PROSLEPTIC PROPOSITIONS AND ARGUMENTS

by

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I

In some ancient writers on logic we find mention of propositions and syllogisms *κατὰ πρόσληψιν*. We shall attempt in this paper to determine the nature and logical relations of these propositions and arguments, which we call for brevity prosleptic. It will be convenient first to set out the evidence, which is rather tantalizingly exiguous. The quoted passages are numbered for ease of reference. In probable order of date and authority they are as follows.

1. In his work *Εἰσαγωγὴ Διαλεκτική* (*Institutio Logica*, ed. Kalbfleisch, pp. 47–8) Galen says:

> Since the Peripatetics have written about the syllogisms called prosleptic as useful, but they seem to me to be superfluous (*περιττοί*), as I have also shown in my work *On Proof*, it would be proper to say something about them. It is not necessary here to go through their number and nature completely, as I have spoken about them in those notes. Their form (*εἶδος*) will be shown in two examples. One form is like this. "Of what so-and-so, also so-and-so. ⟨But so-and-so of so-and-so. So-and-so also⟩ therefore of so-and-so" (***καθʼ οὗ τόδε, καὶ τόδε· ⟨ἀλλά τόδε κατὰ τοῦδε· καὶ τόδε⟩ ἄρα κατὰ τοῦδε***). And in nouns "Of what tree, also plant. Tree of plane. Plant therefore also of plane" (*ἐφʼ οὗ δένδρον, καὶ φυτόν· δένδρον δε ἐπὶ πλατάνου· καὶ φυτὸν ἄρα ἐπὶ πλατάνου*). It is clear that the word "is predicated" (*κατηγορεῖται*) or "is said" (*λέγεται*) must be understood (*προσυπακοῦσαι*) in the argument so that the complete argument would be like this, "Of what tree is predicated, of that plant is predicated. But tree is predicated of plane. Therefore plant will be predicated of plane." Another kind of prosleptic syllogism is "What is of so-and-so is of so-and-so. ⟨But so-and-so is of so-and-so. So that it is also of so-and-so.⟩" In nouns "What is of tree is also of plane. But plant is of tree. Therefore also of plane." ⟨That⟩ such syllogisms are in a

way sketches (ἐπιτομαί τινες) of the categorical and not a kind different from them I have shown in the notes of which I have spoken, and I need say no more about them here.

2. Alexander of Aphrodisias in his Commentary on *Prior Analytics*, I, 23 (41ᵃ37) (*G.I.A.G.*, ii (i), pp. 263–4) when explaining the difference between μεταλαμβανόμενον and προσλαμβαμόνενον as used by the ἀρχαῖοι, i.e. the Aristotelians, says:

> And they say "taken in addition" (προσλαμβανόμενον) in those cases where something is added in addition to what is laid down, which is contained in them in a way potentially (δυνάμει πως) and not in actuality, as it is in the case of syllogisms which arise through proslepsis (ἐπὶ τῶν κατὰ πρόσληψιν γινομένων συλλογισμῶν). For in the expression "Of what β of that α; but β of γ" (καθ' οὗ τὸ *B* κατ' ἐκείνου τὸ *A*, κατὰ δὲ τοῦ *Γ* τὸ *B*) the expression "β of γ" is taken in addition from outside (ἔξωθεν προσειλῆπται). For it was not actually asserted in the premiss "Of what β, of that α" that β was predicated of γ. They used the word πρόσληψις instead of μετάληψις.

3. Alexander further says in his Commentary on *Prior Analytics*, I, 41 (49ᵇ27) (*C.I.A.G.*, ii (i), p. 378):

> What he [Aristotle] says is that in those propositions which have three terms in them potentially, of which he has just given examples, and in general those called prosleptic (κατὰ πρόσληψιν) by Theophrastus—for these have three terms in a sense, since in the expression "α of all of that of all of which β", in the two terms α and β, that is in the determinate (τοῖς ὡρισμένοις) terms, there is in a sense already contained the third of which β is predicated (ἤδη πως περιείληπται καὶ ὁ τρίτος καθ' οὗ τὸ *B* κατηγορεῖται) except that it is not as determinate and plainly revealed as they are—in such propositions, which differ only in expression (τῇ λέξει μόνον) from categoricals, as Theophrastus has shown in his work *On Assertion*. . . .

4. An anonymous scholiast edited by Brandis (*Scholia in Aristotelem*, 189ᵇ43) in commenting on *Prior Analytics*, II, 5 (58ᵃ21) says:

> He [Aristotle] here outlines for us another kind of proposition which Theophrastus calls prosleptic. Such propositions consist of an indeterminate middle and two determinate extreme terms, as for example in the first figure "What of γ, α of it", in the second "What of α, that of β also", in the third "Of what α, of that β". Such propositions seem not to be simple but to contain potentially a syllogism (δυνάμει περιληπτικαί εἶναι συλλογισμοῦ). Theophrastus says that it is

equivalent in force to the categorical for there is no difference between saying "α of no β" (τὸ *A* κατ' οὐδενὸς τοῦ β) and "α of none of that of all of which β" (καθ' οὗ τὸ β παντός, κατ' οὐδενὸς ἐκείνου τὰ *A*) or again between saying "α of all β" and "α of all of that of all of which β".

5. In a later passage the same scholiast, commenting on *Prior Analytics*, II, 6 (58ᵃ25 ff.), says:

> This is the proposition κατὰ πρόσληψιν. It is called κατὰ πρόσληψιν because when the indeterminate term in the compound premiss is determined and taken in addition, the syllogism is completed and the conclusion is drawn. Such a proposition resembles the complex (ὑποθετικός) conditional (συνημμένος) syllogism.

6. A commentary on Aristotle's *Prior Analytics* attributed to Ammonius has the following on I, 23 (41ᵃ39) (*C.I.A.G.*, iv (vi), p. 67):

> Aristotle calls prosleptic the proposition which is equivalent in force (ἰσοδυναμοῦσαν) to a syllogism and has two terms actually expressed and one only potentially, "Of what man, of that animal". . . . Prosleptic syllogisms are reduced (ἀνάγονται) to the three figures. For "Of what α, β also" is of the third figure.

7. Separated from the passage immediately above only by a short section called "On Hypothetical Syllogisms from the Monograph of Ammonius" (ἐκ τοῦ μονοβίβλου Ἀμμωνίου) is a further section headed "On Prosleptic Syllogisms" which seems also to be intended as a summary of the work of Ammonius. The author is concerned to point out the resemblances and differences between categorical, hypothetical, and prosleptic syllogisms, and he writes as follows (op. cit., p. 69):

> These have in common with categorical syllogisms that they occur in all the figures. In the first, "What to all γ, α to all of it", in the second, "What of β, this of all γ", and in the third, "Of that of all of which α, of this β also." But an affirmative conclusion is drawn in the second figure, and a universal in the third, and conclusions are drawn from two negatives in all figures, and belonging is concluded from not belonging. They have in common with hypotheticals the fact that their propositions (προτάσεις) are linked (συνῆφθαι, cf. συνημμένος) and that the one is established by the other. But they are not totally hypothetical, nor are they reducible to the five moods.

8. Another account of prosleptic arguments by a scholiast of the Ammonian school is to be found in a little work called "Of All Kinds of

Syllogism" which has been published by Wallies in his preface to Ammonius' commentary on the *Prior Analytics* (op. cit., p. xii):

> There is a third kind of syllogism besides the categorical and the hypothetical, called in the work of Theophrastus "prosleptic". It is formed according to the three figures as follows:
>
> Figure 1. What is of all man, substance is of all of it. Animal is of all man. Substance, therefore is also of all animal. . . .
>
> Figure 2. What is of all man, that is of all horse. . . .
>
> Figure 3. Of all of which animal, of all of this rational. . . .
>
> In the three figures the middle is entirely and only indeterminate; but they are not syllogisms in all respects, he says, since they break the peculiar rules of syllogisms.

In this passage both the argument in the second figure and that in the third are incomplete, but it is clear from some accompanying diagrams that the extra premiss to be supplied in each case is "Animal is of all man". The "he" referred to in the last sentence is probably Ammonius, since this passage seems to be a comment on our passage 7 above. It goes on to point out the differences between prosleptic and categorical syllogisms, adding, however that some of the peculiarities of the prosleptic are shared by hypothetical syllogisms, i.e. arguments in the *modus ponens*.

9. Philoponus in his commentary on *Prior Analytics*, II, 5–7(*C.I.A.G.*, xiii (ii), pp. 417 ff. produces several examples of prosleptic propositions. On 58^a21 he has:

(a) You cannot prove the minor premiss except by proslepsis, since it is not possible categorically. . . . By proslepsis the minor is proved in this way "To that to which α in no way belongs, to all of that β belongs (ᾧ τὸ A οὐδ' ὅλως ὑπάρχει, τουτῷ το B παντὶ ὑπάρχει). But γ is something to which α in no way belongs, therefore β belongs to all of it." We must know that in prosleptic syllogisms one term which is indeterminate is later made determinate. In our example "To that to none of which α belongs, to all of that β belongs", you see that we have taken as indeterminate the last term and that it is afterwards made determinate when we say "γ is something to none of which α belongs. Therefore β belongs to all of it."

On 58^a26:

(b) β belongs to all of that to none of which α belongs. But α belongs to none of γ. Therefore β belongs to all of it.

On 58^a41:

(c) What belongs to some γ, α belongs to all of it. But β belongs to some γ. Therefore α belongs to all β.

On 58^b6:

(d) β belongs to some of that to some of which α does not belong. But α does not belong to some of γ. Therefore β belongs to some γ.

On 58^b18:

(e) α belongs to all of that to none of which γ belongs. But γ belongs to none of β. Therefore α belongs to all of β.

On 58^b22:

(f) α belongs to all of that to none of which β belongs. But β belongs to no γ. Therefore α belongs to all γ.

On 58^b27:

(g) α belongs to all of that which does not belong to all of γ. But β does not belong to all of γ. Therefore α belongs to all β.

On 58^b33:

(h) α belongs to some of that to which β belongs only partially (οὐ παντὶ). But β belongs to γ only partially. Therefore α belongs to some γ.

On 59^a24:

(i) That belongs to some γ to which α belongs only partially. α belongs only partially to β. Therefore β belongs to some γ.

Each of these arguments is said explicitly to be prosleptic (διὰ προσλήψεως or κατὰ πρόσληψιν). In addition Philoponus implies by his remark on 59^a18 that he recognizes a proslepic proposition of the form "That to which α does not belong, itself belongs to all β."[1]

II

Although these sources are meagre, it is not difficult to discern the main outlines of the doctrine to which they refer. Either Aristotle or his pupil Theophrastus—and probably it was Theophrastus, since he gets the credit in passages 3 and 4 above—invented the description κατὰ πρόσληψιν for arguments of such forms as "Whatever x-ness may be, if every x thing is α, then every x thing is β; but every γ thing is α; therefore every γ thing is β." The description was intended to draw attention to the fact that these arguments proceed by specification of what was at first left indeterminate. In modern terminology they are arguments by substitution and detachment, where the variable for which we make substitution is general rather than individual. Since, however, the pattern of each such argument is fully determined by the nature of its leading premiss, this also is called prosleptic by a very natural extension of usage and assigned to the same figure as its argument.

In working out their account of the various figures of prosleptic argument the Peripatetics seem to have been influenced by a misleading analogy with the theory of the categorical syllogism which was the pride of their school. Assuming wrongly that the indeterminate term of the leading premiss (i.e. what we now call the term-variable) was to be compared with the middle term of a categorical syllogism, they assigned to the first figure those prosleptic premisses in which the indeterminate term occurred as predicate of the antecedent clause and subject of the consequent clause (e.g. "Whatever x-ness may be, if every α thing is x, then every x thing is β"), to the second figure those in which the indeterminate term occurred as predicate of both clauses, and to the third figure those in which it occurred as subject in both clauses. This classification is explained by the anonymous scholiast in quotation 4 above and again by members of the school of Ammonius in passages 6, 7, and 8. Some gratuitous puzzles into which it led them are indicated in passage 7. In the quotation 9 (i) there is to be found a prosleptic proposition which might be assigned on the same principles to a fourth figure in which the indeterminate term occurs as subject of the first clause and predicate of the second. But apparently the Peripatetics recognized no such figure, and on this point at least they were not misled by their analogy between prosleptic arguments and categorical syllogisms. For just as any fourth figure syllogism can be presented as an indirect mood of the first figure, so any fourth figure prosleptic argument can be reduced to the first figure by contraposition. From "Whatever x-ness may be, if every x thing is α, then every β thing is x" we can easily derive "Whatever x-ness may be, if some β thing is not x, then some x thing is not α", and *vice versa*. Such transformations involve use of categorical forms other than the universal affirmative in the two clauses of a prosleptic premiss, but it is clear from the various quotations listed above under 9 that this possibility was recognized by the Peripatetics.

If for the purpose of making a complete survey of all possibilities we allow four figures of prosleptic propositions and recognize that in any figure each clause may have any one of the four categorical forms traditionally indicated by the letters A, E, I, and O, we obtain the 64 distinct types of prosleptic propositions which are expressed shortly in the accompanying table by a combination of modern logical symbolism with the traditional lettering for categorical forms. For brevity we have omitted the universal quantifier that might have been written at the beginning of each formula. That is to say, the free variable x is to be understood in each case as a sign for generalization over kinds.

So far as we know, Professor Lejewski was the first in modern times to recognize the possibility of 64 different types of prosleptic propositions. Whether any of the Peripatetics ever surveyed all the 48 types distinguishable within the three figures which they allowed we cannot tell for certain, since none of our sources mentions a definite total. It is true that passages 1 and 3 above imply claims by some of the Peripatetics to have made a complete survey. But what these passages say about the equivalence of all prosleptic formulae to categorical seems to be incorrect, at least according to the ordinary Aristotelian interpretation of "categorical". Since evidence of such a mistake is obviously important for an assessment of the completeness of the ancient theory, we shall attempt a systematic account of the various possible kinds of prosleptic propositions before going on to consider why and how much they were studied by the Peripatetics. For this purpose we must distinguish four groups according to the methods by which we can prove their equivalence or non-equivalence to categorical propositions.

1. Each of the 26 members of the first group is indicated in our table by simple entry of a categorical equivalent under a prosleptic formula, and each can be reduced to its categorical equivalent without much difficulty. Since a prosleptic formula involves generalization over kinds, it must be understood to entail every proposition expressible by substitution of a common term for its variable. Thus from I (3) by substitution of α for x we can derive the conditional formula $\alpha A \alpha \supset \alpha I \beta$, whose consequent may be affirmed by the *modus ponens* because its antecedent is tautological. Similarly from I (5) by substitution of the negative term $\bar{\alpha}$ for x we can obtain $\alpha E \bar{\alpha} \supset \bar{\alpha} A \beta$, which is reducible in turn to $\bar{\alpha} A \beta$ or $\bar{\beta} A \alpha$. From I (11), on the other hand, we cannot by any substitution produce a conditional formula whose antecedent is tautological, since the first logical constant of this prosleptic formula is a sign of particularity that does not lend itself to such a manoeuvre. Nevertheless by substitution of $\bar{\beta}$ for x we can in this case produce the conditional formula $\alpha I \bar{\beta} \supset \bar{\beta} I \beta$ whose consequent is self-contradictory, and so by the *modus tollens* we can obtain the negation of the antecedent, namely $\alpha E \bar{\beta}$ or $\alpha A \beta$. Obviously any member of our first group which has both a sign of universality in its first part and a sign of particularity in its second part can be reduced by either method indifferently. Thus in I (3) we can, if we wish, substitute $\bar{\beta}$ for x to produce the conditional formula $\alpha A \bar{\beta} \supset \bar{\beta} I \beta$ whose consequent is self-contradictory. But $\alpha O \bar{\beta}$, or $\alpha I \beta$, which we can then obtain by the *modus tollens*, is the result we have already obtained by the other method.

PROSLEPTIC FORMULAE AND THEIR EQUIVALENTS

	I	II	III	IV
1.	$\alpha Ax \supset xA\beta$ $U\beta$	$\alpha Ax \supset \beta Ax$ $\beta A\alpha$	$xA\alpha \supset xA\beta$ $\alpha A\beta$	$xA\alpha \supset \beta Ax$ $N\beta$
2.	$\alpha Ax \supset xE\beta$ $N\beta$	$\alpha Ax \supset \beta Ex$ $N\beta$	$xA\alpha \supset xE\beta$ $\alpha E\beta$	$xA\alpha \supset \beta Ex$ $\beta E\alpha$
3.	$\alpha Ax \supset xI\beta$ $\alpha I\beta$	$\alpha Ax \supset \beta Ix$ $\beta I\alpha$	$xA\alpha \supset xI\beta$ $* \alpha A\beta$	$xA\alpha \supset \beta Ix$ $* \alpha A\beta$
4.	$\alpha Ax \supset xO\beta$ $\alpha O\beta$	$\alpha Ax \supset \beta Ox$ $* \bar{\alpha} A\beta$	$xA\alpha \supset xO\beta$ $* \alpha E\beta$	$xA\alpha \supset \beta Ox$ $\beta O\alpha$
5.	$\alpha Ex \supset xA\beta$ $\beta A\alpha$	$\alpha Ex \supset \beta Ax$ $N\beta$	$xE\alpha \supset xA\beta$ $\bar{\alpha} A\beta$	$xE\alpha \supset \beta Ax$ $N\beta$
6.	$\alpha Ex \supset xE\beta$ $\beta A\alpha$	$\alpha Ex \supset \beta Ex$ $\beta A\alpha$	$xE\alpha \supset xE\beta$ $\beta A\alpha$	$xE\alpha \supset \beta Ex$ $\beta A\alpha$
7.	$\alpha Ex \supset xI\beta$ $* \bar{\alpha} A\beta$	$\alpha Ex \supset \beta Ix$ $* \alpha A\beta$	$xE\alpha \supset xI\beta$ $* \bar{\alpha} A\beta$	$xE\alpha \supset \beta Ix$ $* \bar{\alpha} A\beta$
8.	$\alpha Ex \supset xO\beta$ $* \beta A\alpha$	$\alpha Ex \supset \beta Ox$ $\beta I\alpha$	$xE\alpha \supset xO\beta$ $* \beta A\alpha$	$xE\alpha \supset \beta Ox$ $\beta I\alpha$
9.	$\alpha Ix \supset xA\beta$ $N\alpha \vee U\beta$	$\alpha Ix \supset \beta Ax$ $N\alpha \vee N\beta$	$xI\alpha \supset xA\beta$ $N\alpha \vee U\beta$	$xI\alpha \supset \beta Ax$ $N\alpha \vee N\beta$
10.	$\alpha Ix \supset xE\beta$ $N\alpha \vee N\beta$	$\alpha Ix \supset \beta Ex$ $N\alpha \vee N\beta$	$xI\alpha \supset xE\beta$ $N\alpha \vee N\beta$	$xI\alpha \supset \beta Ex$ $N\alpha \vee N\beta$
11.	$\alpha Ix \supset xI\beta$ $\alpha A\beta$	$\alpha Ix \supset \beta Ix$ $\alpha A\beta$	$xI\alpha \supset xI\beta$ $\alpha A\beta$	$xI\alpha \supset BIx$ $\alpha A\beta$
12.	$\alpha Ix \supset xO\beta$ $\alpha E\beta$	$\alpha Ix \supset \beta Ox$ $N\alpha$	$xI\alpha \supset xO\beta$ $\alpha E\beta$	$xI\alpha \supset \beta Ox$ $N\alpha$
13.	$\alpha Ox \supset xA\beta$ $N\alpha \vee U\beta$	$\alpha Ox \supset \beta Ax$ $N\alpha \vee N\beta$	$xO\alpha \supset xA\beta$ $U\alpha \vee U\beta$	$xO\alpha \supset \beta Ax$ $U\alpha \vee N\beta$
14.	$\alpha Ox \supset xE\beta$ $N\alpha \vee N\beta$	$\alpha Ox \supset \beta Ex$ $N\alpha \vee N\beta$	$xO\alpha \supset xE\beta$ $U\alpha \vee N\beta$	$xO\alpha \supset \beta Ex$ $U\alpha \vee N\beta$
15.	$\alpha Ox \supset xI\beta$ $N\alpha$	$\alpha Ox \supset \beta Ix$ $N\alpha$	$xO\alpha \supset xI\beta$ $\bar{\alpha} A\beta$	$xO\alpha \supset \beta Ix$ $\bar{\alpha} A\beta$
16.	$\alpha Ox \supset xO\beta$ $N\alpha$	$\alpha Ox \supset \beta Ox$ $\alpha A\beta$	$xO\alpha \supset xO\beta$ $\beta A\alpha$	$xO\alpha \supset \beta Ox$ $U\alpha$

In order to show that the categorical formula derivable from a prosleptic formula by substitution and use of the *modus ponens* or the *modus tollens* is not merely a consequence of the prosleptic formula but a deductive equivalent, we must also derive the prosleptic formula from the categorical. For this purpose it is sufficient to note that whatever value be given to the variable in a modern prosleptic formula of the group under consideration, the second part is obtainable as conclusion in a valid syllogism that has for one premiss the first part of the prosleptic formula and for the other the categorical formula we offer as equivalent of the whole. Thus, whatever value we give to x, the second part of I (3), namely $xI\beta$ is obtainable by a syllogism in Disamis from the first part, αAx, taken together with the categorical formula $\alpha I\beta$ which appears below it in our table. That is to say, we have

$$\alpha I\beta,\ \alpha Ax \vdash xI\beta$$

from which it is easy to get

$$\alpha I\beta \vdash \alpha Ax \supset xI\beta$$

by conditionalization. In fact by the two operations of conditionalization and generalization over inessential terms every valid syllogism can be shown to give rise to a prosleptic formula. Moreover, since it does not matter for the purpose of the argument whether the major or the minor premiss of a syllogism is written first, each valid syllogism may be said to give rise to two prosleptic formulae. And conversely, since some syllogisms are reducible to others, a single prosleptic formula may be connected with more than one syllogism. For prosleptic formulae of our first group the most obvious connections with syllogism seem to be the following:

I (3)	Disamis	II (1)	Barbara
(4)	Bocardo	(3)	Darii
(5)	Camenes	(6)	Celarent
(6)	Camenes	(8)	Ferio
(11)	Datisi	(11)	Disamis
(13)	Ferison	(16)	Bocardo
III (1)	Barbara	IV (2)	Camestres
(2)	Celarent	(4)	Baroco
(5)	Barbara	(6)	Cesare
(6)	Camestres	(8)	Festino
(11)	Darii	(11)	Dimaris
(12)	Ferio	(15)	Dimaris
(15)	Darii		
(16)	Baroco		

In short, figures I and IV of prosleptic formulae, which are inter-convertible, are both naturally associated with figures II and IV of syllogism, while figure II of prosleptic formulae goes with figures I and III of syllogism and figure III of prosleptic formulae with figures I and II of syllogism It will be noticed, however, that some of the prosleptic formulae of our first group are equivalent to categorical formulae with negative terms that cannot be eliminated. Obviously none of these can be obtained from its equivalent by conditionalization of any syllogism that Aristotle would have recognized.

2. Each of the ten members of our second group of prosleptic formulae is indicated in the large table by an asterisk before its categorical equivalent. All alike have a sign of universality in the first part and a sign of particularity in the second, but they are distinguished from those members of the first group which share this peculiarity, e.g. I (3), by the fact that application of the *modus ponens* after a substitution which makes the antecedent a tautology does not lead to the same result as an application of the *modus tollens* after a substitution which makes the consequent a self-contradiction. Obviously, therefore, neither of the results obtainable in this way can be the categorical equivalent of the original prosleptic formula. But each such result is a categorical formula genuinely entailed by the original, and careful examination shows that they fall into a curious pattern. For when treated by the methods we have used so far, each prosleptic formula of the second group yields either, like I (7), the pair of categorical formulae $\alpha\mathrm{I}\bar{\beta}$ and $\bar{\alpha}\mathrm{I}\beta$ or, like I (8), the pair $\alpha\mathrm{I}\beta$ and $\bar{\alpha}\mathrm{I}\bar{\beta}$. And each of these pairs of consequences amounts to a requirement that neither α nor β in the prosleptic formula from which it comes be either a null or a universal (i.e. all-comprehensive) term. In short they call for a strictly Aristotelian interpretation of the schematic letters in the prosleptic formulae under consideration. If we suppose a similar requirement to hold for all terms admissible in substitution for the variables of these formulae, we find a new way of determining their categorical equivalents.

From the fact that α and β are Aristotelian terms it does not follow that complex terms such as $\alpha\beta$ and $\alpha \vee \beta$ are also Aristotelian. On the contrary, if neither α nor β is null, proof of the nullity of $\alpha\beta$ is a discovery entitling us to assert the Aristotelian proposition $\alpha\mathrm{E}\beta$. Similarly, if neither α nor β is universal, proof of the universality of $\alpha \vee \beta$ is a discovery entitling us to assert the proposition $\bar{\alpha}\mathrm{A}\beta$, which fails to be Aristotelian only by involving a negative term. We must therefore consider whether the prosleptic formulae of our second group impose on their variables any conditions which amount to rejection of certain complex terms as null or universal, and we find that this is in fact so. If, for example, we substitute $\bar{\alpha}\bar{\beta}$ for x in I (7), we get the conditional formula $\alpha\mathrm{E}\bar{\alpha}\bar{\beta} \supset \bar{\alpha}\bar{\beta}\mathrm{I}\beta$ which must be false because its antecedent is a tautology and its consequent a self-contradiction. From this result it follows that if I (7) is true $\bar{\alpha}\bar{\beta}$ cannot be an admissible term. But even within the restrictions of the Aristotelian scheme $\bar{\alpha}\bar{\beta}$ cannot be excluded from the range of the generalization in I (7) unless it is null, i.e. unless $\bar{\alpha}\mathrm{E}\bar{\beta}$, or more shortly $\bar{\alpha}\mathrm{A}\beta$, is entailed. By similar substitutions of conjunctive terms categorical formulae can be derived from all the other prosleptic formulae of our second group except II (4). This last (which still eluded our grasp when we wrote a sketch of the present theory for the third impression of *The Development of Logic* in 1966) is peculiar in having a first part, $\alpha\mathrm{A}x$, from which we cannot obtain a tautology by substitution of a conjunctive term for x. We must therefore substitute instead the disjunctive term $\alpha \vee \beta$ to get the conditional formula $\alpha\mathrm{A}\alpha \vee \beta \supset \beta\mathrm{O}\alpha \vee \beta$ whose necessary falsity indicates that $\alpha \vee \beta$ is an inadmissible term, to be excluded from the range of the generalization in II (4). Since, however, $\alpha \vee \beta$ cannot be null if α and β are not null, it must be universal. In other words II (4) must entail $\bar{\alpha}\mathrm{A}\beta$ under Aristotelian restrictions. So for all prosleptic formulae of our second group the categorical consequence that we require is to be found only by exclusion of null and universal terms.

When we turn to consider whether the consequences so obtained are deductively equivalent to the original formulae, we find that they are so only under the same restriction to terms that are neither null nor universal. For while in each case the categorical formula presented in our table after an asterisk can be conjoined with the first part of the relevant prosleptic formula to make a syllogism yielding the second part as conclusion, that syllogism is always of the variety called subaltern, i.e. a syllogism with a weakened conclusion assertible only on the assumption of existential import for universal statements. As might be expected, more than one syllogism may be associated with a single prosleptic formula, but the most natural associations seem to be:

I (7)	Barbari	II (4)	Barbari
(8)	Camenos	(7)	Bramantip
III (3)	Barbari	IV (3)	Bramantip
(4)	Celaront	(7)	Bramantip
(7)	Barbari		
(8)	Camestros		

As before, II (4) is somewhat peculiar. For in order to construct an associated argument which looks something like a traditional syllogism we must rewrite

$$\bar{\alpha}A\beta,\ \alpha Ax \vdash \beta Ox$$

in the form

$$\bar{\alpha}A\beta,\ \bar{x}A\bar{\alpha} \vdash \bar{x}I\beta$$

where x as well as α appears each time under a negation sign.

It will be noticed that the list of syllogisms given above contains neither *Darapti* nor *Felapton*, which in Aristotle's scheme are the only two independent syllogisms requiring existential import. The reason for this is that each of the four prosleptic formulae obtainable either from *Darapti* or from *Felapton* can be obtained also from a syllogism with a particular premiss and is in fact deductively equivalent to that premiss. The same holds also for some of the prosleptic formulae obtainable from subaltern syllogisms, but not for the members of our second group, because these do not by themselves provide the distribution of terms necessary for a new syllogism with a particular premiss.

3. Each of the twelve members of the third group of prosleptic formulae is indicated in our table by the entry beneath it of a two-letter formula such as $N\alpha$ or $U\beta$. Six have signs of universality in both parts and six have signs of particularity in both parts. By the methods used for investigation of the first group each can be shown to entail one or other of the categorical formulae $\alpha A\beta$, $\beta A\alpha$, $\alpha E\beta$, but no valid syllogism can be conditionalized in such a way as to show that any of these categorical formulae entails the prosleptic formula from which it is derived. In short none of the prosleptic formulae of our third group has a categorical equivalent of the ordinary kind, and closer examination shows that the categorical formulae which they entail are non-Aristotelian.

For this purpose we proceed by the methods used for investigating the first group, except that where we formerly put a positive term α or β in place of x to make a tautology or contradiction in one part of a prosleptic formula we now put the universal term $\cup$ (which may be understood in the sense of "entity" or taken as short for $\alpha \vee \bar{\alpha}$, $\beta \vee \bar{\beta}$, etc.) and where we formerly put a negative term $\bar{\alpha}$ or $\bar{\beta}$ we now put the null term $\bar{\cup}$ (which may be understood in the sense of "non-entity" or taken as short for $\alpha\bar{\alpha}$, $\beta\bar{\beta}$, etc.). Obviously we can still go on to use the *modus ponens* or the *modus tollens*, as the case may be, but what we get in the end is not any categorical formula that Aristotle would have recognized but something such as $\cup A\alpha$ (for which it is convenient to use the abbreviation $U\alpha$) or $\cup E\alpha$ (for which it is convenient to use the abbreviation $N\alpha$). Thus in I (1) our new procedure leads to $\cup A\beta$, which entails our previous result $\alpha A\beta$ and may be taken as a deductive equivalent of the original prosleptic formula by anyone who insists on trying to translate that into a shorter and more familiar pattern.

4. The 16 prosleptic formulae of our fourth group are those lying in the rows 9, 10, 13, and 14 of the table, which all have a sign of particularity in the first part followed by a sign of universality in the second. Because of their special form they cannot be treated by any of the methods used so far. But it is easy to rewrite each in such a way as to show its equivalence to the formula printed below it in the table. Thus I (9) can be reduced to $\alpha Ex \vee xA\beta$ and this in turn to $\sim(xI\alpha) \vee \sim(xI\bar{\beta})$, which evidently means that there is nothing covered by α or nothing covered by $\bar{\beta}$, i.e. what we convey by the formula $N\alpha \vee U\beta$. Similarly the disjunctive formula $\alpha Ex \vee xA\beta$ can be derived without difficulty from either $N\alpha$ or $U\beta$ and therefore also from the disjunction of these. In this connection it is important to notice that the simplest equivalent for any prosleptic formula of our fourth group is itself a disjunctive formula and therefore not to be called categorical, however widely we interpret that word.

III

We must now enquire how the theory of prosleptic argument first arose in antiquity. Our quotations from Philoponus (9) show clearly that one important source is Aristotle's *Prior Analytics*, II, 5–7. In these chapters he is engaged in the curious intellectual exercise of determining which of the valid syllogistic forms give rise to an equally valid syllogistic form having as premisses the conclusion of the original syllogism and one of its premisses with its terms reversed (the sign of quantity and quality remaining unchanged) and as conclusion the other premiss. In some cases he finds that he can get something like the result he wants by substituting for one of the premisses or the conclusion of the original a prosleptic form in the new syllogism and he gives prosleptic formulae on four occasions in the course of the discussion without offering any general account of his innovation. They are as follows:

(i) *ᾧ τὸ A μηδενὶ ὑπάρχει, τὸ B παντὶ ὑπάρχειν* (58ᵃ29–31), i.e. "β belongs to all of that to none of which α belongs";

(ii) *ᾧ τὸ A τινὶ μὴ ὑπάρχει, τὸ β τινὶ ὑπάρχειν* (58ᵇ9–10), i.e. "β belongs to some of that to some of which α does not belong";

(iii) *ᾧ τὸ B τινὶ μὴ ὑπάρχει, τὸ A τινὶ ὑπάρχειν* (58ᵇ37–38), i.e. "α belongs to some of that to some of which β does not belong";

(iv) *ᾧ τὸ A τινὶ μὴ ὑπάρχει, τὸ Γ τινὶ ὑπάρχειν* (59ᵃ28–29), i.e. "γ belongs to some of that to some of which α does not belong".

It will be noticed that only two distinct forms of proposition are involved, namely III (5) and III (15) in our table, both of which are equivalent to $\bar{\alpha}A\beta$. As Prior points out, it was Aristotle's failure to recognize negative terms which gave him occasion to formulate prosleptic propositions here. Since the connection between these three chapters and prosleptic propositions has been well explained both by Lejewski[2] and by Prior,[3] we need say no more of them except to remark that Philoponus in his commentary tries to improve on his master not only by supplying prosleptic premisses of forms recognized by Aristotle in cases where Aristotle could have thought of them but did not (our quotations 9 (e) (f) and (h)) but also by producing prosleptic forms of his own that were not used by Aristotle, i.e. our quotations 9 (c), (g) and (i). Of these latter, which have respectively the forms I (9), I (13) and IV (15) of our table, two are equivalent to non-Aristotelian propositions while IV (15) is equivalent to III (15) which Aristotle himself uses in the passage on which Philoponus comments.

Another and more interesting source for the theory of prosleptic syllogisms is *Prior Analytics*, I, 41, in connection with which Alexander (our quotation 3) mentions prosleptic propositions. This chapter is undoubtedly difficult and seems to have baffled commentators from Alexander downwards. It has misled some modern commentators[4] into attributing to Aristotle a failure to recognize certain equivalences stated by Theophrastus, i.e. those between our prosleptic forms III (1) and III (5) and the categorical A and E propositions. It seems to us on the other hand that Aristotle shows clearly in this chapter that he recognizes the first of these equivalences.

He begins by saying that there is a difference both in the expressions themselves and in what they mean (οὐκ ἔστι δε ταὐτὸν οὔτ' εἶναι οὔτ εἰπεῖν) between "α belongs to all of that to which β belongs" (ᾧ τὸ B ὑπάρχει, τούτῳ παντὶ τὸ A ὑπάρχει) and "α belongs to all of that to all of which β belongs". Both these seem to be prosleptic expressions corresponding respectively to our forms III (9) and III (1), i.e. with Aristotle's lettering $xI\beta \supset xA\alpha$ and $xA\beta \supset xA\alpha$. He is indeed right in saying that they are different expressions and express different states of affairs, but since it is not likely that he realized fully the peculiar character of an expression of the form III (9), he must have reached his conclusions in some other way. In order to catch the drift of his argument we must realize that, although he begins and ends with the distinction between these two prosleptic forms, his main purpose is to elucidate the meaning of an expression commonly used by him as a substitute for the ordinary universal proposition, i.e. τὸ A λέγεται (κατηγορεῖται) καθ' ὅυ τὸ B παντός or

alternatively τὸ A ὑπάρχει παντὶ καθ' οὗ τὸ B. The artificiality of Aristotle's terminology is such that he probably failed to realize any difference between this and the simple τὸ A κατηγορεῖται κατὰ παντὸς τοῦ B. Reflecting on it, however, he came to realize that it was ambiguous, especially as the phrase καθ' οὗ τὸ B would almost invariably in syllogistic contexts designate a *kind* rather than an individual. Does β belong to that kind simply (ὑπάρχει μόνον) or does it belong to it universally (ὑπάρχει παντί)? In this chapter Aristotle uses the distinction between simple or indeterminate predication and universal predication rather than that between particular and universal. He has already recognized in the *Analytics* that the indeterminate is equivalent to the particular in logical force so that if we represent the indeterminate proposition as $\beta I\alpha$ we do not falsify his doctrine logically, but in order to preserve the flavour of this chapter, we shall use the form $\beta Y\alpha$ for indeterminate predication, the "Y" being intended to suggest the Aristotelian verb ὑπάρχει with the succeeding letter as its grammatical subject.

In interpreting this chapter, it will be convenient to begin with its conclusion (49^b27 ff.) where Aristotle finally elucidates the meaning of τὸ καθ' ὅυ τὸ B παντὸς τὸ A λέγεσθαι. It means, he says, καθ' ὅσων το B λέγεται, κατὰ πάντων λέγεσθαι καὶ τὸ A ("A is said of all those things of which B is said"). The plural indicates that the word πᾶς is not here used, as in many other places in the chapter, to express quantification within a clause of a prosleptic expression but rather to show that the whole expression is to be taken universally, i.e. in modern terminology it expresses the universal closure. "α is said of everything to which β applies", Aristotle tells us, means that α is predicated of all those things, be they kinds or individuals, of which β is predicated. This is, in our symbolism, a declaration of equivalence between $\beta A\alpha$ and $xY\beta \supset xY\alpha$. The result is correct, as the entry III (11) of our table confirms, but Aristotle reaches his conclusion in a different way. He tries to determine the meaning of the universal proposition by considering what can be derived from it in terms of syllogistic. He has realized (by reasoning similar to that outlined in the second part of our paper) that he can derive from the universal proposition in virtue of *Barbara* the prosleptic $xA\beta \supset xA\alpha$ and in virtue of *Darii* the prosleptic $xI\beta \supset xI\alpha$ but that no syllogism would enable him to derive $xI\beta \supset xA\alpha$. For this reason he is convinced of the difference between this expression and $xA\beta \supset xA\alpha$, which he enunciates in the first sentence of the chapter and repeats at the end where he says that, if the universal proposition as elucidated holds, we have $xA\beta \supset xA\alpha$ but not $xI\beta \supset xA\alpha$ (εἰ μὲν κατὰ παντὸς τὸ B, καὶ τὸ A οὕτως· εἰ δὲ μὴ κατὰ παντός, οὐκ ἀνάγκη τὸ A κατὰ παντός) (49^b30–31). In his exposition he assumes a third term γ in place of our variables, but he is well aware that it is permissible to generalize over this, as he shows in the final section of the chapter (49^b33–50^a4) on ἔκθεσις, where he argues that the particular nature of the third term is irrelevant to the argument. The third term would occupy the subject position in both the minor premiss and the conclusion of a first figure syllogism, and for this reason Aristotle considers and uses only Figure III prosleptic forms, which are indeed the most natural.

The intermediate sections of the chapter are not very well arranged and they may be jottings not brought into a properly connected form, but the meaning of each section is clear.

First (lines 16 ff.) Aristotle distinguishes clearly between $\beta A\alpha$ and $\beta Y\alpha$ by means of an example. Then he says correctly that if we have merely $\beta Y\alpha$ no prosleptic form is derivable (εἰ μὲν οὖν τὸ *A* τῷ *B* ὑπάρχει μὴ παντὶ δὲ καθ' οὗ τὸ *B*, οὔτ' εἰ παντὶ τῷ *Γ* το *B*, οὔτ' εἰ μόνον ὑπάρχει, ἀνάγκη τὸ *A* οὐχ ὅτι παντὶ ἀλλ' οὐδ' ὑπάρχει). That is, from $\beta Y\alpha$ we cannot derive $xA\beta \supset xA\alpha$ or $xA\beta \supset xI\alpha$ or $xI\beta \supset xA\alpha$ or $xI\beta \supset xI\alpha$. In the next section, beginning at line 22, he says that $xI\beta \supset xA\alpha$ entails $xA\beta \supset xA\alpha$ (εἰ δε καθ' οὗ ἂν τὸ *B* λέγηται ἀληθῶς τουτῷ παντὶ ὑπάρχει ⟨τὸ *A*⟩, συμβήσεται τὸ *A*, καθ' οὗ παντὸς τὸ *B* λέγεται κατὰ τούτου παντὸς λέγεσθαι). This is correct, but Aristotle is unlikely to have reached his conclusion by consideration of the peculiar nature of propositions of the form III (9). It is more probable that he argued justly that what follows from a weaker proposition of the I form must also follow from the corresponding stronger proposition of the A form.

In the next section (ll. 25 ff.) he states that from $xA\beta \supset xI\alpha$ no other prosleptic form follows (εἰ μέντοι τὸ *A* λέγεται καθ' οὗ ἂν τὸ *B* λέγηται κατὰ παντός, οὐδὲν κωλύει τῷ *Γ* ὑπάρχειν τὸ *B*, μὴ παντὶ δε τὸ *A* ἢ ὅλως μὴ ὑπάρχειν). This is the only statement in the chapter which may be regarded as incorrect; for as we see from our table, III (3) $xA\beta \supset xI\alpha$ is equivalent under Aristotelian assumptions to $\beta A\alpha$, and under these assumptions it should therefore entail the same prosleptic forms as the universal proposition. That Aristotle says what he does here is a sign that he did not fully recognize what was involved in his commitment to existential import. It is in line with his failure to recognize *Barbari*. But the equivalence in question is not an easy one to establish, and Aristotle may well have argued that, since from the combination AI in a first figure syllogism he could get only an I proposition and the I proposition alone yields no prosleptic proposition of the third figure, the same must be true of the AI combination.

To sum up, what Aristotle has established correctly in this chapter are the two points: (a) that $\beta A\alpha$ is equivalent to $xA\beta \supset xA\alpha$ and

also to $xY\beta \supset xY\alpha$, and (b) that $xI\beta \supset xA\alpha$ entails $xA\beta \supset xA\alpha$ but not *vice-versa*.

This is no mean achievement for a logician with the technical resources at his disposal.

IV

We have noticed that Aristotle uses only prosleptic formulae of the third figure. His practice might arise naturally from thinking of the major premiss of a first figure syllogism as a material principle of inference which justifies the passage from the minor premiss to the conclusion. To take Galen's example, if we are asked to justify the inference from "A plane is a tree" to "A plane is a plant", we may do so by citing as a principle the truth "Whatever is a tree is a plant."[5] In modern times we should most naturally take "whatever" (of the x in $xY\alpha \supset xY\beta$) as an *individual* variable, but Aristotle thought that the relation he expressed by ὑπάρχειν could hold between universal and universal, as well as between universal and individual, and his use of prosleptic formulae seems to be no more than a small development of this way of thinking which he found useful in certain contexts.

The name κατὰ πρόσληψιν is ascribed to Theophrastus by two of our authorities (quotations 3 and 4), and it is probable that he worked out the theory of the three figures, which would be in line with his theory of the three figures of totally hypothetical syllogisms.[6] Alexander (quotation 3) and the anonymous scholiast (quotation 4) also state that Theophrastus showed that prosleptic propositions were equivalent to categoricals. If he made this claim quite generally, then, as our table shows, he was wrong, but the only two specific equivalences attributed to him by the anonymous scholiast are correct, and it may be that he did not consider all forty-eight formulae of the complete scheme, but only a few which are in fact equivalent to categoricals, e.g. I (6) and II (6), I (11) and II (11).

We have argued that in recognizing such equivalences. Theophrastus did not differ from Aristotle, but there is one passage in Alexander (not quoted above) which seems to suggest a difference between them. After trying to explain Aristotle's account of the meaning of καθ' οὗ τὸ *B* παντὸς τὸ *A* λέγεσθαι in accordance with his view that the main purpose of the whole chapter is to point out that in a first figure syllogism it is the major premiss that must be universal, he says "Theophrastus, however, in his work *On Assertion* takes 'α of what β' as being equal in force to 'Of all of what β, of all of that α'." (ὁ μέντοι Θεόφραστος ἐν τῷ Περὶ καταφάσεως τὴν 'καθ' οὗ τὸ *B* τὸ *A*' ὡς ἴσον δυναμένην λαμβάνει τῇ καθ οὗ παντὸς τὸ *B*, κατ' ἐκείνου παντὸς τὸ *A*.[7])

This remark is mysterious on any account, as there is no discussion in Aristotle of the simple expression *καθ' οὗ τὸ Β το Α*. Nor is it clear how we should interpret the Greek. If we take it simply as meaning $\beta\mathrm{Y}\alpha$, then Alexander is attributing an absurd mistake to Theophrastus. If, on the other hand, we interpret it as meaning $x\mathrm{Y}\beta \supset x\mathrm{Y}\alpha$, then Theophrastus is insisting on the equivalence of that formula and $x\mathrm{A}\beta \supset x\mathrm{A}\alpha$. But on this point, as we have seen, Aristotle agrees, so that Alexander's adversative *μέντοι* is out of place. Possibly Alexander means that Theophrastus denied that either $x\mathrm{I}\beta \supset x\mathrm{A}\alpha$ or $x\mathrm{A}\beta \supset x\mathrm{I}\alpha$ is a legitimate expansion of the simple *καθ' οὗ τὸ β τὸ Α*. But again there is no reason to suppose that Aristotle would have disagreed with him here.

In general it may be said that Alexander's commentary on this chapter is not helpful. He is obsessed with the categorical syllogism and thinking it necessary to find a simple categorical expression for each prosleptic formula tries to take both $x\mathrm{I}\beta \supset x\mathrm{I}\alpha$ and $x\mathrm{I}\beta \supset x\mathrm{A}\alpha$ as equivalent to $\beta\mathrm{I}\alpha$.[8] This being so, it is not surprising that he makes little sense of Aristotle's exposition of the relations between the different prosleptic formulae.

His misunderstanding may have led to the rather curious examples given by the Ammonian scholiast in our quotation 8. For it seems likely, though not certain, that "What is of all man, that is of all horse" and "Of all of which animal, of all of this rational" are offered as *true* examples of the second and third figure prosleptic formulae $\alpha\mathrm{A}x \supset \beta\mathrm{A}x$ and $x\mathrm{A}\alpha \supset x\mathrm{A}\beta$. Plainly it is not the case either that all horses are men or that all animals are rational, and so it seems that the scholiast may be reading his formulae as existential, i.e. as equivalent to $\exists x(\alpha\mathrm{A}x \,\&\, \beta\mathrm{A}x)$ and $\exists x(x\mathrm{A}\alpha \,\&\, x\mathrm{A}\beta)$. But it is obvious that the addition of a second premiss of the form $\alpha\mathrm{A}\gamma$ or $\gamma\mathrm{A}\alpha$ would yield no conclusion.

It is difficult to tell from the passage we quote how much of the genuine theory Galen understood. If when he says that prosleptic syllogisms are superfluous he means that prosleptic propositions are either equivalent to categoricals or to non-categoricals useless in ordinary discourse, then he is startlingly right. But his characterization of prosleptic syllogisms is not very perceptive; for the fully developed prosleptic syllogism is every bit as explicit as and somewhat longer in expression than the categorical syllogism to which it is equivalent. It may, however, be that he is here expressing rather carelessly the same thought as the anonymous scholiast who says in our quotation 4 that a prosleptic *proposition* contains a syllogism potentially; and, if so, what he says is valuable. For a prosleptic premiss can be regarded as a kind of recipe for constructing the whole argument by finding a suitable substitute for x. The completed prosleptic syllogism can then easily be transformed into the equivalent

categorical syllogism. Galen's examples make this clear. But it is to be noticed that in the categorical syllogism corresponding to his third figure prosleptic syllogism the middle term "tree" is not the substitute for x. His analogy between the middle term of the categorical and the indeterminate term of the prosleptic is wholly misleading.

Galen sets something of a historical puzzle by his remark that the Peripatetics found prosleptic syllogisms useful. As presumably they followed Aristotle and Theophrastus in thinking prosleptic propositions equivalent to categoricals, it is difficult to see why this should be so. But unless this is a simple reference to commentaries on *Prior Analytics*, II, 5–7, it is a remark we cannot interpret on our present evidence.

Philoponus improves somewhat on Alexander in his commentary on *Prior Analytics*, I, 41. He says that the point of the chapter can scarcely be to show that the major premiss of a first figure syllogism must be universal, as Aristotle had proved this abundantly elsewhere,[9] and he suggests, which is nearer the mark, that Aristotle is here concerned with the analysis of enthymemes.[10] Given a minor premiss and a conclusion, what major premiss is necessary to justify the transition from one to the other? This is a possible way of conceiving the discovery of the prosleptic leading premiss. But he does not mention proslepsis in his commentary on this chapter, and when he treats it in detail in his commentaries on II, 5–7, he does so with an unintelligent formalism. In this curious episode in the history of logic Aristotle, as often, shows up better than his followers.

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[1] This is pointed out by C. Lejewski, "On Prosleptic Syllogisms", *Notre Dame Journal of Formal Logic*, II, 3, p. 163.

[2] Op. cit. pp. 165–7.

[3] *Formal Logic*, pp. 124–5.

[4] I. M. Bochenski, *La Logique de Théophraste*, pp. 50–1, and A. N. Prior, op. cit., p. 122.

[5] Strictly speaking, the material principle is "Being a tree entails being a plant", but Aristotle did not make a clear distinction between first- and second-order discourse even when expounding the formal principles of syllogism.

[6] *C.I.A.G.*, ii (i), p. 326.

[7] *C.I.A.G.*, ii (i), p. 379, 9–11.

[8] *C.I.A.G.*, ii (i), p. 375, 18–20.

[9] *C.I.A.G.*, xiii (ii), p. 350.

[10] Ibid., p. 348.