp. 3-4).

143. Malink, Marko. 2022. "Aristotle and Bolzano on Grounding." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 221-243. New York: Oxford University Press
"Marko Malink examines Aristotle's conception of scientific proofs as the historical roots of Bolzano's conception of grounding and compares the two philosophers' views on infinite grounding chains, on the role that generality plays in ground-revealing proofs, and on scientific knowledge." (p. 37)

144. Mancosu, Paolo. 1999. "Bolzano and Cournot on Mathematical Explanation." *Revue d'histoire des sciences* no. 52:429-456
Abstract: "Recent discussions on the topic of « mathematical explanation » have focused on the distinction between explanatory and non-explanatory proofs. The former proofs are supposed to differ from the latter in that they not only establish that a result is true but also show why it is true. This opposition is at the core of the philosophies of mathematics of Bolzano and Cournot. The paper analyzes Bolzano's theory of Grund and Folge, and Cournot' s opposition between the logical and the rational order, emphasizing their relevance to the issue of mathematical explanation. The final part of the paper investigates the shortcomings of Bolzano's and Coumot's theories as explications of mathematical explanation."

145. Mates, Benson. 1992. "Bolzano and Ancient Pyrrhonism." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 121-139. Firenze: Leo S. Olschki
"Bolzano's attempt to refute so-called "radical" or "complete" skepticism is carefully described in Professor Berg's introduction to his edition of the *Wissenschaftslehre* (WL). Two forms of such skepticism are there distinguished. The thesis of the *ontological* form is
(1) No propositions (*Sätz an sich*) is true and that of the *epistemological* form is
(2) No judgment (*Urteil*) is true.
Bolzano's principle arguments against these are roughly as follows. Against (1) he argues that, for any proposition S, either S is true or the proposition that S is false is true. Therefore, at least one proposition is true. The argument against (2) is less clear. Bolzano (WL 40) takes the problem to be that of convincing a radical skeptic that, after all, he must recognize the truth of at least one proposition. After considering various possibilities, he concludes that the skeptic will have to accept as true at least the proposition that he has ideas (*Vorstellungen*), for obviously he confirms this proposition the moment he doubts or denies it. The point, I suppose, is that, just as one cannot doubt that there are men on the moon if one has no idea of what it is to be a man or to be on the moon, so the skeptic, if he has no ideas, is in no position to doubt anything, not even that he has ideas. Bolzano thinks that while the skeptic might refuse publicly to admit the proposition in question, "nevertheless he will surely feel in his innards that it is true... and if he feels this, we have won". Whatever one may think of these arguments, in this paper I am not concerned to evaluate them but only to consider whether they refute Pyrrhonism, as Bolzano seems to suppose." (pp. 121-122)
(...)
The root of Bolzano's failure to appreciate the force of Pyrrhonism is, in my opinion, that he does not realize that its self-referential aspect is essential. This aspect is not something that Sextus is reluctant to admit but is rather a feature that he emphasizes over and over again and that he obviously regards as crucial to the consistency of the skeptic's position. Bolzano's failure to understand this is especially evident at WL 40, where he quotes and discusses one of the many passages in which Sextus points out the self-reference of the skeptic's slogans (*phonai*), i.e., pronouncements like "contrary claims are equal", "no more this than that", "I decide nothing", etc. Bolzano says:
In setting forth the various formulae with which the skeptic is accustomed to express his state of doubt, Sextus Empiricus tries to employ maximal caution so as to protect it from the charge of self-contradiction, but nevertheless he finds himself compelled at the end to admit

"As concerns all the skeptic slogans the following must be understood in advance, namely that we do not maintain their truth in any absolute way, since we say that they themselves are included among the things to which they apply -- just as cathartic drugs do not merely eliminate humor from the body but also expel themselves along with the humors" (*Outlines of Pyrrhonism* I 206).
"This amounts to the reluctant admission", says Bolzano, that the skeptic ceases to be a skeptic as soon as he declares himself to be a skeptic. Only if he keeps silent and makes no judgment, not only in words but also internally, is he a complete doubter; and as long as this condition exists we *others* can say of him truly that he doesn't know a single truth. But as soon as he *himself* says it, the condition ceases and his judgment is therefore false.
But there is no "reluctant admission" here, and the Pyrrhonist doesn't have to be silent if he is to remain a Pyrrhonist. He will say "*It seems to me now that* contrary claims are equal" and "*It seems to me now that* there is no more reason for this than that", and so on. What he refrains from are flat out categorical statements, whether concerning his own skepticism or anything else.
It will be evident that this form of skepticism is not easily refuted. Since the Pyrrhonist agrees only to propositions expressing what *seems* to him at the moment to be the case, it is even unclear what a refutation would be like. But that is a topic for another day". (pp. 138-139).

146. Morscher, Edgar. 1986. "Was Existence Ever a Predicate?" *Grazer Philosophische Studien* no. 25/26:269-284
Abstract: "The question "Was 'existence' ever a predicate?" in a way already suggests its own answer, that this is really the wrong question to ask, because 'existence' has always been a predicate. Even those, such as Kant, who supposedly opposed this view, in fact held it. They merely denied that 'existence' is a "normal" first-order predicate. Not only Kant, but also Bolzano, Frege and Russell claimed that it is a second-order predicate. There is substantive disagreement between Kant and Bolzano on the one hand and Frege and Russell on the other over two issues: the former claim that this second-order predicate applies to no concept analytically and that it can be properly ascribed to a singular concept, whereas the latter deny both of these claims."

147. ———. 1986. "Propositions and States of Affairs in Austrian Philosophy before Wittgenstein." In *From Bolzano to Wittgenstein. The Tradition of Austrian Philosophy*, edited by Nyiri, Janoc Cristof, 75-85. Vienna: Hölder-Pichler-Tempsky.

148. ———. 1987. "Propositions and All That: Ontological and Epistemological Reflections." In *Logos and Pragma. Essays on the Philosophy of Language in Honour of Professor Gabriël Nuchelmans*, edited by Rijk, Lambertus Marie de and Braakhuis, Henk A.G., 241-257. Nijmegen: Ingenium Publishers
"Bernard Bolzano was one of the first philosophers in modern times to develop explicitly a complete theory for entities like propositions, statements and states of affairs. I will first describe and clarify the main features of his theory, and then sketch the subsequent development to our day." (p. 243)
(...)
"Let me now complete my historical sketch. Up to now I have only discussed Bolzano's doctrine of propositions. I concentrated on Bolzano's doctrine because I think that he gave the clearest account, the clearest description of propositions available in his time, and that none of the philosophers who followed, including Frege, has made an essential improvement in this respect. Although Bolzano's doctrine, his description of the propositions and the ontological status he ascribes to them, is far from being satisfactory, because it is insufficiently clear, no other philosopher up to our time has done any better. I have therefore explained Bolzano's doctrine in more detail in order to have one representative traditional doctrine to which I can refer in what follows.
What seems very interesting to me and what I have always been very impressed by is the fact that philosophers with completely different backgrounds and from

different schools developed, at the same time as Bolzano and afterwards, quite similar views, sometimes using almost the same words as Bolzano, without being familiar with his work. Although this is far from being a proof for the truth of his doctrine, it is nevertheless a fact a philosopher cannot pass by because it indicates that this is not the doctrine of an eccentric outsider. On the contrary, it has attracted many philosophers, including such prominent ones as Frege, Wittgenstein and Russell, Husser! and Meinong, Windelband and Rickert. (I have described the views of some of these philosophers and compared them in another paper: Morscher (1972 [*Von Balzano zu Meinong: zur Geschichte des logischen realismus*])." (p. 248)

149. ———. 1997. "Bolzano's Method of Variation: Three Puzzles." *Grazer Philosophische Studien* no. 53:139-165
Abstract: "Bernard Bolzano's most fruitful invention was his method of variation. He used it in defining such fundamental logical concepts as logical consequence, analyticity and probability. The following three puzzles concerning this method of variation seem particularly worth considering. (i) How can we define the range of variation of an idea or the categorial conformity of two ideas without already using the concept of variation? This question was raised by Mark Siebel in his M.A. thesis. (ii) Why must we define analyticity by means of (simultaneous or successive) variation of several ideas rather than by means of replacing a single idea? This problem is suggested by an example due to W.V.O. Quine, John R. MyhiII and Benson Mates. (iii) Must every 'there is ...' sentence be synthetic for Bolzano, as his pupil Franz Prihonsky claims in his booklet *Neuer Anti-Kant,* or can a `there is...' sentence be logically analytic?"

150. ———. 2006. "The Great Divide within Austrian Philosophy. The Synthetic a Priori." In *The Austrian Contribution to Analytic Philosophy*, edited by Textor, Mark, 250-263. New York: Routledge
"In this chapter I will try to show that the divergent Austrian ways of being anti-Kantian do not vanish even when we focus on this single topic. To illustrate this view, I will take as my examples Bernard Bolzano and Rudolf Carnap, who both belong – for different reasons – to the so-called Austrian tradition in philosophy. Both are fully conversant with Kant's work, and both have a critical attitude toward it and are in this sense anti-Kantian. This is also true when it comes to the question of the synthetic a priori: both refute strongly Kant's treatment of the synthetic a priori. However, whereas Carnap denies synthetic sentences a priori altogether, Bolzano does not deny their existence but only the way in which Kant justifies their truth.
What is even more important is that Bolzano not only – contrary to Carnap – accepts Kant's synthetic a priori, but even extends it to the realm of logic. In clear opposition to Kant and Carnap, who take all logical truths to be analytic, there are synthetic truths for Bolzano even in the area of logic. I will try to argue for this claim in the following sections." (p. 250)

151. ———. 2008. *Bernard Bolzano's Life and Work*. Sank Augustin: Academia Verlag
Table of Contents: Preface 9; Introduction 13; 1. Bolzano's Life and Scientific Career 17; 2. Bolzano's Removal from Office and the "Bolzano Trial" 23; 3. A Short Survey of Bolzano's Work 29; 4. Logic 33; 5. Epistemology and Philosophy of Science 75; 6. Ethics 89; 7. Aesthetics 107; 8. Political and Social Philosophy 113; 9. Philosophy of Religion and Theology 125; 10. Metaphysics 135; 11. Philosophy of Nature and of Physics 139; 12. Philosophy of Mathematics 141; 13. Metaphilosophy and History of Philosophy 149; 14. The So-called Bolzano Circle and Bolzano's Influence on the Development of the Sciences and on Intellectual History 151; Appendix: A Formal Reconstruction of Bolzano's Method of Idea-Variation and of his Definitions of Logical Truth and of Logical Consequence 159; Bibliography 169; Index of Names 207-211.
"Despite the enormous increase of interest in Bolzano's philosophy during the last decades, an up-to-date monograph on Bolzano's philosophy is still a desideratum.

The last book that might be called a monograph on Bolzano's philosophy dates from almost 100 years ago; it is Shmuel Hugo Bergmann's *Das philosophische Werk Bernard Bolzanos* (Halle/S. 1909), written in the spirit of the Brentano school, in particular of Bergmann's teacher Anton Marty.
When I was invited by the Editors of the *Stanford Encyclopedia of Philosophy* to contribute the entry on Bernard Bolzano, I took it as a challenge for starting my long-standing plan to write a monograph on Bolzano's philosophy. The present book is, to be clear, merely the first step toward this end." (from the *Preface*)
(...)
"Bolzano's uncommonly versatile work culminated in three extensive main writings in three different areas of knowledge: 1) in theology his four volume *Textbook of the Science of Religion* (Bolzano 1834b), 2) in philosophy the four volume *Theory of Science* (Bolzano 1837a), which provides a new foundation for logic and is at the same time an extensive manual of logic, and 3) in mathematics the *Theory of Quantities*, conceived of as a monumental work, but not completed.
Bolzano's teaching was concerned exclusively with fundamental topics of theology, in addition he worked mainly in logic. Nevertheless, his scientific development began in mathematics. It was mathematics that was the starting point for his scientific work and to which he ultimately returned in order to create a new foundation on which mathematics as a whole could be built; he succeeded in doing this, however, only in bits and pieces." (p. 29)

152. ———. 2022. "The Grounds of Moral 'Truths'." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 343-363. New York: Oxford University Press
"Central to Bolzano's ethics is his Supreme Moral Law, i.e. an ethical truth that grounds all other ethical truths. While Bolzano considers this law to be fundamental in the realm of ethics, he also claims that it is not
an ungrounded, basic truth. Edgaqr Morscher discusses this view in the context of a succinct reconstruction of Bolzano's views on grounding, his ethics, and his deontic logic." (p. 38)

153. Morscher, Edgar, and Simons, Peter. 2014. "From Bolzano via Quine to Fine." In *Joint Ventures in Philosophy*, edited by Morscher, Edgar and Simons, Peter, 137-156. Sankt Augustin: Academia Verlag.

154. Mugnai, Massimo. 1992. "Leibniz and Bolzano on the "Realm of Truths"." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 207-220. Firenze: Leo S. Olschki
"In his article *Propositions and Sentences* (1956) Alonzo Church pointed out -- on the basis of a suggestion made by Joseph Maria Bochenski -- that strong analogies exist between Bolzano's theory of *Satz an sich* and Gregory of Rimini's doctrine of complexe significabile.' In the same essay, Church also pointed out that Bolzano appealed to Leibniz as to a logician who plainly recognized propositions in the abstract sense. After Church's essay, it became very usual to mention Gregory of Rimini in reference to Bolzano's ontological conceptions. Nevertheless, we do not have any evidence of a direct influence of Gregory of Rimini's ideas on Bolzano's philosophy of logic. Bolzano seems to have only a limited acquaintance with the logic of the late medieval period: the credit accorded to Savonarola's Compendium logicae - a standard work which is absolutely lacking in originality - corroborates, I think, this view.2 Yet Bolzano may have benefited by late scholastic inheritance through the intermediation of later works, like those of Campanella, Clauberg, Fonseca, Keckermann, Leibniz and Wolf.' In fact, as already mentioned, Leibniz is the first author whom Bolzano explicitly refers to, in paragraph 21 of the Wissenschaftslehre, as a forerunner of the Satz an sich theory:
"Thus Leibniz uses as equivalent the expressions *proposition* and *cogitatio possibilis (Dial. de Connexion inter Verba a Res* [C. I. Gerhardt, ed. *Philos. Schriften, vol.* VII, p. 190]. This obviously presupposes that by propositions he meant propositions in themselves." (*)

The Leibniz's work on which Bolzano explicitly bases this conviction is the *Dialogus de connexione inter res et verba*, first published by Raspe in 1765 -- a work whose content paradoxically seems to partly disprove Bolzano's interpretation.' Thus Church considers it «
"an exaggeration or a misunderstanding" on Bolzano's part to have attributed to Leibniz's D*ialogus* "the use of the word propositio for proposition in the abstract sense" or *Satz an sich*.(6) The same remarks are repeated by Prof. Berg in his monograph on Bolzano's logic: after having identified Bolzano's *Satz an sich* with Frege's *Gedanke*, Prof. Berg writes:
According to Leibniz a proposition (*propositio*) is a possible thought (*cogitatio possibilis*), which is capable of being true or false... But no thought or reasoning is possible without words or some other kind of signs. And under transformation of a proposition into a different language a certain relationship (*proportio*) among the signs and between the signs and the objective reality is transformed into a similar relationship. The last two conditions fit Aristotle's and Peter of Spain's but not Frege's notion of proposition. Therefore... it must have been a misunderstanding on Bolzano's part to have attributed to Leibniz the use of the world "propositio" for *Satz an sich.*(7)
In what follows, I intend to take up the problem of the correcteness of the interpretation given by Bolzano and then to develop a comparison between the positions of Leibniz and those of Bolzano relative to the notions of idea, proposition and truth.
In the notes I have employed the following abbreviations: WL = B. Bolzano, *Wissenschaftslehre*, in B. Bolzano, Gesamtausgabe, Reihe I, Schriften, Stuttgart-Bad Cannstatt, Friedrich Frommann Verlag 1985 ff; GP = G. W Leibniz, *Die philosophische Schriften*, Hrsg. von C. I. Gerhardt, Berlin, Akademie, 1857-90, vol. I-VII; VE = G. W. Leibniz, *Vorausedition zur Reihe VI - Philosophische Schriften* - Munster, Akademie, 1982 ff.
(1) A. Church, *Propositions and Sentences*, in I. M. Bochenski, A. Church, N. Goodman, *The Problem of the Universals*, Notre Dame, Notre Dame Press, 1956, p. 3.
(2) WL 1, 11/1, pp. 105 ff.
(3) WL 1, 11/1, pp. 234 ff.
(4) WL 1, 11/1, p. 111.
(5) Cfr. *Oeuvres philosophiques latines et francoises de feu Mr. de Leibnitz ...* publiees par Mr. Rud. Eric Raspe, Leipzig, 1765, pp. 505-512.
(6) A. Church, *op. cit*., p. 10.
(7) J. Berg, *Bolzano's Logic*, Stockholm, Almquist and Wiksell, 1962, pp. 51-52.
(*) [cited in German in the original; I cite from the translation of *Wissenschaftslehre* by Rolf George, p. 24]

155. Mulligan, Kevin. 2022. "Logic, Logical Norms, and (Normative) Grounding." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 244-275. New York: Oxford University Press
"While Bolzano's writings were largely ignored by the philosophical community of his time, they later aroused the attention of Franz Brentano and his students, in particular that of Edmund Husserl. Kevin Mulligan examines a range of Husserl's views on grounding and their relation to Bolzano's views. A particular emphasis is laid on Husserl's conception of logic: is logic a normative or a theoretical discipline? Relatedly:
what is the connection between logical norms and logical truths? Husserl argues that logic is a theoretical discipline and that logical truths ground logical norms. In order to understand and evaluate this view, it is compared with Bolzano's account of the grounds of moral truths." (p. 37)

156. Neeman, Ursula. 1970. "Analytic and Synthetic Propositions in Kant and Bolzano." *Ratio* no. 12:1-25
"Whereas Kant regards the structure of being and knowing as identical, Bolzano interprets the Kantian true synthetic propositions as true propositions, in which the

predicate is a characteristic of the subject and not a component of the notion of the subject (characteristic =df. a property of the object, which falls under the concept; component =df. ingredient of the concept). These propositions are analytic in a wider sense, because they render possible an analysis of an object, whereas the logico-analytic propositions render possible only an analysis of their concept. Therefore Bolzano also distinguishes between deductibility (ordo cognoscendi) and ground-consequence relation (ordo essendi) and grounds the latter on the principle of simplicity. A discovery of an objective connection in mathematics is only possible by a strict determination of the basic concepts and by axiomatization, because in opposition to Kant, Bolzano thinks mathematical laws to be discoveries and not creations of the human mind."

157. Otte, Michael. 2008. "Proof and Explanation from a Semiotical Point of View." *Relime*:23-43
Abstract: "A distinction between proofs that prove and proofs that explain has over and again played an important role within recent discussions in epistemology and mathematics education.
The distinction goes back to scholars who, like Bolzano or Dedekind, have tried to reestablish pure mathematics as a purely conceptual and analytical science. These endeavors did in particular argue in favor of a complete elimination of intuitive or perceptual aspects from mathematical activity, arguing that one has to rigorously distinguish between a concept and its representations. Using a semiotical approach which negates such a separation between idea and symbol, we shall argue that mathematics has no explanations in a foundational sense. To explain amounts to exhibiting the meaning of something.
Mathematics has, however, as we shall try to show, no definite meanings, neither in the structural intra-theoretical sense nor with respect to intuitive objectivity. Signs and meanings are processes, as we shall argue along with Peirce."
"Before we can address the issue of proof and explanation we have to get rid of traditional *Bewusstseinsphilosophie* (philosophy of consciousness), that is, popularly speaking, the belief that “meanings are in the head” and knowledge is some sort of mental experience. After Kant epistemology began to ramify and various new philosophies of mathematicsarose in which meaning, rather than mind played the central role. But the view that there exists an epistemologically autarkic or self-sufficient epistemic subject, which serves itself from external sensations and internal experiences or representations (Vorstellungen) to thereby constitute true knowledge, is a myth and should also be abandoned.
In Part I of this paper we try to provide some pertinent arguments to this end, based on Peirce’s semiotics.
“Consciousness is used to denote the I think, the unity of thought; but the unity of thought is nothing but the unity of symbolization” (Peirce CP 7.585). Part II treats the questions of proof and explanation with respect to the ideas of Bolzano on the one hand and Peirce on the other. Part III presents some examples and tries to make a connection with current debates about the issue in mathematical education and cognitive psychology." (p. 25)

158. ———. 2009. "The Analytic/Synthetic Distinction in Kant and Bolzano." In *Relatively and Philosophically E^{a}rnest. Festschrift in Honor of Paul Ernest's 65 Birthday*, edited by Sriraman, Bharath and Goodchild, Simon, 39-56. Missoula: Information Age Publishing.

159. Parsons, Charles. 2012. "Two Studies in the Reception of Kant's Philosophy of Arithmetic." In *From Kant to Husserl: Selected Essays*, 80-99. Harvard: Harvard University Press
"The present essay takes its point of departure from a thought I have had at various times in thinking about interpretations of Kant’s philosophy of mathematics in the literature, in particular that offered by Jaakko Hintikka. That was that if the interpretation is correct, shouldn’t one expect that to show in the way that Kant’s views were understood by others in the early period after the publication of the first

Critique? That reflection suggests a research program that might be of some interest, to investigate how Kant's philosophy of mathematics was read in, say, the first generation from 1781. I have not undertaken such a project. However, I will make some comments about two examples of this kind. In doing so I haven't always kept my eye 0n Kant, because the figures involved are of interest in their own right. The first is Johann Schultz (1739-1805), the disciple of Kant who was professor of mathematics in Köinigsberg.Thc second is Bernard Bolzano (1781-1848), who in an early essay of 1810 (*)
offered a highly critical discussion of Kant's theory of construction of concepts in intuition. In one way, I think the result of this little experiment is negative, in that it does little toward settling disputed questions about the interpretation of Kant. On the other hand, I think it brings out some problems of Kant's views that could be seen either at the time he wrote or not long after." (p. 80)
(*) [*Beyträge zu einer begründeteren Darstellung der Mathematik = Contributions to a Better Grounded Presentation of Mathematics*]

160. Poggiolesi, Francesca. 2022. "Bolzano, (the Appropriate) Relevant Logic, and Grounding Rules for Implication." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 319-342. New York: Oxford University Press
"While Bolzano sharply distinguishes grounding from logical deducibility {in modern terminology: entailment), he also regards the two notions as importantly connected. He sees a particularly close connection
between grounding and exact deducibility (a special case of deducibility). Francesca Poggiolesi examines this latter notion and compares it to notions of relevant entailment. She argues that Neil Tennant's system CR is the best model for Bolzano's ideas, and can in turn also serve as a framework for developing grounding rules for conditionals." (p. 37)

161. Proust, Joëlle. 1981. "Bolzano's Analytic Revisited." *The Monist. An International Quarterly Journal of General Philosophical Inquiry*:214-230
"What I propose is to reconsider the interpretation of Bolzano's concept of analytic propositions which was offered thirty years ago by Bar-Hillel.(1) The claim of Bar-Hillel was that, in a late addition to his book, *The Theory of Science*,(2) Bolzano actually had been radically improving his concept of analyticity, thus creating some inconsistencies with the previous, uncorrected version. This allows us to equate the new Bolzanian definition of analytic with what was to be defined, a century later, as logical truth by W. V. Quine. Bar Hillel's interpretation has been uncritically accepted by commentators, although the historical issue has been rightly challenged by J. Berg. What I want to show is that, in spite of a surface analogy between Bolzano's phrasing of the definition of 'logical analytic' and Quine's definition, certain considerations should lead us to call that parallel into question. Attractive as it may be for a Quinian, such a view of Bolzano's analytic can be shown as incompatible with the leading ideas of his philosophy of logic. Furthermore, there is enough evidence in other sections of the *Theory of Science* to show that Bolzano's criterion of analyticity is grounded on purely semantical properties and is part of a general account of logical properties in terms of the mapping of propositions to corresponding models." (p. 214)
(1) "Bolzano's Definition of Analytic Propositions," *Theoria* 16 (1950): 91-117 and *Methodos* 2, (1950): 32-55; reprinted in *Aspects of Language* Magnes Press 1970, pp. 3-32.
(2) Bernard Bolzano, *Wissenschaftslehre* (Sulzbach 1837, Leipzig 1914); partly translated by Rolf George: *Theory of Science*, (Oxford: Basil Blackwell, 1972). We shall quote this translation whenever available.

162. ———. 1989. *Questions of Form: Logic and the Analytic Proposition from Kant to Carnap*. Minneapolis: University of Minnesota Press
Translated by Anastasios Albert Brenner from the original French: *Questions de forme. Logique et proposition analytique de Kant à Carnap* - Paris, Fayard, 1986.

Section Two: *Bolzano's Renovation of Analiticity,* pp. 49-108.
"The specifically Bolzanian concept of analyticity is brought in at an advanced life, as the maturely formulated answer to a problem that never ceased to appear under different aspects. Only in the *Wissenschaftslehre* of 1837 does what we might call a "revolution" in analyticity occur. Earlier texts strive to adapt the Kantian definition so that it satisfies the new requirements of the anticritical mathematicians. But this definition, often revised, gives rise to growing difficulties. There were so many reasons for abandoning it, but also so many constraints working to shape the new definition. "Revolution," we said; but until the *Wissenschaftslehre*, analyticity was a marginal theme. Its main function was, as in Kant, to reveal the problematic existence of the synthetic a priori. From a theme of preliminary exposition, analyticity becomes in the work of 1837 an "integrated" concept: henceforth it is part of a philosophy and becomes inseparable from a method of identifying logical objects, variation. But this was a "Ptolemaic," not a "Copernican," revolution: instead of statically emphasizing the synthetic a priori, it becomes a notable property of certain propositions whose definition now requires a preliminary examination of other properties such as truth and validity. This definition, however, does not have a purely descriptive interest; the theses of Volume 3 must be taken seriously in order to portray with perfect clarity the deep interest that Bolzano had in his new definition of analyticity." (p. 49)

163. ———. 1999. "Bolzano's Theory of Representation." *Revue d'Histoire des Sciences* no. 52:363-383
Abstract: "Bolzano's theory of representation is one of the most radically intensionalist approaches to representation. It is based on the following three claims: A). A representation is essentially independent of thought and of linguistic expression; B). A representation is structured; C). Such a structure is independent of the objects represented. These claims are both tools and constraints relative to Bolzano's substantive goals. Bolzano ultimately aimed to carry out a deep transformation of mathematical and scientific practice, thanks to a more accurate conception of logic and of the role of logic in scientific exposition. I examine some of the consequences of Bolzano's claims in regard to his conception of mathematical treatises."

164. Raspa, Venanzio. 1996. "Su ciò che non esiste. Da Bolzano a Meinong: un *excursus* nella filosofia austriaca." *Studi Urbinati.B: Scienze Umane e Sociali* no. 67:115-201
1. Ci sono oggetti che non esistono. - 2. Rappresentazioni in sé e rappresentazioni senza oggetto .in B. Bolzano. - 3. La mediazione storica di R. Zimmermann. - 4. Il capovolgimento delle rappresentazioni senza oggetto in K. Twardowski. - 5. · Oggetti non esistenti nella Gegentstandstheorie di A. Meinong. - 6. Aspetti della controversia fra Russell e Meinong.
"L'apparato concettuale di base che, per successivi sviluppi, ci porterà ad una messa a fuoco del nostro discorso, e maturerà nella *Gegenstandstheorie* di Meinong, ci viene offerto da Bernard Bolzano, il primo autore di questa storia. Bolzano assume all'interno del suo universo, in cui si danno anche oggetti non esistenti, una classe di oggetti logici, le cosiddette rappresentazioni senza oggetto [*gegenstandslose Vorstellungen*], vale a dire - nel suo linguaggio - rappresentazioni in sé [*Vorstellungen an sich*] cui non corrisponde nessun oggetto, in quanto gli attribuiscono proprietà fra loro contraddittorie, oppure che non si ritrovano nell'esperienza. Nell'ambiente filosofico austriaco a cavallo fra la seconda metà del XIX e l'inizio del XX sec., le rappresentazioni senza oggetto non vengono accolte nei termini in cui erano state elaborate da Bolzano, ma vengono, in un certo senso, capovolte; al loro posto compaiono gli oggetti non esistenti contraddittori oppure non fattuali. Dalle rappresentazioni senza oggetto di Bolzano si giunge, attraverso una duplice mediazione, quella di Robert Zimmermann e quella ben più determinante di Kazimierz Twardowski, il vero artefice del capovolgimento, agli oggetti non esistenti di Alexius Meinong. Quel che vorrei cercare di ricostruire è la maniera in cui avviene il capovolgimento e gli sviluppi teorici cui esso dà luogo. Procediamo dunque col chiederci: cosa intende specificamente Bolzano per

'rappresentazione in sé' e, quindi, per 'rappresentazione senza oggetto'? Le rappresentazioni in sé possono anche essere denominate concetti (4); se questo può dare un idea del tipo di nozione con cui abbiamo a che fare, restano tuttavia ancora da spiegare le due caratteristiche essenziali corrispondenti alle espressioni di 'in sé [*an sich*]' e 'senza oggetto [*gegenstands los*]'.." (pp. 118-119)
(4) Sulla scelta del termine 'rappresentazione' preferito a quello di 'concetto', cfr._ B. Bolzano, *Wzssenschaftslehre*. Versuch einer ausführlichen und grossentheils neuen Darstellurzg der Logik mit steter Rücksicht auf deren bisherige Bearbeiter, 4 Bde., Sulzbach, J. E. v. Seidelschen Buchhandlung 1837, Bd. I, § 50, p. 222-223 (d'ora in avanti *WL*). Per le citazioni si è tenuta presente l'edizione critica della *Bernard Bolzano-Gesamtausgabe*, Reihe I: Schriften, Bde. 11-14: Wissenschaftslehre, hrsg. von J. Berg, Stuttgart/Bad Cannstatt, Frommann/Holzboog 1985 sgg.

165. Roberts, Mark. 1994. "The Bearer of Truth and Falsity." *Southwest Philosophy Review* no. 10:59-67
Abstract: "Until Bolzano nearly all philosophers believed that truth and falsity are predicated of judgments of beliefs. Bolzano and other philosophers after him argue that propositions are the bearers of truth and falsity and that propositions have a timeless ideal existence: a position which seems to discredit completely their view that propositions are the bearers of truth and falsity. Yet, several arguments can be offered which show that propositions are the bearers of truth and falsity without introducing as a premise the timeless existence of propositions."

166. Rohloff, Waldemar. 2012. "From Ordinary Language to Definition in Kant and Bolzano." *Grazer Philosophische Studien* no. 85:131-149
Abstract: "In this paper I discuss Kant's and Bolzano's differing perspectives on ordinary natural language. I argue that Kant does not see ordinary language as providing semantically organized content and that, as a result, Kant does not believe that ordinary language is sufficiently well-developed to support philosophical analysis and definition. By contrast, for Bolzano, the content given in ordinary language are richly structured entities he calls 'propositions in themselves'. This contrast in views is used to explain Bolzano's criticism of Kant's belief that definition is impossible for philosophical concepts. It is also used to explain Bolzano's criticism of Kant's methods of exposition of philosophical concepts."

167. Rojszczak, Artur. 2005. *From the Act of Judging to the Sentence. The Problem of Truth Bearers from Bolzano to Tarski*. Dordrecht: Springer
Edited by Jan Wolenski.
Chapter 7.1: *Bernard Bolzano (I): Sentences in Themselves*, pp. 111-115.
"I shall not go into the details of the multiplicity of Bolzano’s ideas and their particular influence on the history of semantics. I shall, as I have tried to do with respect to every issue in this study, concentrate on his ideas within the theory of science as it is related to the problem of the truth bearer. In the context of the theory of truth, it is worth noting that Bolzano’s position during his times, i.e. in the first half of the nineteenth century, was quite unusual. Bolzano’s influence on this century was provided by his notion of the objectivity of truth in a way that also remained standard for the next century. Furthermore, the theory which should guarantee the objectivity of truth was, for Bolzano, his theory of sentences in themselves. Only the semantics of the twentieth century sees Bolzano’s theory of sentences in themselves as an anticipation of the contemporary notion of proposition. I shall, however, refer to his *Fundamentallehre* [Theory of Fundamentals], i.e. to the first sections of his *Theory of Science*, which deals with the existence of objective truth and with the possibility of its cognition. I shall omit some elements of this theory that are irrelevant to my purposes; for example, Bolzano’s proof of the existence of truth, his proof of the existence of infinitely many truths or the argument for the cognition of truths. In this part of Bolzano’s argumentation, he focuses on the problem of skepticism, making an attempt to prove the fundamentalist position in epistemology.(1) I shall take the liberty of

presenting Bolzano's ideas as far as truth bearers are concerned as contrasted with the views of Brentano and Twardowski on the objectivity of truth which I shall present in the next sections." (p. 111)
(1) Bolzano 1837, par. 40–43.

168. Rojszczak, Artur, and Smith, Barry. 2003. "Truthmakers, Truthbearers and the Objectivity of Truth." In *Philosophy and Logic in Search of the Polish Tradition: Essays in Honour of Jan Wolenski on the Occasion of His 60th Birthday*, edited by Kijania-Placek, Katarzyna, 229-268. Dordrecht: Kluwer
"The aim of this paper is to show that the account of objective truth taken for granted by logicians at least since the publication in 1933 of Tarski 's 'The Concept of Truth in Formalized Languages' arose out of a tradition of philosophical thinking initiated by Bolzano and Brentano. The paper shows more specifically that certain investigations of states of affairs and other objectual correlates of judging acts, investigations carried out by Austrian and Polish philosophers around the turn of the century, formed part of the background of views that led to standard current accounts of the objectivity of truth. It thus lends support to speculations on the role of Brentano and his heirs in contemporary logical pbilosophy advanced by Jan Woleński in his masterpiece on the *Logic and Philosophy in the Lvov-Warsaw School* of 1989." (p. 229)

169. Rollinger, Robin D. 2004. "Austrian Theories of Judgment: Bolzano, Brentano, Meinong, and Husserl." In *Phenomenology & Analysis. Essays on Central European Philosophy*, edited by Chrudzimski, Arkadiusz and Huemer, Wolfgang, 257-284. Frankfurt: Ontos Verlag
Reprinted in: R. D. Rollinger, *Austrian Phenomenology. Brentano, Husserl, Meinong, and Others on Mind and Object*, Frankfurt: Ontos Verlag 2009, pp. 233-262.
"Introduction
In nineteenth century German philosophy it was among the prevailing views that mental phenomena were to be divided into three classes: thinking, feeling, and willing. In Austria, however, two of the towering philosophers, Bernard Bolzano and Franz Brentano, held that presentations (*Vorstellungen*) and judgments (Urteile) make up two distinct classes of mental phenomena. Moreover, both of these philosophers saw it as an important task to work out a theory of judgment in particular. It is accordingly no surprise that Brentano's two most outstanding pupils, Alexius Meinong and Edmund Husserl, developed theories of judgment, though their results were markedly different from those of their predecessors and from each other's. In the following the line of Austrian philosophy from Bolzano to Husserl will be traced by presenting an overview of the four theories indicated in the title. The topic under consideration in these theories, though apparently little more than a chapter in descriptive psychology, is of great significance because it gives us an intersection for issues in epistemology, ontology, and philosophy of logic." (p. 257)

170. Rootselaar, Bob van. 1970. "Bernard Bolzano." In *Dictionary of Scientific Biography, Vol. 2*, edited by Gillispie, Charles Coulston, 273-279. New York: Charles Scribner's Sons
"Bolzano planned to elaborate the methodology begun in his *Beyträge* and to develop it into a complete lheory of science, of which a treatise on logic was to form the cornerstone. From 1820 on, he worked steadily on it, and his four-volume treatise *Wissenschaftslehre* appeared in 1837. The plan of the W*issenschaftslehre* appears clearly from the following subdivision (see Kambartel, *Bernard Bolzano's Grundlegung der Logik*, pp. 14-17):
(I) Fundamental theory: proof of the existence of abstract truths and of the human ability to judge.
(2) Elementary theory: theory of abstract ideas, propositions, true propositions, and deductions.
(3) Theory of knowledge: condition of the human faculty of judgment.
(4) Heuristics: rules to be observed in human thought in the search for truths,

(5) Proper theory of science: rules to be observed in the division of the set of truths into separate sciences and in their exposition in truly scientific treatises.
The work did not induce a complete revision of science, as Bolzano hoped, but, on the contrary, remained unnoticed and did not exercise perceptible influence on the development of logic. Some of the
innovations in logic contained in the first two volumes did attract attention, as well as excessive praisenotably from Edmund Husserl and Heinrich Scholz (see Berg. op. cit.; Kambanel, op. cit.; and the literature
cited in them).
The rise of logical semantics, initiated by Alfred Tarski in the l 930's, has led to a revival of the study of Bolzano's logic in the light or modem logic (see Berg, op. cit.) and of his theory of an ideal language.
The heart of Bolzano's logic is formed by his concepts of (abstract) proposition (*Satz an sich*), abstract idea (*Vorstellung an sich*), truth, and the notions of derivability (*Ab!eirbarkei*t) and entailment (*Abfolge*)." (pp. 277-278)

171. ———. 1992. "Axiomatics in Bolzano's Logico-Mathematical Research." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 221-230. Florence: Leo S. Olschki
"A discussion of Bolzano's axiomatical considerations requires some care, because his idea of axiomatization differs considerably from axiomatics as it is currently understood.
His *Wissenschaftslehre* is testimony of his concern for the foundation of science in general and in particular of the theoretical sciences. Among the theoretical sciences mathematics is of special interest.
According to Bolzano, the mathematics of his time was based on shaky foundations, and one of his activities was directed toward correction of this situation.
On the other hand he certainly had the intention to recapture essentially the entire body of existing mathematics and present it in full accordance with his newly laid foundations. This is the reason why on several occasions he revised existing proofs of known mathematical theorems." (p. 221)

172. Rosenkoetter, Timothy. 2012. "Kant and Bolzano on the Singularity of Intuitions." *Grazer Philosophische Studien* no. 85:89-129
Abstract: "Kant and Bolzano agree that intuitions are non-accidentally singular, but each offers more than one explanation of why this is the case. One model, exemplified by Bolzano's explication of intuitions as "this"-representations, posits a type of representation which is such that it can only have one object. A very different explanation, prominent in Kant's Transcendental Aesthetic, has recourse to the fact that certain classes of objects (spaces and times) can have only one instance, and argues on this basis that some representations with those contents are singular. This paper surveys various versions of these two explanations and uses each philosopher's answers to shed light on the other's."

173. Roski, Stefan. 2013. "A priori Knowledge in Bolzano: Conceptual Truths and Judgements." In *Judgement and the Epistemic Foundation of Logic*, edited by Schaar, Maria van der, 101-132. Dordrecht: Springer
"According to Kant, a true judgement can be called a priori in case it can take place absolutely (*schlechterdings*) independent of experience. Propositions that are knowable in this way are called a priori propositions by him (Kant, [*Critique of Pure Reason*], 1787 B, 3–4)." (p. 101)
(...)
"[Bolzano] tried to *give* a satisfactory theoretical account of the notion of synthetic *a priori* proposition. Roughly speaking, he located Kant's mistake in the attempt to introduce a distinction among propositions by means of a distinction among judgements. Bolzano reversed this order and aimed instead to explicate the valid core of what Kant tried to capture in epistemic terms entirely in objective, logical ones." (p. 101)

(...)
"Bolzano's explication has two aspects, a logical and an epistemological one. The logical aspect consists in drawing a precise and workable distinction in terms of non-epistemic notions. The epistemological aspect concerns the way in which Bolzano's suggestion might work: What is his account of how one can come to know synthetic truths a priori?
While there have been investigations of Bolzano's objective explication of the notion of a priori proposition (see Textor 1996, chapter 4), the epistemological details have never been examined in great detail.(2) The task of this chapter is thus to tell the epistemological story behind Bolzano's objective explication.
I should note right from the beginning that the aim of the chapter is descriptive and historical. Primarily, I want to make sense of what Bolzano plausibly had in mind, rather than assess its intrinsic plausibility." (p. 102)
(2) An exception is Lapointe (2010).
References
Lapointe, S. 2010. Bolzano, a priori knowledge and the classical model of science. *Synthese* 174:263–281.
Textor, M. 1996. *Bolzanos propositionalismus*. Berlin/New York: Walter De Gruyter.

174. ———. 2017. *Bolzano's Conception of Grounding*. Frankfurt: Vittorio Klostermann
Contents: Preface IX; I. Introduction 1; 2. Objective truth, variation & truth-preservation 19; 3. Explanatory priority: Bolzano's pure logic of grounding 55; 4. Simplicity and economy: Bolzano's impure logic of grounding 109; 5. Bolzano's logic of grounding and the logic of metaphysical grounding 215; 6. Conclusion 233; l ist of abbreviations 251; List of symbols, definitions, and principles 253; Bibliography 257; Index 267-269.
"Overview of the book
As each of the following chapters will be accompanied by a detailed overview of its content and line of argumentation, I will confine myself here to a brief overview of the main line of argumentation of the book.
At the core of Bolzano's theory of grounding lies a set of general principles that express properties the relation exhibits according to him. An analysis of these principles, their interrelation, and their role in Bolzano's methodology will form the main bulk of the book. It is heuristically useful to divide these principles into two classes. The first class contains principles that hold for every case of grounding, irrespective of any specific properties of the relata. These principles capture, as it were, minimal conditions an explanatory relation has to satisfy according to Bolzano. The second class consists of more specific principles that mostly apply only to truths from deductive or a priori sciences. Adapting a distinction by Kit Fine, I will call the former Bolzano's *pure logic of grounding* and the latter his *impure logic of groundin*g.(52) Before we can dive into the details of Bolzano's theory, we will have to gain some familiarity with the nuts and bolts of his logical framework. This will be done in Chapter Two. Chapter Three then discusses Bolzano's pure logic of grounding, while Chapter Four is concerned with the impure logic of grounding. Chapter Five wraps up and draws some connections to the recent debate on grounding. In what follows I will sketch the content of each of these chapters in a little more detail." (p. 16)
(52) Cf. (Fine [The Pure Logic of Ground. *Review of Symbolic Logic* 5(1) 1-25] 2012b). The justification for employing this distinction will be given further below.

175. ———. 2019. "Bolzano and Kim on Grounding and Unification." *Synthese* no. 196:2971-2999
Abstract: "It is sometimes mentioned that Bernard Bolzano's work on grounding anticipates many insights of the current debate on metaphysical grounding. The present paper discusses a certain part of Bolzano's theory of grounding that has thus far not been discussed in the literature. This part does not so much anticipate what are nowadays common assumptions about grounding, but rather goes beyond them. Central to the discussion will be a thesis of Bolzano's by which he tries to establish

a connectionbetween grounding and (deductive) unification. The paper spells out this thesis in detail and discusses the assumptions on which it rests. Next to this mainly historical aim, the paper also presents reasons why philosophers who are not interested in the historical Bolzano should find the thesis interesting by relating it to a certain view on unification and explanation that has been put forward by Kim. A final part of the paper provides a critical evaluation of the thesis against the background of current accounts of grounding."
Reference
Kim Jaegwon (1994) Explanatory Knowledge and Metaphysical Dependence. *Philosophical Issues* 5:51–69-

176. ———. 2020. "Bolzano." In *The Routledge Handbook of Metaphysical Grounding*, edited by Raven, Michael J., 76-89. New York: Routledge
"This chapter provides an overview of Bernard Bolzano's views about grounding. On Bolzano's account, grounding is an objective priority relation among true propositions that has certain explanatory features.The chapter briefly highlights historical influences on Bolzano's account of grounding and subsequently provides an overview of the most important aspects of it. As we shall see, Bolzano's account resembles current accounts of metaphysical grounding in many respects and can thus easily be related to many positions in the current debate.This is going to be a main focus of this chapter. Apart from that, we shall investigate some Bolzanian ideas about grounding that differ from the current orthodoxy but may constitute interesting additions, challenges or inspirations for those working in the current debate." (p. 76)

177. Roski, Stefan, and Rumberg, Antje. 2016. "Simplicity and Economy in Bolzano's Theory of Grounding." *Journal of the History of Philosophy* no. 54:469-496
Abstract: This paper is devoted to Bolzano's theory of grounding (*Abfolge*) in his *Wissenschaftslehre*. Bolzanian grounding is an explanatory consequence relation that is frequently considered an ancestor of the notion of metaphysical grounding. The paper focuses on two principles that concern grounding in the realm of conceptual sciences and relate to traditionally widespread ideas on explanations: the principles, namely, that grounding orders conceptual truths from simple to more complex ones (Simplicity), and that it comes along with a certain theoretical economy among them (Economy). Being spelled out on the basis of Bolzano's notion of deducibility (*Ableitbarkeit*), these principles are revealing for the question to what extent grounding can be considered a formal relation."

178. Roski, Stefan, and Rusnock, Paul. 2014. "Bolzano on Necessary Existence." *Archiv für Geschichte der Philosophie* no. 96:320-359
Abstract: "This paper is devoted to an examination of Bolzano's notion of necessary existence, which has so far received relatively little attention in the literature.
We situate Bolzano's ideas in their historical context and show how he proposed to correct various flaws of his predecessors' definitions. Further, we relate Bolzano's conception to his metaphysical and theological assumptions, arguing that some consequences of his definition which have been deemed counterintuitive by some of his interpreters turn out to be more reasonable given the broadly Leibnizian background of his metaphysics. Finally, we consider some difficulties that arise from Bolzano's evolving views on freedom, which, at least in his early thought, was intimately linked with contingency. In an appendix, we discuss a recent debate on Bolzano's notion of necessary truth between Textor and Rusnock that has some bearing on our overall line of interpretation of Bolzano's notion of *necessary existence*."
References
Rusnock, P. 2012. "On Bolzano's Conception of Necessary Truth". *British Journal of the History of Philosophy* 20, 817-837.
Textor, M. 2013. "Bolzano on the Source of Necessity: A Reply to Rusnock". *British Journal of the History of Philosophy* 21, 381-392.

179. Roski, Stefan, and Schnieder, Benjamin. 2019. "Fundamental Truths and the Principle of Sufficient Reason in Bolzano's Theory of Grounding." *Journal of the History of Philosophy* no. 57:675-706
Abstract: "Bernard Bolzano developed his theory of grounding in opposition to the rationalists' Principle of Sufficient Reason (the PSR). He argued that the PSR fails because there are fundamental, that is, ungrounded truths. The current paper examines Bolzano's views on fundamentality, relating them to ongoing debates about grounding and fundamentality."

180. ———. 2022. "Introdfuction: A Survey of Bolzano's Theory of Grounding." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 4-34. New York: Oxford University Press
"In this survey paper, we pursue three aims:
- First, we briefly sketch the origins of Bolzano's views on grounding and the role that grounding plays in his philosophy.
- Second, we give an overview of Bolzano's mature conception of grounding, focussing on its most detailed exposition, which can be found in his *Theory of Science*.
- Third, we introduce elements and terminology from Bolzano's conceptual framework that are required to understand his theory of grounding." (p. 4)

181. ———, eds. 2022. *Bolzano's Philosophy of Grounding: Translations and Studies*. New York: Oxford University Press
Table of Contents: Acknowledgements IX; List of Tables and Figures XI; List of Contributors XIII; Part I: Stefan Roski, Benjamin Schnieder: Introduction 3; 1. Preamble; 2. A Survey of Bolzano's Theory of Grounding 4; 3. On the Contents of This Volume 35; Part II: Bolzano's Writings on Grounding (in English Translations); 4. Early Period: Scientific Method and the Foundations of Mathematics 45; 5. Middle Period: Theology and Metaphysics 85; 6. Mature Period: A Theory of Grounding 107; Part III: Research Papers on Bolzano's Theory; 7. Mark Malink: Aristotle and Bolzano on Grounding 221; 8. Kevin Mulligan: Logic, Logical Norms, and (Normative) Grounding 244, 9. Kit Fine:Some Remarks on Bolzano on Ground 276; 10. Mark Textor: Grounding, Simplicity, and Repetition 301; 11. Francesca Poggiolesi: Bolzano, (the Appropriate) Relevant Logic, and Grounding Rules for Implication 319; 12. Edgar Morscher: The Grounds of Moral 'Truths' 343; 13. Paul Rusnock: Grounding in Practice: Bolzano's *Purely Analytic Proof* in Light of the Contributions 364; 14. Marc Lange: Bolzano, the Parallelogram of Forces, and Scientific Explanation 394; 15. Benjamin Schnieder: A Fundamental Being: Bolzano's Cosmological Argument and Its Leibnizian Roots 418; Glossary of German Terms 445; Name Index 447; Subject Index 450-458.

182. Rumberg, Antje. 2013. "Bolzano's Concept of Grounding (*Abfolge*) Against the Background of Normal Proofs." *Review of Symbolic Logic* no. 6:424-459
Abstract: "In this paper, I provide a thorough discussion and reconstruction of Bernard Bolzano's theory of grounding and a detailed investigation into the parallels between his concept of grounding and current notions of normal proofs. Grounding (*Abfolge*) is an objective ground-consequence relation among true propositions that is explanatory in nature. The grounding relation plays a crucial role in Bolzano's proof-theory, and it is essential for his views on the ideal buildup of scientific theories. Occasionally, similarities have been pointed out between Bolzano's ideas on grounding and cut-free proofs in Gentzen's sequent calculus. My thesis is, however, that they bear an even stronger resemblance to the normal natural deduction proofs employed in proof-theoretic semantics in the tradition of Dummett and Prawitz."

183. Rusnock, Paul. 1997. "Bolzano and the Traditions of Analysis." *Grazer Philosophische Studien* no. 53:61-85
Abstract: "Russell, in his *History of Western Philosophy,* wrote that modern analytical philosophy had its origins in the construction of modern functional analysis by Weierstrass and others. As it turns out, Bolzano, in the first four decades

of the nineteenth century, had already made important contributions 'to the creation of "Weierstrassian" analysis, some of which were well known to Weierstrass and his circle. In addition, his mathematical research was guided by a methodology which articulated many of the central principles of modern philosophical analysis. That Russell was able to discover philosophical content within mathematical analysis was thus not surprising, for it had been carefully put there in the first place. Bolzano can and should, accordingly, be viewed as a founder of modern analytical philosophy, and not necessarily as an uninfluential one. This paper considers his work in mathematical and philosophical analysis against some of the relevant historical background."

184. ———. 1997. "Remaking Mathematics: Bolzano reads Lagrange." *Acta Analytica* no. 18:51-72
"With Cauchy, Bolzano was among the most thorough and acute of Lagrange's readers, and it is clear that Bolzano had a good deal of respect for him as a mathematician, going through his treatises pencil in hand as soon as they were published, and occupying himself with many of the same questions. Like Lagrange, Bolzano was dissatisfied with the state of the foundations of analysis; like him he sought to provide an autonomous foundation for this branch of mathematics, one free from appeals to infinitesimals, geometry, and motion. Bolzano also appears to have respected Lagrange's opinion on the contents of analysis.
(...)
This broad agreement on content, however, was accompanied by sharp disagreements concerning method. Indeed, Bolzano chose his early subjects in part precisely in order to accentuate these differences. For Lagrange's entire approach to analysis was out of harmony with Bolzano's philosophy of science. And as Lagrange's work was in many ways the highest expression of analysis around the beginning of the nineteenth century, Bolzano's criticisms applied quite generally to the state of mathematics at the time. The difficulties which he found were not of the kind that one could hope to resolve by small changes of detail. They were, rather, systemic. What was required, according to Bolzano, was no less than a "complete transformation" of mathematics, at least of those parts which are not to be rejected as completely incorrect.(3) Not one to make such a statement idly, Bolzano had already been working on the task for over a decade, and would spend a good part of the rest of his life attempting to finish the work, rebuilding mathematics from the ground up in line with his methodology. This led to a detailed confrontation with eighteenth-century and notably Lagrangian mathematics; and it is here, in Bolzano's criticisms, and the alternatives he proposes, that we find the unmistakable imprint of his philosophy." (pp. 2-3)
(3) *Rein analytischer Beweis des Lehrsatzes, daß zwischen je zwey Werthe, die eine entgegengesetzes Resultat gewähren, mindestens eine reele Würzel der Gleichung liege* (Prague, 1817), Preface; English translation by S. B. Russ, *Historia Mathematica* 7 (1980) 156-185.

185. ———. 1999. "Philosophy of Mathematics: Bolzano's Responses to Kant and Lagrange." *Revue d'Histoire des Sciences* no. 52 (3-4):399-428
Summary: "Bolzano's philosophy of mathematics is presented through a consideration of his critical responses to Kant and Lagrange."
"In a late essay, Bolzano describes the philosophy of mathematics as an activity aimed at discovering the objective grounds of propositions which we already know with the greatest certainty and evidence (1). For him, philosophy of mathematics was simply what we would now call foundational research in the broadest sense - that is, it was not just a matter of « ultimate » foundations (for instance set theory, logic, or the like), but also of the foundations of particular mathematical theories (for instance geometry, the calculus, combinatorics...). Bolzano was certainly committed to dealing with questions of ultimate foundations, with developing a unified system of mathematics from first principles - his detailed investigations of set theory and logic bear ample witness to this. He also understood, however, that foundational inquiries could be, at least provisionally, local. One could, as he

explained in the *Contributions to a better-founded presentation of mathematics* of 1810, assume certain propositions as locally primitive, deferring until a later date their proof from more basic principles (2). No sharp line can be drawn to separate such local questions from those of ultimate foundations. Searching for underlying principles, in whatever domain and at whatever level, was an activity he quite plausibly and in line with tradition regarded as philosophical." (pp. 399-400)
(1) Bernard Bolzano, *Was ist Philosophie?* (Wien, 1849), 23.
(2) Bernard Bolzano, *Beyträge zu einer begründeteren Darstellung der Mathematik* (Prag, 1810), part II, § 11 (hereafter: *Beyträge*).

186. ———. 2000. *Bolzano's Philosophy and the Emergence of Modern Mathematics*. Amsterdam: Rodopi
"In his own time, Bolzano was known primarily for his highly public life as a social and religious reformer, one of the leading figures of the Bohemian Enlightenment. In mathematics and logic - the concerns of this book - Bolzano was no less a reformer, developing strikingly modem views on logic, and attempting to recast mathematics in line with the methods set out in this new logic. He pursued this project doggedly,
attempting to carry it through to the last details. The results, although incomplete, are impressive, and worthy of our attention.
I have tried in this book to give an adequate sketch of Bolzano as a philosopher of mathematics and as a philosophical mathematician.
Within his mathematical work, I have chosen to focus on his research in the foundations of real analysis, as it is here where he had the greatest success, and where the positive imprint of his philosophical views is most apparent. Of his vast writings on logic, I have confined my attention mainly to those parts which bear most directly on mathematical method. Much of Bolzano's mathematics and logic will no doubt appear quite familiar, and it is easy to forget just how new and strange this territory was when Bolzano - often on his own - first moved into it. For this reason, I have attempted also to convey something of the historical context of his work." (pp. 4-5)

187. ———. 2011. "Kant and Bolzano on Logical Form." *Kant-Studien* no. 102
Abstract: "In the works of Kant and his followers, the notion of form plays an important role in explaining the apriority, necessity and certainty of logic. Bernard Bolzano (1781–1848), an important early critic of Kant, found the Kantians' definitions of form imprecise and their explanations of the special status of logic deeply unsatisfying. Proposing his own conception of form, Bolzano developed radically different views on logic, truth in virtue of form, and other matters. This essay presents Bolzano's views in the light of his criticisms of the Kantian logicians."

188. ———. 2012. "Remarks on Bolzano’s Conception of Necessary Truth." *British Journal for the History of Philosophy* no. 20:817-837
Abstract: "This essay presents a new interpretation of Bolzano's account of necessary truth as set out in §182 of the *Theory of Science*. According to this interpretation, Bolzano's conception is closely related to that of Leibniz, with some important differences. In the first place, Bolzano's conception of necessary truth embraces not only what Leibniz called metaphysical or brute necessities but also moral necessities (truths grounded in God's choice of the best among all metaphysical possibilities). Second, in marked contrast to Leibniz, Bolzano maintains that there is still plenty of room for contingency even on this broader conception of necessity."

189. ———. 2013. "On Bolzano’s Concept of a Sum." *History and Philosophy of Logic* no. 34:155-169
Abstract: "Alongside his groundbreaking work in logic, Bernard Bolzano (1781–1848) made important contributions to ontology, notably with his theory of collections. Recent work has done much to elucidate Bolzano's conceptions, but his notion of a sum has proved stubbornly resistant to complete understanding. This

paper offers a new interpretation of Bolzano's concept of a sum. I argue that, although Bolzano's presentation is defective, his conception is unexceptionable, and has important applications, notably in his work on the foundations of arithmetic."

190. ———. 2013. "Kant and Bolzano on Analyticity." *Archiv für Geschichte der Philosophie* no. 95:298-335
Abstract: "The history of speculation on a notion or notions called analyticity, now usually characterized as truth in virtue of meanings and independently of fact, is often viewed from the perspective of the Quine-Carnap dispute. Previous characterizations, due to Kant, Frege and others, are then seen as being of a piece with Carnap's various definitions of analyticity, and thus open to Quine's objections. Seen from this point of view, Bolzano's claims about analyticity appear downright bizarre: for on his conception, analyticity is not only non-linguistic, but also independent of both apriority and necessity. In this paper, it is argued that the problem lies not with Bolzano, but rather with the received historical account, especially its interpretation of Kant."

191. ———. 2022. "Grounding in Practice: Bolzano's Purely Analytic Proof in Light of the *Contributions*." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 364-393. New York: Oxford University Press
"Bolzano's best-known mathematical work, the *Rein analytischer Beweis* of 1817, promises to deliver a ground-revealing proof of an important theorem from the theory of equations, which Bolzano shows to follow from (a generalization of) the intermediate value theorem. In his paper Paul Rusnock explains and assesses this promise against the background of Bolzano's early account of mathematical method, in which the idea of grounding plays a central role."

192. Rusnock, Paul, and Burke, Mark. 2010. "Etchemendy and Bolzano on Logical Consequence." *History and Philosophy of Logic* no. 31:3-29
Abstract: "In a series of publications beginning in the 1980s, John Etchemendy has argued that the standard semantical account of logical consequence, due in its essentials to Alfred Tarski, is fundamentally mistaken. He argues that, while Tarski's definition requires us to classify the terms of a language as logical or non-logical, no such division is guaranteed to deliver the correct extension of our pre-theoretical or intuitive consequence relation. In addition, and perhaps more importantly, Tarski's account is claimed to be incapable of explaining an essential modal/epistemological feature of consequence, namely, its necessity and apriority. Bernard Bolzano (1781-1848) is widely recognized as having anticipated Tarski's definition in his *Wissenschaftslehre* (or *Theory of Science*) of 1837. Because of the similarities between his account and Tarski's, Etchemendy's arguments have also been extended to cover Bolzano. The purpose of this article is to consider Bolzano's theory in the light of these criticisms. We argue that, due to important differences between Bolzano's and Tarski's theories, Etchemendy's objections do not apply immediately to Bolzano's account of consequence. Moreover, Bolzano's writings contain the elements of a detailed philosophical response to Etchemendy."

193. Rusnock, Paul, and George, Rolf. 2004. "Bolzano as Logician." In *The Rise of Modern Logic: from Leibniz to Frege*, edited by Gabbay, Dov and Woods, Jean, 177-205. Amsterdam: North-Holland
Handbook of the History of Logic. Vol. 3.
"Bernard Bolzano (1781-1848) stands out with Frege as one of the great logicians of the nineteenth century. His approach to logic, set out in the *Theory of Science* [WL] of 1837, marks a fundamental reorientation of the subject on many fronts, one which is as radical as any in the history of the field. In sharp contrast to many of his contemporaries, Bolzano insisted upon a rigorous separation of logic from psychology. It should be possible, he thought, to characterize propositions, ideas, inferences, and the axiomatic organization of sciences without reference to a thinking subject. Consistently pursuing this approach to logic and methodology, Bolzano developed important accounts of formal semantics and formal axiomatics.

A talented mathematician, Bolzano developed his logic in conjunction with his mathematical research. Among the first to work on the foundations of mathematics in the modern sense of the term, he made a number of key discoveries in analysis, topology, and set theory, and had a significant influence on the development of mathematics in the nineteenth century. In logic, Bolzano is best remembered for his variation logic (section 4.2 below), a surprisingly subtle and rigorous development of formal semantics. In this article, we discuss Bolzano's logic along with some of his work in the foundations of mathematics which has some bearing on logic." (p. 177)

194. Rusnock, Paul, and Šebestik, Jan. 2013. "The *Beyträge* at 200: Bolzano's Quiet Revolution in the Philosophy of Mathematics." *Journal for the History of Analytical Philosophy* no. 1:1-14
Abstract: "This paper surveys Bolzano's *Beyträge zu einer begründeteren Darstellung der Mathematik* (Contributions to a better-grounded presentation of mathematics) on the 200th anniversary of its publication. The first and only published issue presents a definition of mathematics, a classification of its subdisciplines, and an essay on mathematical method, or logic. Though underdeveloped in some areas (including, somewhat surprisingly, in logic), it is nonetheless a radically innovative work, where Bolzano presents a remarkably modern account of axiomatics and the epistemology of the formal sciences. We also discuss the second, unfinished and unpublished issue, where Bolzano develops his views on universal mathematics. Here we find the beginnings of his theory of collections, for him the most fundamental of the mathematical disciplines. Though not exactly the same as the later Cantorian set theory, Bolzano's theory of collections was used in very similar ways in mathematics, notably in analysis. In retrospect, Bolzano's debut in philosophy was a remarkably successful one, though its fruits would only become generally known much later."

195. ———. 2019. *Bernard Bolzano: His Life and Work*. New York: Oxford University Press
"Yet interest in Bolzano’s theoretical work has rarely extended farther than mere curiosity. Where Frege, for instance, has been the subject of many studies, few English-speaking philosophers have felt moved to look into the details of Bolzano’s work. This is more than a pity, since Bolzano did not simply anticipate what others later developed, but has original things to say that are of enduring interest. One of the most remarkable philosophers of the nineteenth century, his works are still very much worth studying today, so solid is their foundation, so meticulous their detail. Quine might have done well, for instance, to have considered what Bolzano had to say about the analytic/synthetic distinction, or about the *a priori*, Putnam and Kripke to what Bolzano had to contribute to their discussions of indexicals and natural kind terms. Frege himself, as Alwin Korselt [*] pointed out in a none-too-friendly exchange over the foundations of geometry, might have learned a few things about logical consequence from him.
(...)
In the English-speaking world, Bolzano is best known for his work in logic and mathematics. There are certainly things of great importance and beauty in these parts of his work. We have already written, each of us, on these matters, and will have more to say about them in this book. But a faithful portrait of Bolzano cannot limit itself to this, for until he was 40 years old, he was only able to pursue these subjects in his spare time. With his considerable gifts in these non-controversial areas, he certainly might have led a distinguished life of speculation as a mathematician or philosopher. Instead he chose quite deliberately to plunge into the turbulent political life of his homeland, applying his formidable intelligence, energy, and determination to the reform of his society and its institutions. It is here that we shall begin." (pp. 2-3)
[*] Korselt, Alwin. "Über die Grundlagen der Geometrie." *Jahresberiehte der Deutschen Mathematikervereinigung*, 12 (1903): 402-407.

196. Schnieder, Benjamin. 2007. "Mere Possibilities: a Bolzanian Approach to Non-Actual Objects." *Journal of the History of Philosophy* no. 45:525-550
"The paper is a detailed reconstruction of Bernard Bolzano's account of merely possible objects, which is a part of his ontology that has been widely ignored in the literature so far. According to Bolzano, there are some objects which are merely possible. While they are neither denizens of space and time nor members of the causal order, they could have been so. Thus, on Bolzano's view there are, for example, merely possible persons, i.e., objects which are neither actual nor persons but which could have been both. In course of the development of Bolzano's views, they are contrasted with the better known theory of his compatriot Alexius Meinong, and it is shown that they have a modern counterpart in the accounts of merely possible objects that were developed by Bernard Linsky and Ed Zalta, and by Timothy Williamson."
"Here is a brief outline of my paper. The first section is dedicated to the clarification of some basic Bolzanian notions, an understanding of which is needed for what follows. In the second section, I set out to establish that Bolzano in fact had the ontological view I attribute to him. That is, he accepted that there are merely possible objects. The third and final section is concerned with the exposition and reconstruction of Bolzano’s account of mere possibilities." (p. 526)

197. ———. 2014. "Bolzano on Causation and Grounding." *Journal of the History of Philosophy* no. 52:309-337
"This paper is an exploration of Bolzano’s views on causation, which have not been thoroughly examined yet. The paper reconstructs Bolzano’s position, with a focus on his analysis of the concept of causation, on its ontological presuppositions, and on how he relates causation to his theory of grounding.(1) A comparison with standard positions from the contemporary debate on causation will prove his views to be quite original. Moreover, they are a valuable addition to the more recent debate on metaphysical grounding,(2) in which grounding is sometimes informally described as something like metaphysical causation with the exact connection of the two notions seldom being elaborated. Bolzano’s theory explicitly addresses the issue and takes an innovative stance. However, it will also be revealed that his account is beset with problems. But even if his position should ultimately not be tenable, discussing it can deepen our understanding of problems raised in the current debates about causation and grounding and shed new light on them." (p. 309)
(1) The paper concentrates on general conceptual and metaphysical issues of causation. It will not discuss Bolzano’s views on the epistemology of causation, nor his views on detailed matters of fact perhaps better to be treated in physics and its philosophy (such as the question of how causal powers are actually distributed in the world, what kind of basic causal powers there are, etc.).
(2) See e.g. Rosen, “Metaphysical Dependence”; Schaffer, “What Grounds”; and Fine, “Guide to Ground.”
References
Fine, Kit. “Guide to Ground.” In *Metaphysical Grounding*, edited by F. Correia and B. Schnieder, 37–80. Cambridge: Cambridge University Press, 2012.
Rosen, Gideon. “Metaphysical Dependence: Grounding and Reduction.” In *Modality*, edited by Bob Hale and Avrid Hoffmann, 109–35. Oxford: Oxford University Press, 2010.
Schaffer, Jonathan. “On What Grounds What.” In *Metametaphysics*, edited by David Chalmers et al., 347–383. Oxford: Oxford University Press, 2009.

198. ———. 2022. "A Fundamental Being: Bolzano's Cosmological Argument and Its Leibnizian Roots." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 418-443. New York: Oxford University Press
"In his ontology, Bolzano uses the notion of grounding to make claims about the dependent and independent existence of entities. In particular, he argues that there must be a fundamental object (in Bolzano's

terminology: an *unconditioned* object), whose existence is not grounded in the existence of any other object. In his paper, Benjamin Schnieder reconstructs Bolzano's argument, explains its historical context, and puts the argument under scrutiny." (p. 38)

199. Scholz, Heinrich. 1961. *Concise History of Logic*. New York: Philosophical Library
On Bolzano see pp. 44-48.
"Modern logic interprets syllogisms as deduction of judgments from other judgments. Obviously, this interpretation is meaningless so long as we do not know what is meant by deducing one judgment from another. Bolzano did find the relevant interpretation which, it must be owned, *also* does not satisfy us all around but is, nevertheless, epoch-making solely because in pursuing his objective Bolzano turned away from statements and returned to the "forms."(154)
These "forms" now appear for the first time explicitly in formal logic so that with their aid Bolzano was able to obtain the most interesting interpretations not only for the derivation but also for the rest of the logically basic relations of compatibility, incompatibility, etc.(155) His charming *Philosophische Grammatik*(156) we have already mentioned. A luminous chapter all by itself contains magnificent discussions of earlier treatments of every topic of logic with special reference to Aristotle and Kant.(157) In these discussions there is invaluable material for any *critical history of logic*." (pp. 46-47)
(154) See above, p. 3-4.
(155) Cf. especially WL, II, paragraph 154 ff., 198ff.; I, paragraph 95 ff.
(156) See above, p. 40.
(157) Cf. the little book of Bolzano's keenly critical pupil which I brought out in 1931 in a new edition together with W. Dubislav and which appeared in the Felix Meiner Verlag in Leipzig. It is F. Prikonsky: *Neuer Anti-Kant oder Prülfung der Kritik der reinen Vernunft nach den in Bolzanos Wissenschaftslehre niedergelegten Begriffen*. Here we also get acquainted with Augustine's anticipation of Bolzano's principles and ideas, a fact hardly commented on to this day.

200. Schubring, Gert. 1993. "Bernard Bolzano -- Not as Unknown to His Contemporaries as Is Commonly Believed?" *Historia Mathematica*:45-53
Abstract: "An unknown review of Bolzano's three important papers from the years 1816 to 1817 written in 1821 by J. J. I. Hoffmann, a mathematician from Southern Germany, is edited and commented."
"According to common historiography, Bolzano's pioneer publications, in particular his contributions to a new rigor in analysis in 1816 to 1817, remained almost unknown to the mathematical community. Only one piece of evidence contradicting the general impression that nobody read Bolzano in his own day is frequently quoted: N. H. Abel's remark in one of his *Paris notebooks*. Having read some of Bolzano's publications during the time he spent in Berlin 1825/1826, he noted enthusiastically "Bolzano is a clever man" (1). Abel's appreciation is taken, however, as an isolated instance, and Hermann Hankel is credited with having been the first to bring Bolzano to the general attention of the mathematical community in 1871 (see [Grattan--Guinness 1970, 51-52]).
(...)
With regard to this desideratum concerning the history of reception of Bolzano's work in his own time, an essay review of Bolzano's three key papers of 1816/1817 in one of the leading German review journals, the *Jenaische Allgemeine Literatur--Zeitung* (JALZ), is a most welcome find. I came across it when analyzing the *JALZ* for its numerous mathematical reviews. As a first contribution to the study of Bolzano's contemporary reception, the essay review is examined in order to explore the reviewer's reading and understanding of Bolzano's work. Moreover, the mathematical education and practice of the reviewer is analyzed, and the role of the transmitting journal is briefly discussed. The essay review itself is also presented, or more precisely, those parts of it that are in the reviewer's own words." (pp. 45-46)
References

Grattan-Guinness, I. 1970. *The development of the foundations of mathematical analysis from Euler to Riemann*. Cambridge, MA: MIT Press.

201. Šebestik, Jan. 1990. "The Archaeology of the Tractatus: Bolzano and Wittgenstein." In *Wittgenstein, eine Neubewertung / Wittgenstein, Towards a Re-Evaluation. Akten des 14. Intemationalen Wittgenstein-Symposiums*, edited by Haller, Rudolf and Brandl, Johannes L., 112-118. Wien: Hölder-Pichler-Tempsky
"In the case of Bolzano, a comparison with Wittgenstein covers not only some specific points, but also the style of their philosophies and the roJe of logic in the construction of the system. I see three main points of comparison:
1. For Bolzano, formal logic is the central discipline of philosophy: a logical system once set up becomes an instrument for all philosophical analysis.
2. Bolzano refutes Kant's transeendental argument the function of which is assumed by a logico-semantical theory which is developed in two different ways:
a) a theory of meaning or sense based on abstract intensional entities, propositions (*Sätze an sich*) and ideas-in-themselves (*Vorstellungen an sich*). The grammatical forms of ordinary language have to be elucidated and amended in order to comply with canonic forms obtained by the logical analysis of language.
b) a theory of reference or denotation, more precisely the logic of classes and the logic of extensional relations between propositions (extensional because defined solely in terms of the truth values of the propositions considered). Particularly important in this respect is the elucidation of fundamentallogical notions: validity, contravalidity, logical consequence (deducibility) and its link with probability.
3. Bolzano' s theory of representation (*Vorstellung*) is not properly speaking a picture theory. According to Bolzano, pictures (*Bilder*) are not ideas; they can at most accompany some ideas. No properly functional relationship, no *Abbildung*, is established between propositions and the world. On the one hand, *Sätze an sich* are not *Sachverhalte*, because no *Sachverhalte* correspond to false propositions. On the other hand, the structure of the propositions, which is derived from the structure of the statements of ordinary language, does not correspond exactly to the structure of objects. The system of all true propositions yields a complete description an sich of the world and of the properties of things within it, but Bolzano refutes the idea of morphism between the propositions and the world. It is nevertheless on the grounds of Bolzanian theories that the first (Polish-)Austrian picture theory was born. In Twardowski 's *Zur Lehre vom Inhalt und Gegenstand der Vorstellungen* (1894), where the author attemps a synthesis between Bolzano 's logic and Brentano 's descriptive psychology, a functional relationship (*Abbildung*) is established between objects and ideas." (p. 113)

202. ———. 1992. "The Construction of Bolzano's Logical System." In *Bolzano's Wissenschaftslehre 1837-1987. International Workshop*, 163-177. Firenze: Leo S. Olschki
"Several reconstructions of Bolzano's logical system have been proposed until now, some of them at the present workshop. They exploit systematically different aspects of Bolzano's logic and interpret it in terms of different XXth century systems. Such an approach has its own rights, as the full force of Bolzano's logic can be measured only by the standards of our contemporary logic. This is precisely the mark of great authors: each important discovery in their field brings to the light some hitherto unnoticed aspects of their work. That such reinterpretations are possible in the case of Bolzano, that his system can be represented in a quite different conceptual frame and translated into modern symbolic notation simply shows how rich and far reaching are his theories. Another argument favours this approach: a XXth century logician can read Bolzano and other logicians of the past only against the background of modern theories. It is in this way that the body of scientific knowledge is continuously being transmitted: by adapting and translating incessantly old theories into the present language. Moreover, the very meaning of past theories can often be understood only in the light of our systems. Already Husserl noticed that he would not have been able to grasp the significance of

Bolzano's logic if he had not previously studied the most advanced contemporary logical theories - which in his case mainly meant the logic of Schroder! Nevertheless, this modernizing approach does not yield full justice to Bolzano. Even if some of his doctrines are definitively obsolete, they have their function in the construction of his system. Like his mathematics, his philosophy and his theology, Bolzano's logic was conceived in a specific historical context and its complete understanding requires a close attention to the logical and philosophical theories of his time. This is why a complementary approach seems necessary, namely a historical analysis which would trace the links between his system and the logical doctrines of his contemporaries as well as with great logical theories of the past.
My intention is to explain the formation and the structure of his logical system whose core is propositional logic. Bolzano's system of extensional relations between propositions represents one of the decisive innovations in the history of logic. It has no historical antecedents. It is nevertheless connected with logical theories of the late XVIIIth and early XIXth century and my paper tries to elucidate the genesis of Bolzano's system against this historical background. This approach will not only show the originality of Bolzano's achievement in full light, but also give a perhaps unexpected insight into the structure of his logical system.
In my reconstruction, I intend to remain *within* Bolzano's logic, using only conceptual tools which he himself has designed. Therefore, I shall neither attempt to translate his definitions into some XXth century notation, nor confront his logic with our systems. One of the advantages of this approach is to give a presentation of Bolzano's logic which is as simple as possible and has no recourse either to symbolic language (except for elementary set-theoretical notational devices) or to sophisticated semantic framework. Those who have tried to explain Bolzano's logical theories to non-specialists or even to students of modern logic may test the advantage of such an approach." (pp.163-164).

203. ———. 1997. "Bolzano, Exner and the Origins of Analytical Philosophy." *Grazer Philosophische Studien* no. 53:33-59
Abstract: "Analytical philosophy begins with the first mathematical and philosophical works of Bolzano published between 1804 and 1817. There, Bolzano set out a project for the global reform of mathematics by means of the axiomatic method. Having completed the *Wissenschaftslehre,* Bolzano wrote a summary of his logic for the *Grossenlehre,* which he sent to Exner in 1833. The correspondence between Bolzano and Exner covered some of the main subjects treated by analytical philosophy: the status of abstract objects (propositions and objective ideas), intuitions, objectless ideas, the concept of object and many others. While Bolzano argued in favor of abstract entities independent of mind and of language, Exner considered them as abstractions obtained from the subjective judgments and representations. During the XIXth century, Bolzano's philosophy spread over Bohemia and Austria through manuscripts and through the first edition of Zimmermann's textbook of philosophy. The most important Brenta-n ians, Kerry, Twardowski, Meinong and Husserl, discussed his doctrines which may also have influenced Wittgenstein and the Polish school."

204. ———. 2003. "Husserl Reader of Bolzano." In *Husserl's Logical Investigations Reconsidered*, edited by Fisette, Denis. Dordrecht: Kluwer
"The incredible soundness of Husserl's judgment in the matter of logic is unique among his contemporaries - only Frege's insight is on par with it, if not superior. This is due to the lesson of Bolzano whose logic is the truth itself. Husserl adapted his logical system so that it became the logical basis of phenomenology. He adopted Bolzano's main ideas: the extension of logic to the theory of science, the theory of ideal meanings, the distinction between mental act, linguistic expression, meaning and denoted object, the concept of analyticity. Independently of Bolzano and consonant with later mathematical theories, Husserl developed his formal analytics along two lines, apophantic and formal ontology.

Bolzano, however, had articulated the domain of conceptual truths in the same manner: he constructed his logical system as a theory of meaning and his mathematics as a theory of object in general or *Etwas überhaupt*. Both set theory and mereology have their origin here. By his theory of science, Bolzano gave a new impetus to philosophy and logic. For the first time in modern thought, such questions as the nature of logical objects, the problems of meaning and reference, the relation between logic and language became central issues of philosophy." (p. 80)

205. ———. 2014. "Bolzano's Lehrjahre." In *Mind, Values, and Metaphysics: Philosophical Essays in Honor of Kevin Mulligan. Vol. 1*, edited by Reboul, Anne, 289-293. Dordrecht: Springer
Abstract: "The paper will discuss some changes in Bolzano's definition of mathematics attested in several quotations from the *Beyträge, Wissenschaftslehre* and *Grössenlehre*: is mathematics a theory of forms or a theory of quantities? Several issues that are maintained throughout Bolzano's works will be distinguished from others that were accepted in the *Beyträge* and abandoned in the *Grössenlehre*. Changes will be interpreted as a consequence of the new logical theory of truth introduced in the *Wissenschaftslehre*, but also as a consequence of the overcome of Kant's terminology, and of the radicalization of Bolzano's anti-Kantianism. It will be argued that Bolzano's evolution can be understood as a coherent move, if one compares the criticism expressed in the *Beyträge* on the notion of quantity with a different and larger notion of quantity that Bolzano developed already in 1816. This discussion is based on the discovery that two unknown texts mentioned by Bolzano can be identified with works by von Spaun and Vieth respectively. Bolzano's evolution will be interpreted as a radicalization of the criticism of the Kantian definition of mathematics and as an effect of Bolzano's unaltered interest in the Leibnizian notion of *mathesis universalis*. As a conclusion, it will be argued that Bolzano never abandoned his original idea of considering mathematics as a *scientia universalis*, i.e. as the science of quantities in general, and it will be suggested that the question of ideal elements in mathematics, which has been interpreted as a main reason for the development of a new logical theory, can also be considered as a main reason for developing a different definition of quantity. "

206. Shapiro, Stewart. 2011. "Varieties of Pluralism and Relativism for Logic." In *A Companion to Relativism*, edited by Hales, Steven D., 526-552. Malden: Wiley-Blackwell
Abstract: "My purpose is to articulate a number of different senses in which one can be a pluralist and/or a relativist concerning logical consequence. I propose, first, that logical consequence is either polysemous or it denotes a cluster concept. In other words, there are a number of different notions that go by that name, often run together, or else there are several aspects of the notions, with varying weights. The different notions, or aspects, of consequence, turn on matters of modality, semantics, effectiveness, justification, rationality, and form. Second, most of the articulations of the pre - theoretic notions(s) of logical consequence make essential use of a boundary between logical and non - logical terminology. This suggests a sort of relativism/pluralism explicitly noted by Bernard Bolzano and Alfred Tarski: logical consequence is relative to the logical/non - logical boundary. An argument may be valid on one collection of logical terms, invalid on another. Third, it is possible that at least some aspects of the notion of logical consequence are vague: there may be borderline cases of valid arguments. If so, we have to turn to what the correct account of vagueness is. On some theories of vagueness, consequence ends up as relative to something, such as a sharpening or a conversational context, and on others, we end up with a kind of pluralism. Finally, there are a number of interesting and important mathematical theories that employ a non - classical logic, and are rendered inconsistent if classical logic is imposed. This suggests a fourth kind of relativism/pluralism: relativity to structure." (p. 526)

207. Siebel, Mark. 1997. "Variation, Derivability, and Necessity." *Grazer Philosophische Studien* no. 53:117-137
Abstract: "In Bolzano's view, a proposition is necessarily true iff it is derivable from true propositions that include no intuition (Anschauung). This analysis is historically important because it displays close similarities to Quine's and Kripke's ideas. Its systematic significance, however, is reduced by the fact that derivability is defined with recourse to the method of variation, which we are allowed to apply even to propositions containing none of the respective variables. This liberality leads to the result that, according to Bolzano's analysis, every truth is necessarily true. Even by introducing his condition of relevance (shared variables), Bolzano cannot avoid that some propositions come out as necessarily true which are merely contingently true."

208. ———. 2002. "Bolzano's Concept of Consequence." *The Monist. An International Quarterly Journal of General Philosophical Inquiry* no. 85:581-601
"In the second volume of his *Wissenschaftslehre* (2) from 1837, the Bohemian philosopher, theologian, and mathematician Bernard Bolzano (1781-1848) introduced his concept of consequence, named *derivability* (*Ableitbarkeit*), together with a variety of theorems and further considerations. Derivability is an implication relation between *sentences in themselves* (*Sätze an sich*), which are not meant to be linguistic symbols but the *contents* of declarative sentences as well as of certain mental episodes. When Schmidt utters the sentence 'Schnee ist weiss', and Jones judges that snow is white, the sentence in itself expressed by Schmidt is the same as the one to which Jones agrees in thought. This sentence in itself is an abstract entity: in some sense, it exists; but it is unreal insofar as it lacks a position in space and time, does not stand in causal relationships, and is independent of the existence of thinking beings and languages. (3)" (p. 581)
(*) On the whole, this contribution is a summary of my book *Der Begriff der Ableitbarkeit bei Bolzano* (Siebel 1996).
(2) I refer to it by 'WL' plus number of volume, section, and page. It is partly translated by Rolf George: *Theory of Science*, Oxford 1972; but here translations are mine.
(3) Cf. WL I, § 19, pp. 77f.; § 22, p. 90; § 25, p. 112; § 28, p. 121; WL II, § 122, 4.

209. ———. 2011. "“It Falls Somewhat Short of Logical Precision.” Bolzano on Kant's Definition of Analyticity." *Grazer Philosophische Studien* no. 82:91-127
"Kant's famous definition of analyticity states that a judgement is analytic if its subject contains its predicate. Bolzano objects that (i) Kant's definiens permits an interpretation too wide, (ii) the definiens is too narrow, (iii) the definiendum is too limited, and (iv) the definition does not capture the proper essence of analyticity. Objections (i), (iii) and (iv) can be countered. Objection (ii) remains because, among other things, the Kantian definition has an eye only for an analysis of the subject within a judgement."

210. ———. 2019. "Bolzano’s Theory of Judgment." In *The Act and Object of Judgment: Historical and Philosophical Perspectives*, edited by Ball, Brian and Schuringa, Christoph, 110-128. New York: Routledge
"Section 2 presents one of the many places where Bolzano anticipates Frege’s anti-psychologistic notion of a third realm, which complements the inner realm of mental appearances and the outer realm of perceivable objects. In particular, Bolzano strictly distinguishes between judgments as mental acts and the contents of such acts. In section 3, it is shown how he tries to draw the line between judgments and acts of merely entertaining a thought. Section 4 focuses on the formation of judgments. Of prime importance is the distinction between mediated and unmediated judgments because it is intimately connected with epistemic issues. Section 5 deals with intrinsic qualities of judgments, such as vividness, degree of confi dence, clarity vs obscurity, and distinctness vs confusedness.(1)
The notion of judgment occupies centre stage in Bolzano’s analyses of epistemic concepts. It is not only crucial to his explication of belief (*Meinung*) as a

disposition to judge but also to his explications of cognition (*Erkenntnis*) as true judgment and conviction (*Überzeugung*) and knowledge (*Wissen*) as attitudes towards judgments. In the interest of brevity, I will not go into this conceptual enterprise. Instead, it will be pointed out that Bolzano's theory of judgment includes ingredients one would hardly expect when being told that he anticipated Frege's antipsychologistic views." (pp. 110-111)
(1) Some of the following considerations may also be found in Siebel (1999) and Siebel (2004).
References
Siebel, M. (1999), "Bolzanos Erkenntnistheorie", in E. Morscher (ed.), *Bernard Bolzanos geistiges Erbe für das 21. Jahrhundert*. Sankt Augustin: Academia.
Siebel, M. (2004), "Bolzanos Urteilslehre", Archiv für Geschichte der Philosophie 86: 56–87.

211. Simons, Peter. 1987. "Bolzano, Tarski, and the Limits of Logic." *Philosophia Naturalis*:378-405
Reprinted in: Peter Simons, *Philosophy and logic in Central Europe from Bolzano to Tarski. Selected Essays,* Dordrecht, Kluwer 1992, pp. 13-40.
Abstract: "Both Bolzano and Tarski were unsure what counts as logic. This means that Bolzano's concept of logical analyticity, like Tarski's of logical consequence, is not completely determinate.
In a posthumously published paper, Tarski offers a proposal for demarcating the logical objects in a type-hierarchy, based on the idea of invariance under arbitrary permutations of the domain of individuals. In this paper I comment on and extend Tarski's proposal and show how to combine it with Bolzano's procedure of variation among concepts, to obtain a definition of logical constants in a logically significant fragment of a purported Bolzanian realm of meanings in themselves. I conclude with doubts about the propriety and utility of such a realm."

212. ———. 1997. "Bolzano on Collections." *Grazer Philosophische Studien* no. 53:87-108
Abstract: "Bolzano's theory of Collections (*Inbegriffe*) has usually been taken as a rudimentary set theory. More recently, Frank Krickel has claimed it is a mereology. [*] I find both interpretations wanting. Bolzano's theory is, as I show, extremely broad in scope; it is in fact a general theory of collective entities, including the concrete wholes of mereology, classes-as-many, and many empirical collections. By extending Bolzano's ideas to embrace the three factors of kind, components and mode of combination, one may develop a coherent general account of collections. But it is most difficult to take Bolzano's view to fit modern set theory. So while Krickel's positive thesis is rejected, his negative thesis is confirmed."
F. Krickel, *Teil und Inbegriff. Bernard Bolzanos Mereologie*, 1995.

213. ———. 1999. "Bolzano, Brentano and Meinong: Three Austrian Realists." In *German Philosophy Since Kant*, edited by O'Hear, Anthony, 109-136. Cambridge: Cambridge University Press
"Bolzano's work will in due course be wholly accessible in print and should present relatively few problems of interpretation. I foresee a steadily growing reputation, but whether he comes to his just recognition will depend on attracting sufficiently many interested and talented commentators. The most promising centre of Bolzano studies is currently Hamburg, where a number of young enthusiasts have gathered around Wolfgang Künne.
Of the three philosophers I have mentioned, Bolzano is without doubt the most considerable. Meinong's theories are in the end unacceptably extreme and Brentano's work is often unclear in its implications, though both say things which are of much value to present-day discussions. On the other hand, whether one agrees with his semantic Platonism or not, Bolzano's views are up to the highest standards of contemporary discussion and in their clarity above much of it. His correspondence with Ferdinand Exner has been called the first text of modern analytical philosophy. Most work has to date concentrated on his logic and

semantics, but his ethics, political philosophy, philosophy of religion and philosophy of mathematics all deserve greater exposure. The Complete Edition will serve as a definitive textual basis, but it is very expensive, and we badly need cheap study texts in English and German to complement it, and a good introduction to Bolzano in English. We also need to revise our histories of nineteenth-century philosophy to take adequate account of its greatest representative." (p. 126)

214. ———. 2006. "Austrian Philosophers on Truth." In *The Austrian Contribution to Analytic Philosophy*, edited by Textor, Mark, 159-183. New York: Routledge
"In this chapter, I shall consider what the principal Austrian philosophers from Bolzano to Popper have had to say on the subject of truth. Since I shall cover a fair number of philosophers and theories, my considerations will be mainly confined to two linked questions:
What – according to the philosopher in question – is the nature of truth?
What ontology is required to explicate truth according to their account?
Further questions concerned with our access to and knowledge of the truth will only be considered as necessary, since they lead into a tangle of issues for which I shall not have the space here. Neither shall I justify my selection of this or that philosopher as 'Austrian', but simply press on." (p. 159)

215. ———. 2011. "Bolzano's Logic."1-19
Available on the website academia.edu
Original translated by Giorgio Volpe and published in Italian as "Bolzano e la logica" in S. Besoli, L. Guidetti and V. Raspa, eds., Bernard *Bolzano e la tradizione filosofica*. Macerata: Quodlibet. = *Discipline filosofiche* XXI, 2, 2011, 321–342.
Abstract: "Bolzano's Wissenschaftslehre (1837) is one of the two most important works in logic between Leibniz and Frege. In it, Bolzano revolutionised logic by placing it for the first time on a firm semantic footing, employing the concepts of objective, abstract propositions and ideas. The chief instrument in his account of logic is the variation of ideas, which enabled him to define a wide range of logical concepts, and further allowed him to merge deductive logic with a logical conception of probability. This article summarizes the main points of Bolzano's logic and indicates ways in which they relate to post-Fregean logic."

216. ———. 2015. "Bolzano's Monadology." *British Journal for the History of Philosophy* no. 23:1074-1084
Abstract: "Bernard Bolzano (1781–1848), known in his lifetime as 'the Bohemian Leibniz', is best known as a logician and mathematician, but he also developed a monadology in which the monads, which he called
'atoms', have spatial location and physical properties. This essay summarizes and assesses his monadology."

217. Smart, Harold R. 1944. "Bolzano's Logic." *The Philosophical Review* no. 53:513-533
"Contemporary advocates of Husserl's phenomenological approach to the problems of philosophy tend, consciously or unconsciously, to convey the impression that there is only slight connection between Bernard Bolzano's logical theories and those of their Master. Unfortunately their attitude on this matter encourages the common belief that Bolzano may be safely ignored by students of logic-that his work in this field is of little consequence at the present time. Yet in Husserl's own estimation Bolzano was one of the greatest logicians of all times, and historians of philosophy have called him a "Leibniz auf b6hmischen Boden".
He was at all events one of the staunchest opponents of the metaphysical logicians following Kant, as well as of Kant himself.
His *Wissenschaftslehre* (1837), a compendious work in four volumes totalling nearly 2500 pages, draws much of its inspiration from Augustinian and Leibnizian sources, and in turn has served as a basis for certain theories of Brentano, Husserl, Meinong, and others. Like Leibniz he zealously occupied himself with both mathematics and philosophy from early youth, and again like Leibniz he is rightly famous for his distinguished work in both fields. Indeed his *Paradoxien der*

Unendlichen (posth. 1850) is said to have started the great Cantor on his researches in the realm of the mathematical infinite. And he is another of the few thinkers whose chief philosophical writings are in the field of logic. For the rest, his writings are shot through with references to his predecessors, both ancient and modern, and with critical remarks on their doctrines." (p. 513)

218. Stang, Nicholas F. 2013. "A Kantian Reply to Bolzano's Critique of Kant's Analytic-Synthetic Distinction." *Grazer Philosophische Studien* no. 85:33-61
Summary: "One of Bolzano's objections to Kant's way of drawing the analytic-synthetic distinction is that it only applies to judgments within a narrow range of syntactic forms, namely, universal affirmative judgments. According to Bolzano, Kant cannot account for judgments of other syntactic forms that, intuitively, are analytic. A recent paper by Ian Proops also attributes to Kant the view that analytic judgments beyond a limited range of syntactic forms are impossible.
I argue that, correctly understood, Kant's conception of analyticity allows for analytic judgments of a wider range of syntactic forms."

219. ———. 2014. "Kant, Bolzano, and the Formality of Logic." In *New Anti-Kant*, edited by Lapointe, Sandra and Tolley, Clinton, 192-234. London: Palgrave Macmillan
"In §12 of his 1837 *magnum opus*, the *Wissenschaftslehre*, Bolzano remarks that "In the new logic textbooks one reads almost constantly that 'in logic one must consider not the material of thought but the mere form of thought, for which reason logic deserves the title of a purely formal science'" (*WL* §12, 46).(1) The sentence Bolzano quotes is his own summary of others' philosophical views; he goes on to cite Jakob, Hoffbauer, Metz, and Krug as examples of thinkers who held that logic abstracts from thematter of thought and considers only its form. Although Bolzano does not mention Kant by name here, Kant does of course hold that "pure general logic", what Bolzano would consider logic in the traditional sense (the theory of propositions, representations, inferences, etc.), is formal.
(...)
In recent work, both John MacFarlane and Sandra Lapointe have argued that this 'formality thesis' is original to Kant; according to them, no one in the pre-Kantian, Leibnizian logical tradition held that logic
is about the form of thinking.(3) As MacFarlane points out, the claim that logic is formal is now so widespread that it is often simply asserted without argument. So in criticizing the formality thesis in these post-Kantian figures (whom Lapointe aptly dubs 'Kantian logicians') Bolzano is really targeting one of Kant's most influential ideas in the philosophy of logic." (pp. 192-193)
(1) References to the *Wissenschaftslehre* (WL) are to Bolzano (1837); it is cited by section number and page.
(3) MacFarlane (2002) and Lapointe (2012).
References
Lapointe, S. (2012). 'Is Logic Formal? Bolzano, Kant and the Kantian Logicians', *Grazer Philosophische Studien*, 85, 11–32.
MacFarlane, J. (2002). 'Frege, Kant, and the Logic in Logicism', *The Philosophical Review*, 111, 25–65.

220. Stelzner, Werner. 2002. "Compatibility and Relevance: Bolzano and Orlov." *Logic and Logical Philosophy* no. 10:137-171
"Ivan Orlov (1886 - not later 1936) is the author of "The Logic of Compatibility of Propositions", *Matematicheskii Sbornik* 35, 1928, pp. 263-86 (in Russian), "the first precisely elaborated modern system of relevance logic" (p. 137)
"In Bernard Bolzano Orlov had a great predecessor in the attempt of deriving the concept of logical consequence, and indeed of relevant consequence, from the concept of compatibility of sentences. It is appropriate, therefore, to turn to Bolzano in order to check out parallels and divergences in the treatment and role of the compatibility of sentences in Bolzano's and Orlov's logical projects." (p. 142)

221. Sundholm, Göran. 1994. "Ontologic versus Epistemologic: some Strands in the Development of Logic 1837-1957." In *Logic and Philosophy of Science in Uppsala*, edited by Prawitz, dag and Westerstähl, Dag, 373-384. Dodrecht: Kluwer
"Inferences, that is, acts of passage in which a certain judgement, the conclusion of the inference, is drawn on the basis of certain already made judgements, the premisses of the inference, have yielded their central place at the hard core of logic to relations of logical consequence between propositions that serve as contents of the judgements involved, or even more commonly, between well-formed formulae, that is, between meta-mathematical objects of an uninterpreted formal language. In the present paper I intend to review some of the steps in the process whereby this came about, as well as mention a couple of philosophical corollaries.
Quine, in 1952, held that 'logic is an old subject and since 1879 it has been a great one'.(1) No one reasonably informed concerning the development of logic could possibly object to the first part of this statement, but I want to take mild exception to the second: logic was great also prior to to the appearance of Frege's *Begriffsschrift*.(2) From the perspective I am concerned to develop here, 1837 is as important a year as 1879. In that year Bernhard Bolzano's *Wissenschaftslehre* made its appearance in four mighty tomes.(3)" (pp. 373-374)
(1) *Methods of Logic*, Holt and Co., N.Y. 1950, p. vii.
(2) Louis Nebert, Halle, Jena 1879.
(3) J. von Seidel, Sulzbach.

222. ———. 1998. "MacColl on Judgement and Inference." *Nordic Journal of Philosophical Logic* no. 3:119-132
"The theme of our conference is that of Hugh MacColl and the logical tradition. From any point of view, surely, judgement and inference are (possibly *the*) central components of the logical tradition. However, they do not occur as such in MacColl's Symbolical reasoning(s).
(...).
Accordingly, I begin with a rational reconstruction of what I see as the pivotal moment in the 19th century logical tradition, namely Bolzano's introduction of a novel form of judgement, which will be used to take the measure of the early MacColl with respect to judgement and inference." (p. 119)
(...)
!Why does this Bohemian priest [Bolzano] deserve pride of place over and above such luminaries as Boole, Peirce and Frege? For more than two thousand years, logic has been concerned with how to effect valid acts of inference from judgements known to other judgements that become known through the inference in question. Basically, these judgements take the subject/copula/predicate form [S is P]. Bolzano now has the courage to break with this traditional pattern and uses instead the unary form
(1) A is true;
where A is a *Satz an sich*, or a *Gedanke*, in the later alternative terminology of Frege. The latter term was translated into English as *proposition* by Moore and Russell, with an unusually confusing ambiguity as a result: prior to 1900 a "proposition" stood for a judgement (made), whereas later it came to stand for the propositional content of such a judgement." (p. 120)

223. ———. 1999. "When, and Why, did Frege read Bolzano?" In *Logica Yearbook 1999*, 164-174
"Michael Dummett wrote:
The only nineteenth-century philosopher of whom it would be reasonable to guess, just from the content of his writings and those of Frege, that he had influenced Frege, is Bernhard Bolzano, who died in the year Frege was born; but there is no evidence whatever that Frege ever read Bolzano(1)
Subsequently he was taken to task by Wolfgang Künne for having made the 'grave mistake' of misspelling 'Bernard', the first name of Bolzano.
However, in my opinion, this is not the only mistake in the quote from Dummett. In the present note I wish to dispute that 'there is no evidence whatever that Frege ever

read Bolzano'. On the contrary, by combining two well-known sets of facts, I shall argue, one obtains strong evidence that Frege did read Bolzano late 1905 or early 1906." (p. 164, a note omitted)
(...)
"On the strength of internal evidence I have argued that Frege did read Bolzano. Was it in fact possible for him to do so? It certainly was, as Dr. Uwe Dathe, of the Philosophical Institute at Jena University, has been kind enough to check.(26) The University Library at Jena owns a set of Bolzano's collected works from 1882. The acquisition is not dated, but from the library stamp and binding it is clear that the set must have been obtained shortly after its appearance.
Unfortunately, the library ledgers for the years 1821-1899, which have miraculously been retained, are in too bad a state to allow for any conclusion whether Frege actually borrowed the work during that period.(27)
Finally, if, as I aver, Frege did read Bolzano, why does he not simply say so? The answer here surely lies in his character: throughout his career Frege *never* acknowledges, but always disagrees.(28) His sprit seems to have been essentially adversarial. He is the typical *Gegner* who only attacks, but who cannot be bothered to agree." (p. 172)
(1) *Frege and Other Philosophers*, Clarendon Press, Oxford, 1991, p. VII.
(2) 'Propositions in Bolzano and in Frege', in: *Grazer Philosophische Studien* (Bolzano and Analytic Philosophy; edited by Wolfgang KUnne, Mark Siebel and Mark Textor) 53 (1997), pp. 202-240, at p. 203.
(26) Private letter, November 26,1998.
(27) Of course, if I am right, a later loan, in 1905 or 1906, outside the period of the ledgers, would be more likely.

224. ———. 2002. "A Century of Inference: 1837-1936." In *In the Scope of Logic, Methodology and Philosophy of Science. Vol. II*, edited by Gardenfors, Peter, Wolenski, Jan and Kijania-Placek, Katarzyna, 565-580. Dordrecht: Kluwer
"The first serious breach in the traditional logical fortress was broached by one thoroughly steeped in the Scholastic patrimony, namely Bernard Bolzano, in another *Wissenschaftslehre* from 1837. This, however, is no puny pamphlet,but a monumental four-volume tome.(5) Like all good ideas the basic idea behind Bolzano's magisterial change is essentially simple: Bolzano revolutionizes logical theory by "objectivizing" the middle column of the traditional diagram. This objectivization consists in severing the left - and right-hand links to mind and language, thereby obtaining objective "Platonist" logical notions, for which Bolzano ironically adopts the Kantian 'an sich' idiom.
Thus, the (mental) terms become objective "ideas-in-themselves" (*Vorstellungen an sich*). (6) The judgements made, that is, the mental propositions, become propositions-in-themselves (*Satze an sich*), that is, propositions in the modem, post-Russellian sense.(7) Finally, the mental inferences are replaced by *Ableitbarkeiten*, that is, relations of (logical) consequence between propositions-inthemselves.
The resulting change with respect to the form of judgement is particularly interesting. In place of the traditional bipartite Subject/copula/Predicate form Bolzano uses the unary form
C is true,
where *C* is a *Satz an sich*, that is, a proposition that serves as content of the judgement in question (WL §34). The form of the proposition *C*, on the other hand, stays close to the traditional [*S* is *P*]. Bolzano uses [*A* has *b*], where *A* and *b* are *Vorstellungen an sich*, that is, (what corresponds to) objectivizations of the mental products of simple apprehensions, as canonical form for the objective propositions. Thus, he converts the traditional form of judgement into a form of content:
The proposition that the rose has redness is true
instead of
The rose is red." (pp. 567-568. a note omitted)
(7) Russell ([*Principles of Mathematics*] 1903, Appendix A) might be responsible for sanctioning the unfortunate use of the term Russell (1903, Appendix A) might

be responsible for sanctioning the unfortunate use of the term *proposition* for the Fregean *Gedanken*.

225. ———. 2009. "A Century of Judgement and Inference: 1837-1936. Some Strands in the Development of Logic." In *The Development of Modern Logic*, edited by Haaparanta, Leila, 263-318. New York: Oxford University Press
§ 3. Revolution: Bolzano's *Annus Mirabilis*, pp. 269-273.
"My office in the present chapter is to tell how, within a century, the notions of judgment and inference were driven out of logical theory and replaced by propositions and (logical) consequence. Systematic considerations guide the treatment. My history is unashamedly Whiggish: A current position will be shown as the outcome, or even culmination, of a historical development. No apology is offered, nor, in my opinion, is one needed." (p. 263)
(...)
"Bolzano's revolution with respect to the traditional picture is threefold.
First, the middle ("product") column of the traditional schema is objectified.
The mental links are severed, and thus, in particular, the traditional notions mental term (concept, idea) and mental proposition (judgment) are turned into their ideal, or Platonist, counterparts idea-in-itself (*Vorstellung an sich*) and proposition-in-itself (*Satz an sich*).(23) Second, the pivotal middle square of the diagram is altered: The judgment made no longer takes the traditional (*S* is *P*) form. Logic is no longer term logic. Instead Bolzano uses the propositional, unary form of judgment that was canvassed above, with his *Sätze an sich* taking the role of judgable contents:
The *Satz an sich S* is true.(24)
Third, Bolzano bases his logical theory, not on inference (from judgments known to judgment made), but on (logical) consequence between propositions.(25)
Judgment is dethroned and its content now holds pride of place in logical theory." (pp. 269-270)
(23) The English rendering of Bolzano's *Satz an sich* is a matter of some delicacy. The modern, Moore-Russell notion of proposition, being an English counterpart of the Fregean Thought (German *Gedanke*), really is an *an sich* notion, and, for our purposes, essentially the same as Bolzano's *Satz an sich*. Thus, proposition-in-itself is pleonastic: The in-itself component is already included in the proposition. Furthermore, the mental propositions and their linguistic signs, that is, written or spoken propositions, as explained, carry assertoric force, whereas Bolzano's *Sätze an sich* manifestly do not, serving, as they do, in the role of judgmental content. (...)
(24) WL, §34.
(25) Occasionally I shall permit myself to drop the "in-itself" idiom in the interest of perspicuity and readability and speak just of "propositions."

226. Świętorzecka, Kordula. 2017. "Bolzano's Argument for the Existence of Substances: a Formalization with Two Types of Predication." *Acta Analytica* no. 32:411-426
Abstract: "The topic of our analysis is the argument for the existence of substances given by Bernard Bolzano in *Athanasia* (1827), where he essentially employs two ontological categories: substance and adherence. Bolzano considers the real and conditioned *Inbegriff* of all adherences, which are *wirklich* and *nicht selbst bestehen*.
He claims that the formed collection is dependent on something external and nonadherential, which therefore is a substance. Bolzano's argumentation turns out to be structurally similar to his argument for the existence of God from *Lehrbuch der Religionswissenschaft* (1834), but in each of these reasonings, we find different plausible interpretations of the key concept "Inbegriff". The latter argumentation refers to the mereological totality of existentially conditioned objects. We propose the explication of the Bolzanian *Inbegriff* of all adherences using two types of predication: we consider its extension as composed of certain intensional counterparts of adherences.
In our approach, we use a fragment of the theory of abstract objects formulated by E. Zalta (1983), describing two different relations between individuals and

properties: extensional exemplification and intensional encoding. We put our reconstruction in a wider context of Bolzano's ontology, formulating the needed axioms with two primitive predicates of second order ... is an adherence, ... is conditioned by something real as well as the conditionally introduced first order predicate constant *In* for *Inbegriff of all adherential ideas.* Finally, we sketch a model for our theory."
References
Zalta, E. (1983). *Abstract object: an introduction to axiomatic metaphysics*, D. Dordrecht: Reidel.

227. ———. 2019. "Two Formal Interpretations of Bolzano's Theory of Substances and Adherences." *Axiomathes* no. 29:265-284
Abstract: "Our research concerns a formal representation of Bolzano's original concepts of *Substanz* and *Adhärenz*. The formalized intensional theory enables to articulate a question about the consistency of a part of Bolzano's metaphysics and to suggest an answer to it in terms of contemporary model theory. The formalism is built as an extension of Zalta's theory of abstract objects, describing two types of predication, viz. attribution and representation. Bolzano was aware about this distinction.
We focus on the consistency of this formalism and the description of its semantics. Firstly, we explore the possibility to reconstruct a Russellian antinomy based on the concept of the Bolzano's *Inbegriff* of all adherences. (Bolzano's theory of ideas is often suspected of antinomial consequences.) Our aim is to show limitations of his theory that prevent a contradiction when the *Inbegriff* consists of non-selfreferential adherences. Next, we discuss two competing semantics for the proposed theory: Scott's and Aczel's semantics. The first one yields a problematic result, that there are no models for the considered theory, containing a non-empty collection of all adherences. This is due to the fact that Scott's structures verify the formula on reloading abstracts in extensional contexts. We show that Aczel's semantics does not contain this difficulty. There are described Aczel's models with a non-empty set of all adherences. The self-referentiality of such a collection becomes irrelevant here.
Finally, we show that there are Aczel's structures verifying the formula on reloading abstracts and we exclude them from the class of models intended for our theory."
References
Zalta, E. (1983). *Abstract object: an introduction to axiomatic metaphysics*, D. Dordrecht: Reidel

228. Tarski, Adam. 2002. "On the Concept of Following Logically." *History and Philosophy of Logic* no. 23:155-196
Translated from the Polish and German by Magda Stroinka and David Hitchcock.
"We provide for the first time an exact translation into English of the Polish version of Alfred Tarski's classic 1936 paper, whose title we translate as 'On the concept of following logically'.
We also provide in footnotes an exact translation of all respects in which the German version, used as the basis of the previously published and rather inexact English translation, differs from the Polish. Although the two versions are basically identical, to an extent that is even uncanny, we note more than 400 differences. Several dozen of these are substantive differences due to revisions by Tarski to the Polish version which he did not incorporate in the German version.
With respect to these revisions the Polish version should be regarded as more authoritative than the German. Hence scholars limited to an English translation should use ours." (p. 1)
"After the original of this paper had appeared in print, H. Scholz in his article 'Die Wissenschaftslehre Bolzanos, Eine Jahrhundert-Betrachtung', Abhandlungen der Fries'schen Schule, new series, vol. 6, pp. 399-472 (see in particular p. 472, footnote 58) pointed out a far-reaching analogy between this definition of consequence and the one suggested by B. Bolzano about a hundred years earlier." [Note added by Tarski in English in Tarski (1956, 1983).] (p. 67).

229. Tatzel, Armin. 2002. "Bolzano's Theory of Ground and Consequence." *Notre Dame Journal of Formal Logic* no. 43:1-25
Abstract: "The aim of the paper is to present and evaluate Bolzano's theory of grounding, that is, his theory of the concept expressed and the relation brought into play by 'because'. In the first part of the paper (Sections 1-4) the concept of grounding is distinguished from and related to three other concepts: the concept of an epistemic reason}, the concept of causality, and the concept of deducibility (i.e., logical consequence). In its second part (Sections 5-7) Bolzano's positive account of grounding is reconstructed in axiomatic form and critically discussed."

230. Textor, Mark. 1997. "Bolzano's Sententialism." *Grazer Philosophische Studien* no. 53:181-202
Abstract: "Bolzano holds that every sentence can be paraphrased into a sentence of the form "A has b". Bolzano's arguments for this claim are reconstructed and discussed. Since they crucially rely on Bolzano's notion of paraphrase, this notion is investigated in detail. Bolzano has usually been taken to require that in a correct paraphrase the sentence to be paraphrased and the paraphrasing sentence express the same proposition. In view of Bolzano's texts and systematical considerations this interpretation is rejected: Bolzano only holds that the sentence to be paraphrased and the paraphrasing sentence must be equipollent ("*gleichgeltend*"). It is shown that even this modest view of paraphrase does not help Bolzano in sustaining his claim that all sentences have the form "A has b"."

231. ———. 2001. "Logically Analytic Propositions *A Posteriori*?" *History of Philosophy Quarterly* no. 18:91-113
"In this paper I will be concerned with Bolzano's explication of logical analyticity or I-analyticity for short. Nowadays Balzano is often seen as a forerunner of the so-called substitutionalist account (Etchemendy) of I-analyticity for sentences, the property that distinguishes logical truths (falsehoods) from "ordinary" truths (falsehoods). I will argue that Bolzano's explication does not correspond closely to the modern account. My reason for this heterodox view is not that Balzano tries to define what makes a *proposition*, roughly, the meaning of a sentence, I-analytic. The problem I am interested in will also arise for an account of I-analyticity for sentences that follows Bolzano's lead. My reason is an epistemological one: Bolzano's account does not allow him to say that I-analytic propositions can be known *a priori.* But according to most philosophers' understanding of I-analyticity this epistemological feature is central to the notion of logical truth. Hence, Bolzano's account does not capture an important feature of the concept of a logical truth or the broader concept of an I-analytic proposition." (p. 91)

232. ———. 2003. ""Caius-at-Noon" or Bolzano on Tense and Persistence." *History of Philosophy Quarterly* no. 20:81-102
Translated in French as: "Bolzano sur le temps et la persistence", *Philosophiques*, 30, 2003, pp. 105-125.
"Bolzano's fame among contemporary analytic philosophers is mainly due to his achievements in the philosophy of logic.
(...)
What is less well known is that Bolzano also uses histheory of propositions to define a variety of epistemological and metaphysical notions. Among the metaphysical notions so defined is the notion of time. Crucial for his definition of time is Bolzano 's thesis that
a tensed natural language sentence attributing a substantial property to an actual thing expresses only a complete proposition if it contains an expression like "in (at) t" as part of its *subject-term*.
Bolzano consequently rejects the Aristotelian idea that tense attaches to predicables. (1) Bolzano's proposal is of interest for contemporary philosophers, because it bears a striking resemblance to contemporary theories in which expressions like "Caius at noon" refer to temporal parts. This paper is primarily concerned with a reconstruction and evaluation of the part of Bolzano's doctrine of propositions that

is the basis of his definition of time. The definition itself will be a topic for another occasion. First things first. The following sketch of Bolzano's definitional strategy and its rationale shall introduce the reader to Bolzano's general project, which connects tense and time." (pp. 81-82)

(1) Aristotle puts his view forward in De Interpretalione 16^b6 and 16^b8.
For recent defenses see P. T. Geach, *Reference and Generality* (Ithaca, N.Y.: Cornell University Press, 1962), §27, and D. Wiggins "Substance," in *Philosophy*, ed. A. C. Grayling (Oxford: Oxford University Press, 1995), p. 232.

233. ———. 2013. "Bolzano on the Source of Necessity: A Reply to Rusnock." *British Journal for the History of Philosophy* no. 21:381-392
Abstract: "According to Bolzano, an object has necessary being if, and only if, there is a conceptual truth that ascribes being to it. I (Textor, 1996, chapter 5) proposed that the notion of conceptual truth bears the explanatory weight in Bolzano's theory of necessity because, ultimately, the truth of such a proposition depends only on the nature of the concepts it contains. Rusnock (2012) argues against this interpretation and proposes, in turn, that for Bolzano necessity and contingency are tied to free choice. In this article I will provide conceptual and historical background for Bolzano's view of necessity and use it to motivate my interpretation as well as to rebut Rusnock's criticism."
References
Rusnock, P. 'Remarks on Bolzano's Conception of Necessary Truth', *British Journal for the History of Philosophy*, 20, 817-837, (2012).
Textor, M. *Bolzanos Propositionalismus* (Berlin/New York: De Gruyter, 1996.

234. ———. 2013. "Bolzano's Anti-Kantianism: From a Priori Cognitions to Conceptual Truths." In *The Oxford Handbook of The History of Analytic Philosophy*, edited by Beaney, Michael, 227-250. New York: Oxford University Press
Abstract: "Bernard Bolzano was born in 1781, the year of the publication of the first edition of Kant's *Critique of Pure Reason*; he died in 1848, the year of Gottlob Frege's birth. These dates are symbolic. Bolzano's work is a link between Kant's philosophy and early analytic philosophy of which Frege is a key exponent. In this chapter I will discuss how Bolzano's criticism of Kant shapes Bolzano's theory of propositions. In connection with this I will outline how Bolzano discovered the method of variation and give an overview of his results in employing this method."

235. ———. 2013. "Bolzano on Conceptual and Intuitive Truth: the Point and Purpose of the Distinction." *Canadian Journal of Philosophy* no. 43:13-36
Abstract: "Bolzano incorporated Kant's distinction between intuitions and concepts into the doctrine of propositions by distinguishing between conceptual (*Begriffssätze an sich*) and intuitive propositions (*Anschauungssätze an sich*). An intuitive proposition contains at least one objective intuition, that is, a simple idea that represents exactly one object; a conceptual proposition contains no objective intuition. After Bolzano, philosophers dispensed with the distinction between conceptual and intuitive propositions. So why did Bolzano attach philosophical importance to it? I will argue that, ultimately, the value of the distinction lies in the fact that conceptual and intuitive truths have different objective grounds: if a conceptual truth is grounded at all, its ground is a conceptual truth. The difference in grounds between conceptual and intuitive truths motivates Bolzano's criticism of Kant's view that intuition plays the fundamental role in mathematics, a conceptual science by Bolzano's lights."

236. ———. 2022. "Grounding, Simplicity, and Repetition." In *Bolzano's Philosophy of Grounding: Translations and Studies*, edited by Roski, Stefan and Schnieder, Benjamin, 301-318. New York: Oxford University Press
"For Bolzano, grounding often goes along with a reduction of *propositional complexity*, where he takes the complexity of a proposition to depend not only on *how many* ideas occur in it, but also by how *often* each of them occurs (so that the proposition *that Ann is wise* is less complex than the proposition *that Ann isn't*

unwise). But this raises the Repetition Problem, which Mark Textor explores in his paper: how can a whole contain one and the same entity more than once?" (p. 37)

237. Thompson, Paul B. 1981. "Bolzano's Deducibility and Tarski's Logical Consequence." *History and Philosophy of Logic* no. 2:11-20
Abstract: "In this paper I argue that Bolzano's concept of deducibility and Tarski's concept of logical consequence differ with respect to their philosophical intent. I distinguish between epistemic and ontic approaches to logic, and argue that Bolzano's deducibility presupposes an epistemic approach, while Tarski's logical consequence presupposes an ontic approach."

238. Tolley, Clinton. 2012. "Bolzano and Kant on the Place of Subjectivity in a *Wissenschaftslehre*." *Grazer Philosophische Studien* no. 85:63-88
Summary: "Throughout his career, Bolzano presents his account of knowledge and science as an alternative to 'the Critical philosophy' of Kant and his followers. The aim of this essay is to evaluate the success of Bolzano's own account—and especially, its heavy emphasis on the objectivity of cognitive content—in enabling him to escape what he takes to be the chief shortcomings of the 'subjective idealist philosophy'. I argue that, because Bolzano's own position can be seen to be beset by problems that are both recognizably similar to, and possibly even worse than, those that he takes to afflict Kant's account of the elements of our knowledge, Bolzano's attempt to fully overcome the alleged vices of Kant's idealism by 'extruding' semantic content from the mind must be judged to be less than satisfactory."

239. ———. 2013. "Bolzano and Kant on the Nature of Logic." *History and Philosophy of Logic* no. 33:307-327
Abstract: "Here I revisit Bolzano's criticisms of Kant on the nature of logic. I argue that while Bolzano is correct in taking Kant to conceive of the traditional logic as a science of the activity of thinking rather than the content of thought, he is wrong to charge Kant with a failure to identify and examine this content itself within logic as such. This neglects Kant's own insistence that traditional logic does not exhaust logic as such, since it must be supplemented by a transcendental logic that will in fact study nothing other than thought's content. Once this feature of Kant's views is brought to light, a much deeper accord emerges between the two thinkers than has hitherto been appreciated, on both the nature of the content that is at issue in logic and the sense of logic's generality and formality."

240. ———. 2014. "Bolzano and Kant on Space and Outer Intuition." In *New Anti-Kant*, edited by Lapointe, Sandra and Tolley, Clinton, 157-191. London: Palgrave Macmillan
"Challenges to Kant's account of geometry appear already in some of Bolzano's earliest publications (cf. Bolzano 1810), and are developed more sustainedly in his later discussions of Kant in the 1837 *Wissenschaftslehre* (' *WL* ') and those recorded by Příhonský in the 1850 *New Anti-Kant* (' NAK '). Bolzano argues, against Kant, that it is possible to define the representation of space through mere concepts alone, without this definition including any representations whatsoever drawn from intuition (cf. WL §79.6, I.366; §79 *Anm*, I. 369–370; NAK 74). In this respect, Bolzano thereby puts forward a form of geometrical 'logicism' *avant la lettre*.(4) In fact, Bolzano's criticisms go considerably further, insofar as he argues that the very idea of a pure intuition is essentially incoherent (as we will see below, cf. §§4–5). Yet while existing treatments of Bolzano's criticism of Kant on space have focused primarily on Bolzano's contrasting account of knowledge in geometry and mathematics more broadly, much less attention has been paid to the consequences that Bolzano's rejection of pure intuition has for Bolzano's own account of our intuitions of external objects – representations that Bolzano himself also calls 'outer intuitions'.(5) This will be my focus in what follows." (p. 158)
(4) Cf. Coffa 1991, 27f.; Sebestik 2003, 54f.; cf. Palagyi 1902, iii.
(5) An early start on this topic can be found in Palagyi 1902, chapter VI (esp. §18). Some more recent helpful treatments of related topics can be found in George 2003 and Rosenkoetter 2012. For a discussion of Bolzano's rejection of Kant's

doctrine of the pure intuition of time that is in key ways complementary to what follows, see George 1987.
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Bolzano, Bernard. (1810). *Beiträge zu einer begründeteren Darstellung der Mathematik*. Prague: C. Widtmann.
Coffa, Alberto. (1991). The Semantic Tradition from Kant to Carnap. Cambridge: Cambridge University Press.
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Rosenkoetter, Timothy. (2012). 'Kant and Bolzano on the singularity of intuitions', *Grazer Philosophische Studien*, 85, 89–129.
Sebestik, Jan. (2003). 'La dispute de Bolzano avec Kant', Philosophiques, 30.1 (Printemps), 47–66.

241. Trlifajová, Katerina. 2018. "Bolzano's Infinite Quantities." *Foundations of Science* no. 23:681-704
Abstract: "In his *Foundations of a General Theory of Manifolds,* Georg Cantor praised Bernard Bolzano as a clear defender of actual infinity who had the courage to work with infinite numbers. At the same time, he sharply criticized the way Bolzano dealt with them.
Cantor's concept was based on the existence of a one-to-one correspondence, while Bolzano insisted on Euclid's Axiom of the whole being greater than a part. Cantor's set theory has eventually prevailed, and became a formal basis of contemporary mathematics, while Bolzano's approach is generally considered a step in the wrong direction. In the present paper, we demonstrate that a fragment of Bolzano's theory of infinite quantities retaining the part-whole principle can be extended to a consistent mathematical structure.
It can be interpreted in several possible ways. We obtain either a linearly ordered ring of finite and infinitely great quantities, or a partially ordered ring containing infinitely small, finite and infinitely great quantities. These structures can be used as a basis of the infinitesimal calculus similarly as in non-standard analysis, whether in its full version employing ultrafilters due to Abraham Robinson, or in the recent ''cheap version'' avoiding ultrafilters due to Terence Tao."

242. van der Schaar, Maria. 2007. "Bolzano on Judgement and Error." In *The Logica Yearbook 2006*, edited by Tomala, O and Honzi, R., 211-221. Prague: Filosofia
"Keeler (1934) ends his history of the problem of error with Kant, and Balduin Schwarz, in his article on 'Irrtum' in the *Historisches Wörterbuch der Philosophie*, only mentions 'the important analysis' of error given by Bolzano. In the less known third part of the *Wissenschaftslehre* (1837), the 'Erkenntnislehre', there are several chapters on judgement, knowledge and truth, with a special section on error.
Besides the logical / conceptual question how error is possible, Bolzano also asks the epistemological / psychological question what the causes of error are, how error arises in us.
With respect to the concept of error, one has to distinguish between act and product. 'Error' and the German term 'Irrtum' stand for the product, resulting from an act of erring ('das Irren'). The distinction is a special case of the distinction between the act of judgement and the judgement product. Both act and product need to be distinguished from the proposition, which Bolzano also calls an error, if it is false but held true.
Because Bolzano explains error primarily as incorrect judgement (WL, I, § 36), the question what judgement is comes first (section 2). To understand the concept of error, one also needs to understand what knowledge is (section 3).
In my analysis of Bolzano's notions of judgement and knowledge I have profited from Mark Siebel's two recent articles on these topics (Siebel, 1999 and 2004). In section 4 Bolzano's concept of error will be dealt with." (p. 212)
References

Keeler, L. W. (1934). *The Problem of Error from Plato to Kant,* Rome.
Siebel, M. (1999). Bolzanos Erkenntnistheorie. In E. Morscher (Ed.). *Bernard Bolzanos geistiges Erbe für das 21. Jahrhundert* (59–96). Sankt Augustin: Academia.
Siebel, M. (2004). Bolzanos Urteilslehre. In *Archiv für Geschichte der Philosophie*, 86, 56–87.

243. Waldegg, Guillermina. 2001. "Ontological Convictions and Epistemological Obstacles in Bolzano's Elementary Geometry." *Science and Education* no. 10:409-418
Abstract: "Bernard Bolzano (1781-1848) was a contemporary of the founders of non-Euclidean geometry and of the renovation of projective geometry. However, he did not participate in the movement transforming concepts and methods which crystallized in a new order of geometry at the beginning of the nineteenth century. On the contrary, throughout his life Bolzano tried to demonstrate Euclid's postulate of parallel lines.
Two ontological convictions played the role of epistemological obstacle for Bolzano and prevented him even from imagining the possibility that non-Euclidean geometries might exist. In the first place, Bolzano thought that Euclidean geometry had an intrinsic structure and thus geometrical space must be intrinsically Euclidean. Secondly, the description of this structure contained the existence of an "objective" connection between geometrical truths; a basic truth was, by its nature, "simple and general".
This article forms part of the body of work aimed at identifying obstacles in the history of mathematics in order to confront them with obstacles to learning and to establish their epistemological character."

244. ———. 2005. "Bolzano's Approach to the Paradoxes of Infinity: Implications for Teaching." *Science & Education* no. 14:559-577
Abstract.: "In this paper we analyze excerpts of *Paradoxes of the Infinite*, the posthumous work of Bernard Bolzano (1781–1848), in order to show that Georg Cantor's (1845–1918) approach to the problem of defining actual mathematical infinity is not the most natural. In fact, Bolzano's approach to the paradoxes of infinity is more intuitive, while remaining internally coherent. Bolzano's approach, however, had limitations. We discuss implications for teaching, which include a better understanding of the responses of students to situations involving actual mathematical infinity, for it is possible to draw a kind of parallel between these responses and Bolzano's reasoning."

245. Wedberg, Anders. 1984. "Perfection and Innovation: Bernard Bolzano." In *A History of Philosophy. Vol. 3: From Bolzano to Wittgenstein*, 51-85. Oxford: Oxford University Press.

246. Winner, Thomas G. 1994. "Peirce and Bolzano." In *Living Doubt. Essays Concerning the Epistemology of Charles Sanders Peirce*, edited by Debrock, Guy and Hulswit, Menno, 157-169. Dordrecht: Reidel
"Like Peirce, whom he preceded by roughly half a century, Bernard Bolzano (1781–1848), the brilliant mathematician, logician and semiotician who taught and wrote in Prague, was little recognized in his lifetime. Like Peirce, he endured persecution for his uncompromising attitudes, in his case both in science and political-religious life: also Bolzano's teaching career, like Peirce's, was cut short, in Bolzano's case because of official displeasure of the Vatican and the Vienna court over his resolute and unwavering liberalism in religious, social and political matters and towards the relation of Czechs and Germans in the Bohemian crownlands of the Austro-Hungarian monarchy. Bolzano's principal scientific contribution was, like Peirce's, in the area of mathematics and logic; and Bolzano's logic, like Peirce's, contained major contributions to semiotics, which Bolzano called the theory of signs (*Zeichenlehre*) and *Semiotik*, though Bolzano's *Zeichenlehre* was certainly not as comprehensive and systematic as Peirce's semeiotic. Unlike Peirce, Bolzano is

known primarily to logicians and to specialists in Catholic theology, while his semiotics has received relatively little attention." (p. 157)

247. Wrinch, Dorothy Maud. 1917. "Bernard Bolzano (1781-1848)." *The Monist.An International Quarterly Journal of General Philosophical Inquiry* no. 27:83-107
"In Bolzano we find the virtues of human sympathy and insight coupled with the austerer virtues of the metaphysician and logician. He was a man of action as well as a man of ideas. He was well known for his kindly disposition and his broadmindedness. He possessed not only the sympathy with the poor necessary for a social reformer, but the ability to develop his ideas of social reconstruction on practical lines. Not only did he elaborate a theory of an ideal state, but he also introduced numerous reforms in the actual state of which he was a member. He studied theology very earnestly as a young man and later wrote a great deal on the subject. Even though his liberal views brought him into collision with those on whom his livelihood depended, yet he courageously continued his teaching and writing, always making it his aim to seek for truth. He was a metaphysician of some importance and his treatises on metaphysics are valuable, not only for the original thought which they contain, but also for his important criticisms of Kant. In esthetics his work is by no means without interest, and to the psychology and ethics of his day he made very valuable contributions. But preeminently he was a mathematician and logician." (p. 83)