Boethius' Contribution to Quadrivium. Selected Bibliography

BIBLIOGRAPHY


2. Bower, Calvin M. 1978. "Boethius and Nichomachus: An Essay Concerning the Sources of *De institutione musica*." *Vivarium* no. 16:1-45. "I begin this inquiry with two underlying assumptions. First, Boethius was principally a translator when putting together the *De institutione musica*. The treatise seems to follow the *De institutione arithmetica* in the chronology of Boethius' works, and the arithmetical treatise is recognized to be a translation of Nicomachus' *εισαγωγή άριθμητική*. (...) Boethius' method of composing in his early works is that of compiling through translation with some commentary; the arithmetical treatise and the logical works clearly demonstrate this point. Thus inquiring into Boethius' sources I am trying to determine which Greek treatise Boethius was translating when he compiled his musical treatise. My second assumption is that Boethius was a conscientious and competent translator." (pp. 1-2) (...) "Since this essay is somewhat expository in nature, its organization must largely follow that of Boethius' treatise. Seven principal sections will be designated as follows: I. Pattern of citation in the mathematical works II. Books I and II III. Book III IV. Book IV V. Unity of Books I-IV VI. Nicomachus and Ptolemy VII. Book V and the original scope of *De institutione musica* In that my conclusions concerning the first three books are similar to those of Pizzani (*) and other writers, these sections may be brief and concise. Since my treatment of Book IV stands in sharp contrast to previous scholarship, that section must be the most detailed and extended." (pp. 3-4 notes omitted) (*) Ubaldo Pizzani 'Studi sulle fonti del " De Institutione Musica" di Boezio', Sacris erudiri, 16 (1965), 5-164.

3. ———. 1981. "The Role of Boethius' *De Institutione Musica* in the Speculative Tradition of Western Musical Thought." In *Boethius and the Liberal Arts. A Collection of Essays*, edited by Masi, Michael, 157-174. Bern: Peter Lang. "An attempt to define the role of Boethius' *De institutione musica* in the speculative tradition of Western musical thought may appear to be an awesome and even pretentious task, especially in context of a study as brief as the present one. My limitations may seem even more severe in that I will confine my discussion to musical writings before the year 1100. The centuries immediately prior to 1100 saw the birth of that sphere of study which has come to be called musical theory; the nature of the thought which has grown and developed within this sphere was largely shaped during the years between about 500, when Boethius compiled his *De institutione musica*, and the year 1000, when the first full flowering of medieval musical theory was completed in the works of such theorists as Guido of Arezzo and Herman of Reichenau. If my temporal and spatial boundaries for this study seem small, I would like my consideration of the word "speculative" to seem large. By "speculative tradition" I do not mean what Boethius would term *musica mundana* or even *musica humana*; I mean rather man's verbal reflections and meditations..."
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Boethius, following Pythagorean and neo-Platonic authors before him, held that quantity was divided into two basic genera: discrete quantity – or multitude; and continuous quantity – or magnitude. The monad, or unity, was the source of discrete quantity, and this genus could increase into infinite multitude; yet its basic element, unity, remained indivisible. Magnitude, or continuous quantity, might be represented by the line or a shape, which was delimited with respect to increasing and growth, but could be infinitely divided. The two basic genera of quantity were, in turn, subdivided into two species: multitude is best represented by number, and every number can be considered in and of itself (even, odd, perfect, square, cube, etc), or it can be considered in relation to another (in ratios and proportions – e.g., 2:1, 3:2, or 6:4:2); magnitude is best represented by shapes, and some shapes are fixed and immobile (e.g., a line, a triangle, a cube), while others are in motion (e.g., the sun, the moon, the heavenly spheres). Four areas of study were thus defined by the very nature of quantity: arithmetic pursued number in and of itself; music examined number in ratios and proportions; geometry considered immobile magnitudes; astronomy investigated magnitudes in motion. Boethius described these four disciplines as the quadrivium, the fourfold path by which the soul was led from the slavery of sensual knowledge to the mastery of knowing immutable essences. Musica thus became a necessary prerequisite to the study of philosophy. (19) (pp. 141-142)


"Boethius stands at the very end of a long tradition of mathematical philosophy that had extended for nearly a thousand years. (1) His work on music is much the more original of the two treatises to be considered in this chapter, and it is to that that the greater part of our attention will be devoted; but Boethius also wrote on arithmetic at least of the other mathematical arts, and his contribution must be considered in the light of a tendency in antiquity to isolate the four speculative mathematical subjects — arithmetic, music, geometry and astronomy — and to group them together as a unit which Boethius was apparently the first to call the 'quadrivium' or 'quadrivium'. (2) Opposed to this of course was the trivium of grammar, rhetoric and dialectic, the whole forming the group of seven liberal arts’ which combined with the ‘three philosophies’ made up the staple diet of the medieval university curriculum. With these larger units we are not here concerned. But the development of the quadrivium, and the contribution of Boethius to it, demands a closer look." (p. 135)

(1) In the East, however, writers such as Michael Psellus, Bryennios and Pachymeres reproduced ancient musical theory in their treatises of the Byzantine middle ages, just as Boethian philosophy is reproduced in the works of Hucbald, Regino of Prüm, Johannes de Mûris and innumerable others of the West. But even if Boethius himself were proved to be a wholly unoriginal thinker in music, his work would still belong to the very end of the long cultural tradition to which it relates, and from which the works of Cassiodorus and Isidore already mark a significant break.

(2) Arith. 1: i: pp. 7. 25, 9. 28.


9. Erickson, Raymond. 1992. "Eriugena, Boethius, and the Neoplatonism of Musica and Scolica Enchiriadis." In Musical Humanism and Its Legacy: Essays in Honor of Claude V. Palisca, edited by Kovaleff Baker, Nancy and Russano Hanning, Barbara, 53-78. Stuyvesant, NY: Pendragon Press. "Because Eriugena’s writings include discussions of the liberal arts and music, they have attracted the interest of music historians. And because Eriugena uses certain terms that are found in the Enchiriadis treatises, scholars for the past 150 years have argued for, or at least accepted the likelihood of, links between Eriugena and the treatises. Indeed, no less eminent a scholar than Jacques Handschin was certain that Eriugena knew Musica enchiriadis. (8) Furthermore, none of today’s leading authorities on Carolingian theory — even when a proposed point of contact has been disputed — has argued against such links altogether. (9) It is the intention here, however, to do just that: to assert that there is no likely connection between Eriugena and the Enchiriadis treatises and to demonstrate that the concepts and vocabulary of the Enchiriadis treatises that have been attributed to Eriugena (and indirectly to Greek Neoplatonism) are all explainable in terms of the earlier Latin tradition, and Boethius in particular, to that end, we will undertake a brief review of previous scholarship on this question and a more detailed critique of new, as yet unevaluated claims for Eriugena’s influence." (pp. 56-57)

9. (8) "Die Musikanschauung des Johannes Scotus (Eriigena)," Deutsche Vierteljahresschrift für Literaturwissenschaft und Geistesgeschichte V (1927) 339.

10. Evans, Gillian R. 1978. "Introductions to Boethius's "Arithmetica" of the Tenth to the Fourteenth Century." History of Science no. 16:22-41. "The much-neglected introductions to arithmetical texts — especially Boethius’s Arithmetica — which are to be found in a number of manuscripts of the tenth to the fourteenth century, are essentially teaching-aids. Even if they were not necessarily used on every occasion in the actual presence of a master, they preserved his spoken comments for the use of the student reading alone. In whatever way they were employed, their purpose is primarily instructional; they complement the text to be studied, and they seem to have no pretensions to stand as works of literature in their own right. It is striking that the authors of many of these introductions, commentaries and glosses have often found it convenient to explain arithmetic by means of notions drawn from grammar, logic or rhetoric. Grammar and logic in particular were subjects undergoing active development during these centuries." (p. 22)

11. ———. 1981. "Boethius' Geometry and the four Ways." Centaurus no. 25:161-165. "It is evident on the most cursory reading that neither of the Geometries which have survived under Boethius’ name are, as they stand, authentic works. They contain material which places them, at the earliest, in the tenth century; they are disorderly and repetitious. Nevertheless, there is every reason to believe that Boethius wrote a Geometria, and it is not, on the face of it, impossible that some fragments of the original may survive, embedded in these fraudulent treatises. Some work has been done upon the Euclidean translations which are found in both versions, but little attempt has been made to answer the question: what should we expect the Geometria to have contained? Boethius himself provides some evidence on this point which has been largely overlooked." (p. 161)

12. Folkerts, Menso. 1981. "The Importance of the Pseudo-Boethian Geometria during the Middle Age." In Boethius and the Liberal Arts. A Collection of Essays, edited by Masi, Michael, 187-209. Bern: Peter Lang. "Compared to the other writings of Boethius on the Trivium and Quadrivium, his Geometria takes a special place. We do not have this work in its original form, but only in two later adaptations: both contain only part of the original but on the other hand they are enlarged through a variety of insertions. Therefore any study of the importance of Boethius’ Geometria in the Middle Ages should not only try to show the influence of geometrical writings which were transmitted under the name of Boethius, but also should try to understand the origin of such compilations. From these two demands, then, arises the organization of my essay: the first section will be concerned with the scanty evidence known about the authentic Geometria as well as with the contents and origins of both extant compilations. In the second part of my essay I will attempt to show the dissemination and impact of both writings during the Middle Ages. For this purpose I will make use of entries of the Geometria in medieval library catalogues and of
allusions to it in other medieval writings. A broader understanding of the importance of these two writings and of their comprehension in this period can be gained from an analysis of the scholia to one of these compilations which have been neglected up to now. Such a study should make it possible to indicate the value of these treatises in comparison with the other two, authentic, works of the Boethian quadrivium - the *Arithmetica* and *Musica.*"  

(10) “Albinus quoque de eisdem rebus scripsisse perhibetur, cuius ego geometricos quidem libros editos

"Conclusion. The books of the *Consolation* form a step-by-step ascent from the lower part of the soul to the higher. For Boethius, the ascent passes from sense (book 1) and imagination (book 2) to reason (books 3–4), but it ends with a glimpse of what is beyond reason (book 5). Intellect and not reason characterizes the divine life and is its mode of knowing all the lower modes in a simple way. Boethius moves the reader from the lower modes of knowing to the higher by showing the limits of each mode. At the end of each book there is an opposition or a contradiction that cannot be resolved by the mode and points to the need to adopt a higher mode of knowing. The *Consolation* not only advances from lower modes of knowing to higher, but also presents an ascent through the levels of being. When Philosophy appears to the Prisoner her height is ambiguous (Consolation 1,1,1), and she appears at one moment to “confine herself to the ordinary measure of man” (Consolation 1,1,2), while at another moment it appears that “the crown of her head touched the heavens” (Consolation 1,1,2) and at yet another she appears to have “penetrated the heavens themselves” (Consolation 1,1,2) and passed beyond the reach of human vision. The three heights of Lady Philosophy represent the terrestrial world of process, the spheres of the heavens that circumscribe the natural world, and the transcendent divinity. For Boethius, consolation requires an ascent from the lower, human perspective to the highest, divine perspective, even if this divine perspective is only intimated or adumbrated." (p. 19)  

Abstract: "The first systematic and well-developed treatise on the mathematical subject in the Roman world, the *De institutione arithmetica* should be considered as the first fruits of the intellectual activity of the young Boethius. It is from the pen of Boethius in the *De institutione arithmetica* that we find the first use of the word *quadriuium* to designate the meeting of the four mathematical sciences: arithmetic, music, geometry, and astronomy. Boethius was intrigued at least as much by the philosophical aspect of the study of arithmetic as by its strictly technical and scientific aspects in the modern sense of the term. Without a doubt, what Boethius and Nicomachus understood by arithmetic would be much better expressed today by the term arithmology. The arithmetic treatise of Boethius profoundly marked the intellectual evolution of the West during the High Middle Ages and up to the beginning of the Renaissance."  


Translated from the German *Musica naturalis,* Stuttgart: Franz Steiner, 2008 by Michael J. Curley and Steven Kendall.  

I. To understand the *De institutione musica,* first we have to see clearly the intentions of the writer. Boethius has few Latin antecedants. Both Cassiodorus and he himself mention the work of the 4th century Albinus. (9) Albinus wrote a geometry and probably a dialectics, too. (10) Cassiodorus esteemed high of Gaudentius’ work *Introductio harmonica* (11) and wrote that it was translated to Latin by his friend, Mutianus. 12 It is not clear whether Boethius could know this or not. Besides these, all the musico logical works written in Latin are a few Vitruvius-excursions, (13) the 9th book of Martianus Capella’s illustrious *De nuptiis* and some chapters of *De die natali* by Censorinus. (14) Augustinus’ *De musica* deals only with rhythmics and metrics, and we also know about Apuleius’ work on music, (15) though it is lost. So Boethius was right in thinking that there was no comprehensive work in Latin which could contain high-level theoretical matter and from which Greek music — theory could be thoroughly learnt. Especially the interpretation of the Pythagorean theory was missing, which used mathematical means, for Censorinus and Martianus Capella borrowed a lot from Aristides Quintilianus’ work, who belonged to Aristoxenus’ school. So Boethius’ intentions were clearly to supply this basic lack.” (pp. 6-7) (9) Cf. Cassiodorus, *Institutiones* 5, 10, p. 19. (Gerbert).  
(10) “Albinus quoque de eisdem rebus scripsisse perhibetur, cuius ego geometricos quidem libros editos
scio, de dialectica vero diu multumque quasitos reperire non valui." (De interpretatione ed. secunda I. PL 64, 394 A).
(11) “Habetis hie (sc. in Bibliotheca Romae) Gaudentium Mutiani Latinum, quem si sollicita intentione relegitis, huius scientiae (sc. musicae) vobis atria patefacit.” (Cass.: Inst. 5, 10.)
(13) Vitruvius, De architectura 5, 3.
(14) Censorinus, De die natali 10; 12; 13.
(15) Cf. Cassiodorus I. c. 1b


"In another essay of this collection, Professor Ubaldo Pizzani has made a study of the Boethian De Musica and how it was disseminated throughout Europe until the High Middle Ages. He has made clear that the history of the De Musica was closely tied with the spread of the De Arithmetica which seems to have been intended - or at least to have served in the medieval schools - as an introduction to the music theory. It is my intention in this chapter of the collection to extend Professor Pizzani’s survey to the late history of the De Arithmetica. We should be able to see that the history of the Boethian mathematics underwent several interesting developments, most of these quite apart from its connection with the music theory. First and most significantly it must be noted that the Boethian arithmetic did not lose its importance after the influx of Arabic mathematics and the re-introduction of Greek number theory. If we are to judge from the number of extant manuscripts which contain the De Arithmetica, by the frequency of citations in other treatises (with or without the mention of Boethius' name) and by the number of early printed editions through the 16th century (at least 25), (1) we must conclude that the Boethian mathematics enjoyed an extraordinary increase in popularity and influence between 1200 and 1600.

That Boethius’ mathematics should have become so widely used may seem surprising since the superior texts of Euclid, Nicomachus (Boethius’ source) and Archimedes were available, could be read by many scholars, and were being translated. Moreover, as all students of the period are aware, the nature of mathematics was undergoing thorough and wide reaching changes at this time. The needs of a growing merchant trade which demanded efficient bookkeeping were responded to by an increasingly more sophisticated computational mathematics couched in the recently adapted Arabic number system. Though the new mathematical technique were initially slow to grow in European soil, by the 15th century hundreds of computational works (2) were available for those who wanted to learn. The ascendancy of the Boethian mathematics in this context indicates a far more interesting aspect about the development of mathematics in the Middle Ages than that it was simply evolving into modern algebra and trigonometry.

An examination of the various texts dealing with mathematics shows that this discipline was becoming highly diversified in nature by the late Middle Ages. Until the late Middle Ages, a large portion of the mathematics studied in the schools and universitês was a carry-over of earlier number theory, unoriginal and impractical. It was a mathematics which oriented the student to philosophical study and was imbued with the terminology of logic. As a preparation for higher philosophical study, it had once served its purpose well, but it had long since ceased to grow by the 15th century. But practical and computational mathematics slowly broke with the old number theory and began a new strain of mathematics. This break occurred outside the universities and probably began very early in the Middle Ages. It was a new strain of mathematics that lived in the counting rooms of merchants and its greatest exponent was Fibonacci, the son of a trader. Certainly much of the computational mathematics was as servile to the merchant as the older number theory was to the philosopher. Some few thinkers, subtle and perceptive in their study of both Greek and Arabic numbers, such as Bradwardine, Nicholas of Oresme and Fibonacci, achieved a scope of mathematical vision not fully appreciated until recent times.

By the late Middle Ages the De Arithmetica had become moribund, and the widespread popularity of Boethius served, perhaps, to slow down the progress of mathematical innovation. The text of the De Arithmetica was inherited by the universities as a scrap from that vast learning of Greek thinkers and had become fossilized as part of the Liberal Arts curriculum, a once vital program of studies. But, for a small number of original thinkers (whose proportion among mathematicians has perhaps remained a constant even to our own times) Boethian definitions of numbers, the classification of number and ratio, and the definitions of the types of proportionals were the starting points for new understandings.

I have accordingly divided my study into two parts. Initially I will survey the evidence which makes the Boethian treatise the best known mathematical work of the Middle Ages. Without attempting to evaluate the works cited, I will review a few treatises which show the way in which Boethius was adapted. When the De Arithmetica was not used directly as a text, it appeared in shortened form (an epitome) or merely excerpted to reduce its length. In the second part of the essay I will attempt to demonstrate how Boethian ideas provided seeds for more original thought in the works of a small number of innovative mathematicians. The most important of these are Thomas Bradwardine, Albert of Saxony (whom he influenced), Roger Bacon and Nicholas of Oresme." (pp. 81-82)

(1) In the introduction to my translation of the De Arithmetica (to be published soon by Rodopi,
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"Boethius' motives in embarking upon his intellectual projects were not as grandiose as has sometimes been argued: from the evidence which remains he cannot be regarded as having conceived a 'Hellenist Renaissance' on a significant scale. He saw his role as one of enriching the Latin intellectual tradition with the transfiguration of Greek material, in the mould of many Latin intellectuals before him. He believed that he had an audience, interested in Greek culture if not skilled in the Greek language, and the opposition to such an enterprise (if indeed contemporaries recognised his intellectual activities as having the coherence this word implies) should not be overemphasised. Boethius' intention in his philosophical programme was, and this cannot be overstressed, a highly technical one, that of reconciling the philosophical schemes of Plato and Aristotle, and he himself makes no claim for it to have any greater significance than this." (p. 59)


"To anyone who reads the texts of the quadrivium, it is obvious that for Boethius proper order was very important.

(...)

Each discipline is logically prior to the following and each is dependent on the previous for its principles of procedure. The four arts of the quadrivium must be studied in the given order and they ultimately serve as preparation for a study of philosophy; the conclusion of their study is an approach to the wisdom of the Consolation of Philosophy. Indeed, without the study of the quadrivium, true wisdom is not available to the student.

(...)

"The second point in my exposition, while extending the scope of the discussion, at the same time involves it in a simple but important philosophical consideration. Basic to the nature of the quadrivium is an understanding of the purpose of the arrangement of the disciplines. It is the matter of the nature, arrangement, and purpose of the disciplines where we see the increasing schematization of the allegories of the liberal arts during the late Middle Ages." (pp. 58-59 note mitomitted)

(...)

"At this point it is important to review the conceptual binding force within the quadrivium which enables its disciplines to be extended universally to all the arts and to the diverse modes of thought beyond them. The logical principles of the disciplines in the quadrivium are drawn from number theory, which is explained in considerable detail and with some application by Boethius in the De Arithmetica."

(...)

The proportional definitions of the De Arithmetica are extended to the relationships between sounds in the De Musica, second discipline of the quadrivium.

(...)

From the De Musica, the student of the liberal arts proceeded to the De Geometria. The science of geometry applies the concepts of proportion and order to the dimensions of planes and to the shapes of figures extended into solids. To these geometrical figures are applied the rules of harmonic proportion as outlined in the De Arithmetica and realized in sounds by the De Musica.

(...)

In the study of astronomy, the principles of order and proportion, the metaphor of harmony, and the ideals of the proper arrangement of parts receive their broadest extensions. Man was thought to embark on the highest order of learning when he undertook a survey of the heavens. In the context of the liberal arts, we find this larger meaning in the arrangement of celestial spheres in Dante's Paradiso." (pp. 65-66)


"During the last three or four centuries, the name of Boethius has been linked almost exclusively with the De Consolatione Philosophiae. However, as it has been recently recognized among Boethian scholars more generally, Boethius was recognized during the Middle Ages as the author of a variety of important works on logic, music, theology and mathematics (1). Some of these works became standard texts for the curriculum of the seven liberal arts. In recent years there has been considerable research into the writings and texts of Boethius as seen in the context of medieval culture (2). These more recent studies and re-evaluations of Boethius' work have been important because they demonstrate the complexity of Boethius' influence - notably that of his mathematics. His De Arithmetica had an influence not only on
writers of mathematics but also on those who were concerned with the principles of music, literature, ethics, and architecture. In this essay I would like to examine the Boethian relationship with other medieval mathematical writing, and in so doing hopefully I might demonstrate his authoritative standing among writers of such texts. But, as well, in order to show the full scope of his work I would like to touch briefly on works outside the field of mathematics, although it will be possible to do so in only the most sketchy manner."

(1) See, for example, the entry under "Boezeio", Dizionario Biografico degli Italiani, vol. 11 (1969), 142-165.


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"In another essay of this collection, Professor Ubaldo Pizzani has made a study of the Boethian De Musica and how it was disseminated throughout Europe until the High Middle Ages. He has made clear that the history of the De Musica was closely tied with the spread of the De Arithmetica which seems to have been intended - or at least to have served in the medieval schools - as an introduction to the music theory. It is my intention in this chapter of the collection to extend Professor Pizzani's survey to the late history of the De Arithmetica. We should be able to see that the history of the Boethian mathematics underwent several interesting developments, most of these quite apart from its connection with the music theory. First and most significantly it must be noted that the Boethian arithmetic did not lose its importance after the influx of Arabic mathematics and the re-introduction of Greek number theory. If we are to judge from the number of extant manuscripts which contain the De Arithmetica, by the frequency of citations in other treatises (with or without the mention of Boethius' name) and by the number of early printed editions through the 16th century (at least 25), (1) we must conclude that the Boethian mathematics enjoyed an extraordinary increase in popularity and influence between 1200 and 1600.

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become fossilized as part of the Liberal Arts curriculum, a once vital program of studies. But, for a small number of original thinkers (whose proportion among mathematicians has perhaps remained a constant even to our own times) Boethian definitions of numbers, the classification of number and ratio, and the definitions of the types of proportionalities were the starting points for new understandings.

I have accordingly divided my study into two parts. Initially I will survey the evidence which makes the Boethian treatise the best known mathematical work of the Middle Ages. Without attempting to evaluate the works cited, I will review a few treatises which show the way in which Boethius was adapted. When the De Arithmetica was not used directly as a text, it appeared in shortened form (an epitome) or merely excerpted to reduce its length. In the second part of the essay I will attempt to demonstrate how Boethian ideas provided seeds for more original thought in the works of a small number of innovative mathematicians. The most important of these are Thomas Bradwardine, Albert of Saxony (whom he influenced), Roger Bacon and Nicholas of Oresme."

(1) In the introduction to my translation of the De Arithmetica (to be published soon by Rodopi, Amsterdam [1983]), I have listed almost 200 manuscripts of the De Arithmetica extant. A list of printed editions may be found in David Eugene Smith, Rara Arithmetica (Boston: Ginn & Co., 1908), p. 27.

(2) Descriptions of many such works may be found in Smith’s Rara Arithmetica.


"Cassiodorus in a letter he wrote to Boethius on behalf of Theoderic in about 507, (1) attributes to his young correspondent translations of Pythagoras on music, of Ptolemy on astronomy (for which see below), of Nicomachus on arithmetic, and of Euclid on geometry; later on, in the Institutiones (2), he refers again to Boethius' translation of Euclid." (p. 155)

(...) "It has been argued that only Boethius could and did translate Euclid from the Greek before the twelfth century. However, this cannot be stated with certainty, for it is clear from the Praeceptum Canonis Ptolemei discussed below that reasonably competent Latin translations of Greek works on the exact sciences were still being made in the sixth century, after Boethius’ death. There survive, in fact, two fragments of quite different translations of Euclid that cannot be demonstrated to have anything to do with either Boethius or with M, though one is connected with Luceuil, the other with Corbie." (p. 156)

(...) "Cassiodorus in his letter to Boethius, as we have seen, attributes to the latter a translation of Ptolemy on astronomy. In the Institutiones (29) he gives a description of the contents of Ptolemy’s Canones which corresponds quite well with the Praeceptum Canonis Ptolemei which survives in some eight manuscripts copied between the late tenth and early thirteenth centuries. (30) Naturally, these references by Cassiodorus raise the possibility that the Praeceptum is the translation made by Boethius. This, however, cannot be the case. For the Praeceptum, which comprises two reasonably competent translations of instructions for the use of the Ptolemaic Handy Tables, mentions the year 535 as current. (31) It was, perhaps, intended for use in the Christian school that Cassiodorus and Pope Agapetus hoped to establish in Rome in just that year or the next. (32) Its existence proves that Boethius was not the only translator of Greek texts belonging to the quadrivium in the late fifth and early sixth centuries — a conclusion toward which the Verona fragments of Euclid seem also to point. Thereby the argument that M most probably represents Boethius’ translation because we know of no other translator (33) loses some of its force, though Boethius certainly remains the chief suspect." (p. 159)


(3) An edition is being prepared by N. Swerdlow and the present writer.

(31) Praeceptum II. 1: a Diocletiano usque nunc anni sunt CCLI; cf. II. 11. In Praeceptum I. 23 is mentioned Augustus 383=ad 354.

(32) Inst. I praef. 1, ed. cit. (note 2 above), p. 3. Agapetus was pope from 13 May 535 till 22 April 536.


"Among those writings of Boethius which transmitted to the Middle Ages so much of the ancient culture, the De Institutione Musica must certainly occupy a place of special prominence. The second treatise of the quadrivium, according to the outline apparently sketched by Boethius himself in the first chapter of the De Institutione Arithmetica (1) its fame is attested not only by the great number of manuscripts in which it has reached us, (2) but also (and above all) by the imposing mass of scholia - largely unpublished (3) -preserved in those manuscripts, as well as by the flourishing medieval musicological production that found inspiration in Boethius as the most learned and reliable authority on the ancient musical culture. (4)

It is mainly to this last aspect that scholars as a rule have directed their attention, (5) owing partly to the fact that Boethius, with his approximations and misunderstandings, did not always have a positive and stimulating effect on the musical theories of the Middle Ages. (6) But there is another factor, no less important, in the continuous good fortune of this treatise: its influence, together with that of the De Institutione Arithmetica, and the lost treatises De Institutione Geometrica and De Institutione Astronomica, (7) on the didactic applications of the quadrivium. Here the medieval schools did not have in Boethius their exclusive and unchallenged master, but turned, as well, to the great syntheses of such teachers as Martianus Capella, Cassiodorus, and Isidor of Seville, not to mention the De Musica of St. Augustine. Nevertheless, there is no doubt that the Boethian corpus, with its greater amplitude, and with the prestige conferred upon its author by his logical and theological works, played a decisive and preeminent role. This is true especially in the Carolingian (8) and post-Carolingian periods, whence derive most of the codices bearing the treatises in question. (9) It is more difficult to determine the role they played in the nearly three centuries intervening between their composition and the Renaissance prompted by Charlemagne.

The fact is that the influence of Boethius on the authors of that period - particularly Cassiodorus and Isidor of Seville, who both dealt with the ars of the quadrivium - is not conclusively demonstrable, particularly as far as music is concerned. An exceptional case is that of the Venerable Bede, under whose name we have a Musica Theorica that is definitely linked to Boethius, but which, as I have demonstrated elsewhere (10) and will illustrate later in this essay, came into being under very particular circumstances that do not allow us to attribute it to the Monk of Jarrow sic et simpliciter. An investigation into this matter, culminating, as we shall see, in a proposal for a new form of edition for that unique work, will give us an opportunity to survey the mass of scholia relating to the De Institutione Musica (for, indeed, the disiecta membra of the so-called Musica Theorica are nothing more than scholia); to examine some of the complex problems attending the preparation of such an edition and the identification of sources; and lastly, to ascertain the potential role of these scholia in clarifying in what forms and ways the Boethian text was used in medieval scholastic activity. At that point, with a brief survey of the coeval musicological production, we shall bring our investigation to a close. Our purpose is not so much to obtain definitive solutions to individual problems as to define their terms and to establish the areas of research and the methodological criteria while trying to trace an outline - however tentative and problematical - of the fortune of the De Institutione Musica from Boethius to the scholastic activity of Gerbert d'Aurillac. In the absence of a thorough documentation, it is the history up to the 9th century, at least, that will prove hard to grasp in even its most essential stages. But the problems that it poses, whether in philology, strictly speaking, or in the history of culture, cannot be evaded by anyone who wishes to understand through what adventures the De Institutione Musica passed so decisively into the scholastic and musicological traditions of the Middle Ages - after a period, as it would appear, of total eclipse." (pp. 97-98)

Note: the editor is grateful to Mr. Peter Gimpel who translated this essay from the Italian, working in close collaboration with the author.

(1) De Institutione Arithmetica I, 1. All citations from the Boethian treatises on arithmetic and music are taken from the edition of G. Friedlein (Leipzig, 1867; reprinted, 1966) and cited hereafter as DIA and DIM.

(2) A nearly complete list has been furnished recently by M. Masi, "Manuscripts Containing the De Musica of Boethius," Manuscripta, 15 (1971), 89-95 and "A Newberry Diagram of the Liberal Arts," Gesta, XI/2 (1973), 56.

(3) As far as I know, the only scholia to have yet been published are two by Gerbert d'Aurillac (N. Bubnov, Gerberti Opera Mathematica [Berolini, 1899] pp. 28-31), one on the harmony of the spheres (R. Bragard, "L'harmonie des sphéres selon Boëce," Speculum, 4 [1929], pp. 206-13, and some others in my article, "Uno pseudo-trattato dello pseudo-Beda," (Maia, I [1957], 36-48). It should be noted, however, that a substantial mass of scholia has been packed into this so-called treatise attributed to the Venerable Bede, as we shall see in the course of this study.

(4) The texts are assembled in good part in the fundamental though somewhat obsolete collections of M. Gerbert, Scriptores Ecclesiastici de Musica Sacra Potissimum (Typis San Blasianis, 1784; reprinted, 1931) and E. Coussemaker, Scriptores de Musica Mediæ Aevi, 4 vols. (Paris, 1864-76). More up-to-date
on the critical and textual level is the *Corpus Scriptorum de Musica*, but the latter is still far from completion.

(5) See the ample bibliography after the article on Boethius by R. Wagner in the great German encyclopedia, *Die Musik in Geschichte und Gegenwart*, Band II (Kassel und Basel, 1952), col. 49-57.

(6) See for the bibliographical references the above-cited article by R. Wagner, coll. 54-55; see also my observations in "Studi sulle fonti del De Institutione Musica di Boezio," *Sacris Erudiri*, 16 (1965), 87, 128 ff.

(7) That Boethius completed the entire cycle of the quadrivium is deducted from incontestable evidence. (See the article, "Severino Boezio" in the *Dizionario Biografico degli Italiani*, 11 [Roma, 1969], 142-65.)

(8) Our most ancient voice, in that sense, is perhaps that of Aurélien de Moutier-St. Jean (Aurelianus Reomensis), who lived in the first half of the 9th century and was the author of a *Musica Disciplina*, largely inspired by Boethius, whom he cites and exalts as *vir doctissimus* (p. 41, Gerbert) and *eruditissimus* (Gerbert, p. 32).

(9) None of the extant manuscripts of the *De Institutione Musica* dates from before the 9th century.

(10) U. Pizzani, "Uno pseudo-trattato."


31. Schrade, Leo. 1947. "Music in the Philosophy of Boethius." *The Musical Quarterly* no. 33:188-200. "In the doctrine of music that Boethius formulated in his youth two elements, both of ethical nature, converge, and in this conjunction the ethical value of music surpasses that of any other discipline in the Quadrivium. For music as the art of sound exerts in all events and by its very nature an influence upon the moral state of man, or, in the words of Boethius himself, music is capable of "improving or degrading the morals of men". In addition to this, however, music as part of mathematics shares in those educational ethics that are inherent in the disciplines of the Quadrivium. It contributes to the training of the intellect, which in the end must be totally free from all bodily impediments. This is the meaning of the education in which music assists in liberating the human mind. The music Boethius described at the beginning of his literary activity is of Platonic-Pythagorean origin. It has no direct contact with the Aristotelian system of philosophy. Music stands before philosophy; and the student of music is driven by the ethical impulse to learn how to benefit intellectually from the instrument that holds the key to the "purer reason of the mind" in philosophy. With the assumption of a pre-philosophic position of music, with the thesis of its ethical function in the process of education, and finally with the denial that music as a "science" could be part of philosophy proper, Boethius gives evidence that he wrote his works on the Quadrivium essentially as a Platonist. In it he had no intention — and no need— of reconciling the Aristotelian and Platonic schools of thought with each other.

(...) It may be puzzling to find that an incomplete book on music, written by a man of only twenty, exercised the most extraordinary influence upon centuries to come. It is less puzzling when we take into account both the ethical function and the position music was given in relation to philosophy. And it seems to be very characteristic of this school of thought that many a later philosopher actually did what Boethius had done, that is to say, started any work in philosophy with a treatise on music as a primary necessity, without ever returning to music again. This procedure is by no means accidental; it bears all the marks of the situation in which Boethius’ work on music originated. The significance of this fact has been completely overlooked. Let us think of Augustine, to name only one author of the Latin world of humanities. The first work Augustine wrote is his treatise on music which, however, he did not complete in his youth. Though in later years he returned to the subject of music — in his commentaries on the Psalms — he did so merely for reasons of a religious nature which had nothing to do with the Musica as a discipline of the Quadrivium. When investigating the work of philosophers through the centuries we are surprised how often we find music opening the course of philosophical studies. Even Descartes, in 1618, still begins with an *Essay on Algebra* and the *Compendium of Music*. The theorists of music proper, also, in antiquity, the Middle Ages, through the 16th century (e.g., Glareanus) often first presented an “introduction” to music. But the reason for this would require a special discussion." (pp. 193-194)


"The nature of musical meaning, considered through its physicality, is also often clouded by frameworks built upon aesthetic categories and principles. These provide a lens through which the arts in general are viewed. Such broad-sweep approaches tend towards a certain homogenization across their range and thereby result in distortion within accounts of individual arts. The uniqueness of the arts is not fully attended to. The specific category that will provide the focus for the task at hand is beauty. Beauty features within current aesthetic and theological discussion (standing more peripherally within the domain of philosophical aesthetics). Historically, however, it has been a dominant concept, acting as a cipher for underlying presuppositions that themselves comprise broader frameworks, both philosophical and theological.

It is on this basis that the two central figures of our narrative present themselves: Anicius Manlius Severinus Boethius (c.480-c.525) and Immanuel Kant (1724-1804).” (p. XVII)

(3) Initially, then, we will start with Boethius for whom the world is knowable and for whom, as a result, the physical world is of value. Here, within an integrally theological framework, beauty is understood as harmony and as such is constitutive of the world: it is the principle by which the world coheres as a whole and a property of the material world. I will show how granting beauty this objectivity allows it a cosmic meaning or “resonance” which extends both between and beyond subjects. I will then demonstrate how the Boethian account of music illustrates his understanding of the material world and beauty. Considering beauty as the principle of harmony grants music significance in relation to both the intellectual and the material for, as physical sensation, music offers knowledge of the world. However, I shall show that ultimately Boethius stresses the intellectual to the detriment of the material, using the physical experience of music as merely a stepping-stone to intellectual perception through form (with form finding its ideal location in God). By virtue of the satisfaction and pleasure imparted by music’s physicality, Boethius’ attention is re-invigorated and he is encouraged to re-focus on the world and, specifically, himself as part of the world. Ultimately, however, musical indeterminacy gives way to and is surpassed by the conceptual truths of reason.” (p. XVI)


"At the close of the Consolatio, God is looking down upon the cosmos while the newly liberated and restored Boethius, with the recovery of mens at the end of his philosophical return, is looking up as far as possible toward the Divine Light, who judges His well-ordered world and perceives everything though He Himself is beyond understanding. The whole situation is now the diametrical opposite of what it had been when Boethius, at the beginning, was indulging his grief with his elegiac Muses. The preceding example is interwoven with the important judicial metaphor in the Consolatio. In passing judgment on God, Boethius had been influenced at the beginning by his personal experience of suffering shameful injustice at the hands if men, experience of which the autobiographical material in Book 1 provides ample evidence. By the end, however, Boethius has ascended beyond the limits of reason to accept on faith the goodness of the Divine Judge whom, even in his affliction, he had continued to recognize as Creator and Ruler (I. P.6. 4).

It brings about the changes so dramatically portrayed in these opening and closing scenes, the Liberal Arts make a contribution which it is the main purpose of this paper to examine." (p. 19)


"The quadrivium’ designated the mathematical arts of arithmetic, music, geometry and astronomy. (2) Boethius appears to have been the first writer to use the term to delineate these areas of knowledge existing in the timeless world of being. For these arts were seen not merely as isolated sources of skills and information, but as an indispensable path to abstract knowledge. The created world, Boethius says, appears to have been formed according to number, ‘for this was the principal design in the mind of the Creator’. (3) So the arts which consider number in its pure or solid form, static or mobile, provide paths through the creation to its source in the incorporeal world wherein lies true wisdom. This was the direction of Christian contemplation as much as Neoplatonic. It found a classic source in the verse ‘Thou hast ordered all things in number, measure and weight’ (Wisd. 11:21). Knowledge of the rational, numerical structure of the universe would lead to knowledge of the divine nature and apprehension of God himself. (4)

Although Boethius set his mathematical works in the framework of Neoplatonic and Christian concepts of true wisdom, his immediate aim was severely practical: to make available to the Latin world the treasures of Greek learning. His De Arithmetica is chiefly a translation of a treatise by Nicomachus of Gerasa (2nd cent a.d.); his music draws on several Greek sources. (5) It seems likely too that Boethius made a translation of part of Euclid's Elements which was incorporated into medieval works bearing Boethius’ name. (6) But the complex history of its transmission is largely independent of that of the De Arithmetica and De Musica, and it cannot be considered a definable, independent Boethian quadrivial text. It therefore lies outside the scope of this study. There is no surviving work by Boethius on astronomy; some scholars have pointed to possible traces of such a treatise, but the evidence is at best
Boethius' approach to arithmetic and music was speculative and mathematical. 'Arithmetica' was the science of number, not calculation; 'musica' was harmonic theory based on Pythagorean mathematics of proportion, not music-making. This fact was firmly pointed out in the eleventh century by Guido of Arezzo, who declared that Boethius' De Musica was useful not to musicians, but only to philosophers. But this is one of the cardinal reasons for the durability of his treatises: the science of number and proportion could serve as the foundation of diverse practical applications as long as the fundamental assumptions on which they were based were not challenged. Throughout the Middle Ages, Boethius' De Arithmetica and De Musica remained standard texts for these two arts of the quadrivium, sometimes supplemented by other medieval works, but never ousted from the canon.  


(5) H. Potiron, Boèce, théoricien de la musique grecque (Paris, 1961) ; J. Caldwell, pp. 135-54 above [The De Institutione Arithmetica and the De Institutione Musica].


(7) W. H. Stahl (op. cit., note 2 above, p. 173 and n. 6) and B. L. Ullman (art. cit. note 6 above, 278) suggested that it might be found in Naples, Bibi. Naz., V. A. 13 (s. x), but the Naples catalogue (Naples, 1827), p. 258, and N. Bubnov, Gerberti Opera Mathematica (Berlin, 1899), pp. 478-9, ascribe the material to Isidore, Cassiodorus and Bede. The Bobbio catalogue of the tenth century includes 'Libros Boetii III de aritmetica' (nos 384-6) 'et alterum de astronomia' (no. 387) ; the twelfth-century catalogue of St Bertin mentions 'Boetius de geometria et astronomia'. See G. Becker, Catalogi Bibliothecarum Antiqui (Bonn, 1885), pp. 69 and 182. But the 'astronomiae' are not explicitly identified as Boethius' works.
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