
"Recent discussions on the topic of « mathematical explanation » have focused on the distinction between explanatory and non-explanatory proofs. The former proofs are supposed to differ from the latter in that they not only establish that a result is true but also show why it is true. This opposition is at the core of the philosophies of mathematics of Bolzano and Cournot. The paper analyzes Bolzano's theory of Grund und Folge, and Cournot's opposition between the logical and the rational order, emphasizing their relevance to the issue of mathematical explanation. The final part of the paper investigates the shortcomings of Bolzano's and Cournot's theories as explications of mathematical explanation."


"Bolzano's attempt to refute so-called "radical" or "complete" skepticism is carefully described in Professor Berg's introduction to his edition of the Wissenschaftslehre (WL). Two forms of such skepticism are there distinguished. The thesis of the ontological form is

(1) No propositions (Sätze an sich) is true
and that of the epistemological form is
(2) No judgment (Urteil) is true.

Bolzano's principle arguments against these are roughly as follows. Against (1) he argues that, for any proposition S, either S is true or the proposition that S is false is true. Therefore, at least one proposition is true. The argument against (2) is less clear. Bolzano (WL 40) takes the problem to be that of convincing a radical skeptic that, after all, he must recognize the truth of at least one proposition. After considering various possibilities, he concludes that the skeptic will have to accept as true at least the proposition that he has ideas (Vorstellungen), for obviously he confirms this proposition the moment he doubts or denies it. The point, I suppose, is that, just as one cannot doubt that there are men on the moon if one has no idea of what it is to be a man or to be on the moon, so the skeptic, if he has no ideas, is in no position to doubt anything, not even that he has ideas. Bolzano thinks that while the skeptic might refuse publicly to admit the proposition in question, "nevertheless he will surely feel in his innards that it is true... and if he feels this, we have won".

Whatever one may think of these arguments, in this paper I am not concerned to evaluate them but only to consider whether they refute Pyrrhonism, as Bolzano seems to suppose." (pp. 121-122)

(...) The root of Bolzano's failure to appreciate the force of Pyrrhonism is, in my opinion, that he does not realize that its self-referential aspect is essential. This aspect is not something that Sextus is reluctant to admit but is rather a feature that he emphasizes over and over again and that he obviously regards as crucial to the consistency of the skeptic's position. Bolzano's failure to understand this is especially evident at WL 40, where he quotes and discusses one of the many passages in which Sextus points out the self-reference of the skeptic's slogans (phonai), i.e., pronouncements like "contrary claims are equal", "no more this than that", "I decide nothing", etc. Bolzano says:

In setting forth the various formulae with which the skeptic is accustomed to express his state of doubt, Sextus Empiricus tries to employ maximal caution so as to protect it from the charge of self-contradiction, but nevertheless he finds himself compelled at the end to admit
"As concerns all the skeptic slogans the following must be understood in advance, namely that we do not maintain their truth in any absolute way, since we say that they themselves are included among the things to which they apply -- just as cathartic drugs do not merely eliminate humor from the body but also expel themselves along with the humors" (Outlines of Pyrrhonism I 206).

"This amounts to the reluctant admission", says Bolzano, that the skeptic ceases to be a skeptic as soon as he declares himself to be a skeptic. Only if he keeps silent and makes no judgment, not only in words but also internally, is he a complete doubter; and as long as this condition exists we others can say of him truly that he doesn't know a single truth. But as soon as he himself says it, the condition ceases and his judgment is therefore false.

But there is no "reluctant admission" here, and the Pyrrhonist doesn't have to be silent if he is to remain a Pyrrhonist. He will say "It seems to me now that contrary claims are equal" and "It seems to me now that there is no more reason for this than that", and so on. What he refrains from are flat out categorical statements, whether concerning his own skepticism or anything else.

It will be evident that this form of skepticism is not easily refuted. Since the Pyrrhonist agrees only to propositions expressing what seems to him at the moment to be the case, it is even unclear what a refutation would be like. But that is a topic for another day". (pp. 138-139).


"The question "Was 'existence' ever a predicate?" in a way already suggests its own answer, that this is really the wrong question to ask, because 'existence' has always been a predicate. Even those, such as Kant, who supposedly opposed this view, in fact held it. They merely denied that 'existence' is a "normal" first-order predicate. Not only Kant, but also Bolzano, Frege and Russell claimed that it is a second-order predicate. There is substantive disagreement between Kant and Bolzano on the one hand and Frege and Russell on the other over two issues: the former claim that this second-order predicate applies to no concept analytically and that it can be properly ascribed to a singular concept, whereas the latter deny both of these claims."


"Bernard Bolzano was one of the first philosophers in modern times to develop explicitly a complete theory for entities like propositions, statements and states of affairs. I will first describe and clarify the main features of his theory, and then sketch the subsequent development to our day." p. 243

"Postscript. Professor Nuchelmans' important work on propositions covers the long history from ancient and medieval period (Theories of the proposition. Ancient and medieval conceptions of the bearers of truth and falsity, 1973) through late-scholastic and humanist times (Late-scholastic and humanist theories of the proposition, 1980) to modern time from Descartes to Kant (Judgment and proposition from Descartes to Kant, 1983). My own work starts where Professor Nuchelmans' work published to date ends, i.e., after Kant. I am pleased and honoured to have my paper included in the Festschrift for Professor Nuchelmans, and I dedicate it to him with great respect." (pp. 256-257).


"Bernard Bolzano's most fruitful invention was his method of variation. He used it in defining such fundamental logical concepts as logical consequence, analyticity and probability. The following three puzzles concerning this method of variation seem particularly worth considering. (i) How can we define the range of variation of an idea or the categorial conformity of two ideas without already using the concept of variation? This question was raised by Mark Siebel in his M.A. thesis. (ii) Why must we define analyticity by means of (simultaneous or successive) variation of several ideas rather than by
means of replacing a single idea? This problem is suggested by an example due to W.V.O. Quine, John R. Myhill and Benson Mates. (iii) Must every 'there is...' sentence be synthetic for Bolzano, as his pupil Franz Prihonsky claims in his booklet Neuer Anti-Kant, or can a 'there is...' sentence be logically analytic?"
Gregory of Rimini's ideas on Bolzano's philosophy of logic. Bolzano seems to have only a limited acquaintance with the logic of the late medieval period: the credit accorded to Savonarola's Compendium logicae - a standard work which is absolutely lacking in originality - corroborates, I think, this view. Yet Bolzano may have benefited by late scholastic inheritance through the intermediation of later works, like those of Campanella, Clauberg, Fonseca, Keckermann, Leibniz and Wolf.' In fact, as already mentioned, Leibniz is the first author whom Bolzano explicitly refers to, in paragraph 21 of the Wissenschaftslehre, as a forerunner of the Satz an sich theory:

"Thus Leibniz uses as equivalent the expressions proposition and cogitatio possibilis (Dia

The Leibniz's work on which Bolzano explicitly bases this conviction is the Dialogus de connexione inter res et verba, first published by Raspe in 1765 -- a work whose content paradoxically seems to partly disprove Bolzano's interpretation.' Thus Church considers it «an exaggeration or a misunderstanding" on Bolzano's part to have attributed to Leibniz's Dialogus "the use of the word propositio for proposition in the abstract sense" or Satz an sich.(6) The same remarks are repeated by Prof. Berg in his monograph on Bolzano's logic: after having identified Bolzano's Satz an sich with Frege's Gedanke, Prof. Berg writes:

According to Leibniz a proposition (propositio) is a possible thought (cogitatio possibilis), which is capable of being true or false... But no thought or reasoning is possible without words or some other kind of signs. And under transformation of a proposition into a different language a certain relationship (proportio) among the signs and between the signs and the objective reality is transformed into a similar relationship. The last two conditions fit Aristotle's and Peter of Spain's but not Frege's notion of proposition. Therefore... it must have been a misunderstanding on Bolzano's part to have attributed to Leibniz the use of the world "propositio" for Satz an sich.(7)

In what follows, I intend to take up the problem of the correctness of the interpretation given by Bolzano and then to develop a comparison between the positions of Leibniz and those of Bolzano relative to the notions of idea, proposition and truth.


(1) A. Church, Propositions and Sentences, in I. M. Bochenski, A. Church, N. Goodman, The Problem of the Universals, Notre Dame, Notre Dame Press, 1956, p. 3.
(2) WL 1, 11/1, pp. 105 ff.
(3) WL 1, 11/1, pp. 234 ff.
(4) WL 1, 11/1, p. 111.
(5) Cfr. œuvres philosophiques latines et francaises de feu Mr. de Leibniz ... publiees par Mr. Rud. Eric Raspe, Leipzig, 1765, pp. 505-512.
(6) A. CHURCH, O. Cit., p. 10.
(*) [cited in German in the original; I cite from the translation of Wissenschaftslehre by Rolf George, p. 24]


think the result of this little experiment is negative, in that it does little toward settling disputed questions about the interpretation of Kant. On the other hand, I think it brings out some problems of Kant's views that could be seen either at the time he wrote or not long after." (p. 80)

(*) [Beyträäge zu einer begründeteren Darstellung der Mathematik = Contributions to a Better Grounded Presentation of Mathematics]


"This article offers a new interpretation of what Bolzano had in mind with the concept of 'analytic proposition'. In Bolzano's terms, an analytic proposition is a proposition in which there is at least one constituent that can be arbitrarily changed without altering the truth value of the original proposition. The author shows that a proper understanding of this criterion cannot be reached if one ignores the text in which a full account of the extensional properties of the variable constituent is provided by Bolzano. The completed criterion fits more sharply the Bolzanian epistemology, and is free from the inconsistencies inherent to the so-called 'Quinean' interpretation of Bolzano's analytic."


See the Third Chapter: Bolzano's Renovation of Analyticity, pp. 49-108.


"Bolzano's theory of representation is one of the most radically intensionalist approaches to representation. It is based on the following three claims A. A representation is essentially independent of thought and of linguistic expression; B. A representation is structured; C. Such a structure is independent of the objects represented. These claims are both tools and constraints relative to Bolzano's substantive goals. Bolzano ultimately aimed to carry out a deep transformation of mathematical and scientific practice, thanks to a more accurate conception of logic and of the role of logic in scientific exposition. I examine some of the consequences of Bolzano's claims in regard to his conception of mathematical treatises."


"Until Bolzano nearly all philosophers believed that truth and falsity are predicated of judgments of beliefs. Bolzano and other philosophers after him argue that propositions are the bearers of truth and falsity and that propositions have a timeless ideal existence: a position which seems to discredit completely their view that propositions are the bearers of truth and falsity. Yet, several arguments can be offered which show that propositions are the bearers of truth and falsity without introducing as a premise the timeless existence of propositions."


"In this paper I discuss Kant's and Bolzano's differing perspectives on ordinary natural language. I argue that Kant does not see ordinary language as providing semantically organized content and that, as a result, Kant does not believe that ordinary language is sufficiently well-developed to support philosophical analysis and definition. By contrast, for Bolzano, the content given in ordinary language are richly structured entities he calls 'propositions in themselves'. This contrast in views is used to explain Bolzano's criticism of Kant's belief that definition is impossible for philosophical concepts. It is also used to explain Bolzano's criticism of Kant's methods of exposition of philosophical concepts."


Edited by Jan Wolenski.


"According to Kant, a true judgement can be called a priori in case it can take place absolutely (schlechterdings) independent of experience. Propositions that are knowable in this way are called a priori propositions by him (Kant, Critique of Pure Reason, 1787 B, 3–4). As is well known, the class of those a priori propositions that are synthetic was particularly important for Kant. In contrast to analytic propositions, they are supposed to contain nontrivial information about the world and yet be irrefutable by experience. Not many of his critics were satisfied with Kant’s way of drawing this distinction. Peter Strawson, for example, writes in his commentary on the Critique of Pure Reason."


"In this paper, I provide a thorough discussion and reconstruction of Bernard Bolzano’s theory of grounding and a detailed investigation into the parallels between his concept of grounding and current notions of normal proofs. Grounding (Abfolge) is an objective ground-consequence relation among true propositions that is explanatory in nature. The grounding relation plays a crucial role in Bolzano’s proof-theory, and it is essential for his views on the ideal buildup of scientific theories. Occasionally, similarities have been pointed out between Bolzano’s ideas on grounding and cut-free proofs in Gentzen’s sequent calculus. My thesis is, however, that they bear an even stronger resemblance to the normal natural deduction proofs employed in proof-theoretic semantics in the tradition of Dummett and Prawitz."


"Russell, in his History of Western Philosophy, wrote that modern analytical philosophy had its origins in the construction of modern functional analysis by Weierstrass and others. As it turns out, Bolzano, in the first four decades of the nineteenth century, had already made important contributions 'to the creation of "Weierstrassian" analysis, some of which were well known to Weierstrass and his circle. In addition, his mathematical research was guided by a methodology which articulated many of the central principles of modern philosophical analysis. That Russell was able to discover philosophical content within mathematical analysis was thus not surprising, for it had been carefully put there in the first place. Bolzano can and should, accordingly, be viewed as a founder of modern analytical philosophy, and not necessarily as an uninfluential one. This paper considers his work in mathematical and philosophical analysis against some of the relevant historical background."


"Bolzano's philosophy of mathematics is presented through a consideration of his critical responses to Kant and Lagrange."


"In the works of Kant and his followers, the notion of form plays an important role in explaining the apriority, necessity and certainty of logic. Bernard Bolzano (1781–1848), an important early critic of
Kant, found the Kantians' definitions of form imprecise and their explanations of the special status of logic deeply unsatisfying. Proposing his own conception of form, Bolzano developed radically different views on logic, truth in virtue of form, and other matters. This essay presents Bolzano's views in the light of his criticisms of the Kantian logicians.


"This essay presents a new interpretation of Bolzano's account of necessary truth as set out in §182 of the Theory of Science. According to this interpretation, Bolzano's conception is closely related to that of Leibniz, with some important differences. In the first place, Bolzano's conception of necessary truth embraces not only what Leibniz called metaphysical or brute necessities but also moral necessities (truths grounded in God's choice of the best among all metaphysical possibilities). Second, in marked contrast to Leibniz, Bolzano maintains that there is still plenty of room for contingency even on this broader conception of necessity."


"Alongside his groundbreaking work in logic, Bernard Bolzano (1781–1848) made important contributions to ontology, notably with his theory of collections. Recent work has done much to elucidate Bolzano's conceptions, but his notion of a sum has proved stubbornly resistant to complete understanding. This paper offers a new interpretation of Bolzano's concept of a sum. I argue that, although Bolzano's presentation is defective, his conception is unexceptionable, and has important applications, notably in his work on the foundations of arithmetic."


"The history of speculation on a notion or notions called analyticity, now usually characterized as truth in virtue of meanings and independently of fact, is often viewed from the perspective of the Quine-Carnap dispute. Previous characterizations, due to Kant, Frege and others, are then seen as being of a piece with Carnap's various definitions of analyticity, and thus open to Quine's objections. Seen from this point of view, Bolzano's claims about analyticity appear downright bizarre: for on his conception, analyticity is not only non-linguistic, but also independent of both apriority and necessity. In this paper, it is argued that the problem lies not with Bolzano, but rather with the received historical account, especially its interpretation of Kant."


"In a series of publications beginning in the 1980s, John Etchemendy has argued that the standard semantical account of logical consequence, due in its essentials to Alfred Tarski, is fundamentally mistaken. He argues that, while Tarski's definition requires us to classify the terms of a language as logical or non-logical, no such division is guaranteed to deliver the correct extension of our pre-theoretical or intuitive consequence relation. In addition, and perhaps more importantly, Tarski's account is claimed to be incapable of explaining an essential modal/epistemological feature of consequence, namely, its necessity and apriority. Bernard Bolzano (1781-1848) is widely recognized as having anticipated Tarski's definition in his Wissenschaftslehre (or Theory of Science) of 1837. Because of the similarities between his account and Tarski's, Etchemendy's arguments have also been extended to cover Bolzano. The purpose of this article is to consider Bolzano's theory in the light of these criticisms. We argue that, due to important differences between Bolzano's and Tarski's theories, Etchemendy's objections do not apply immediately to Bolzano's account of consequence. Moreover, Bolzano's writings contain the elements of a detailed philosophical response to Etchemendy."


"Bernard Bolzano (1781-1848) stands out with Frege as one of the great logicians of the nineteenth century. His approach to logic, set out in the Theory of Science [WL] of 1837, marks a fundamental reorientation of the subject on many fronts, one which is as radical as any in the history of the field. In
sharp contrast to many of his contemporaries, Bolzano insisted upon a rigorous separation of logic from psychology. It should be possible, he thought, to characterize propositions, ideas, inferences, and the axiomatic organization of sciences without reference to a thinking subject. Consistently pursuing this approach to logic and methodology, Bolzano developed important accounts of formal semantics and formal axiomatics.

A talented mathematician, Bolzano developed his logic in conjunction with his mathematical research. Among the first to work on the foundations of mathematics in the modern sense of the term, he made a number of key discoveries in analysis, topology, and set theory, and had a significant influence on the development of mathematics in the nineteenth century. In logic, Bolzano is best remembered for his variation logic (section 4.2 below), a surprisingly subtle and rigorous development of formal semantics. In this article, we discuss Bolzano's logic along with some of his work in the foundations of mathematics which has some bearing on logic." p. 177


"This paper surveys Bolzano's Beyträge zu einer begründeteren Darstellung der Mathematik (Contributions to a better-grounded presentation of mathematics) on the 200th anniversary of its publication. The first and only published issue presents a definition of mathematics, a classification of its subdisciplines, and an essay on mathematical method, or logic. Though underdeveloped in some areas (including, somewhat surprisingly, in logic), it is nonetheless a radically innovative work, where Bolzano presents a remarkably modern account of axiomatics and the epistemology of the formal sciences. We also discuss the second, unfinished and unpublished issue, where Bolzano develops his views on universal mathematics. Here we find the beginnings of his theory of collections, for him the most fundamental of the mathematical disciplines. Though not exactly the same as the later Cantorian set theory, Bolzano's theory of collections was used in very similar ways in mathematics, notably in analysis. In retrospect, Bolzano's debut in philosophy was a remarkably successful one, though its fruits would only become generally known much later."


"The paper is a detailed reconstruction of Bernard Bolzano's account of merely possible objects, which is a part of his ontology that has been widely ignored in the literature so far. According to Bolzano, there are some objects which are merely possible. While they are neither denizens of space and time nor members of the causal order, they could have been so. Thus, on Bolzano's view there are, for example, merely possible persons, i.e., objects which are neither actual nor persons but which could have been both. In course of the development of Bolzano's views, they are contrasted with the better known theory of his compatriot Alexius Meinong, and it is shown that they have a modern counterpart in the accounts of merely possible objects that were developed by Bernard Linsky and Ed Zalta, and by Timothy Williamson."

"Here is a brief outline of my paper. The first section is dedicated to the clarification of some basic Bolzanian notions, an understanding of which is needed for what follows. In the second section, I set out to establish that Bolzano in fact had the ontological view I attribute to him. That is, he accepted that there are merely possible objects. The third and final section is concerned with the exposition and reconstruction of Bolzano's account of mere possibilities." (p. 526)


"Several reconstructions of Bolzano's logical system have been proposed until now, some of them at the present workshop. They exploit systematically different aspects of Bolzano's logic and interpret it in terms of different XXth century systems. Such an approach has its own rights, as the full force of Bolzano's logic can be measured only by the standards of our contemporary logic. This is precisely the mark of great authors: each important discovery in their field brings to the light some hitherto unnoticed aspects of their work. That such reinterpretations are possible in the case of Bolzano, that his system can be represented in a quite different conceptual frame and translated into modern symbolic notation simply shows how rich and far reaching are his theories. Another argument favours this approach: a XXth century logician can read Bolzano and other logicians of the past only against the background of modern theories. It is in this way that the body of scientific knowledge is continuously being transmitted: by adapting and translating incessantly old theories into the present language. Moreover, the very meaning of past theories can often be understood only in the light of our systems. Already Husserl noticed that he would not have been able to grasp the significance of Bolzano's logic if he had not previously studied the most advanced contemporary logical theories - which in his case mainly meant the logic of Schroder! Nevertheless, this modernizing approach does not yield full justice to Bolzano. Even if some of his doctrines are definitively obsolete, they have their function in the construction of his system. Like his mathematics, his philosophy and his theology, Bolzano's logic was conceived in a specific historical context and its complete understanding requires a close attention to the logical and philosophical theories of his time. This is why a complementary approach seems necessary, namely a historical analysis which would trace the links between his system and the logical doctrines of his contemporaries as well as with great logical theories of the past.

My intention is to explain the formation and the structure of his logical system whose core is propositional logic. Bolzano's system of extensional relations between propositions represents one of the decisive innovations in the history of logic. It has no historical antecedents. It is nevertheless connected with logical theories of the late XVIIIth and early XIXth century and my paper tries to elucidate the genesis of Bolzano's system against this historical background. This approach will not only show the originality of Bolzano's achievement in full light, but also give a perhaps unexpected insight into the structure of his logical system.

In my reconstruction, I intend to remain within Bolzano's logic, using only conceptual tools which he himself has designed. Therefore, I shall neither attempt to translate his definitions into some XXth century notation, nor confront his logic with our systems. One of the advantages of this approach is to give a presentation of Bolzano's logic which is as simple as possible and has no recourse either to symbolic language (except for elementary set-theoretical notational devices) or to sophisticated semantic framework. Those who have tried to explain Bolzano's logical theories to non-specialists or even to students of modern logic may test the advantage of such an approach." (pp. 163-164).


"Analytical philosophy begins with the first mathematical and philosophical works of Bolzano published between 1804 and 1817. There, Bolzano set out a project for the global reform of mathematics by means of the axiomatic method. Having completed the Wissenschaftslehre, Bolzano wrote a summary of his logic for the Grossenlehre, which he sent to Exner in 1833. The correspondence between Bolzano and Exner covered some of the main subjects treated by analytical philosophy: the status of abstract objects (propositions and objective ideas), intuitions, objectless ideas, the concept of object and many others. While Bolzano argued in favor of abstract entities independent of mind and of language, Exner considered them as abstractions obtained from the subjective judgments and representations. During the XIXth century, Bolzano's philosophy spread over Bohemia and Austria through manuscripts and through the first edition of Zimmermann's textbook of philosophy. The most important Brenta-n ians, Kerry, Twardowski, Meinong and Husserl, discussed his doctrines which may also have influenced Wittgenstein and the Polish school."


"The incredible soundness of Husserl's judgment in the matter of logic is unique among his contemporaries - only Frege's insight is on par with it, if not superior. This is due to the lesson of Bolzano whose logic is the truth itself. Husserl adapted his logical system so that it became the logical basis of phenomenology. He adopted Bolzano's main ideas: the extension of logic to the theory of science, the
theory of ideal meanings, the distinction between mental act, linguistic expression, meaning and denoted object, the concept of analyticity. Independently of Bolzano and consonant with later mathematical theories, Husserl developed his formal analytics along two lines, apophatic and formal ontology. Bolzano, however, had articulated the domain of conceptual truths in the same manner: he constructed his logical system as a theory of meaning and his mathematics as a theory of object in general or Etwas überhaupt. Both set theory and mereology have their origin here. By his theory of science, Bolzano gave a new impetus to philosophy and logic. For the first time in modern thought, such questions as the nature of logical objects, the problems of meaning and reference, the relation between logic and language became central issues of philosophy."


"My purpose is to articulate a number of different senses in which one can be a pluralist and/or a relativist concerning logical consequence. I propose, first, that logical consequence is either polysemous or it denotes a cluster concept. In other words, there are a number of different notions that go by that name, often run together, or else there are several aspects of the notions, with varying weights. The different notions, or aspects, of consequence, turn on matters of modality, semantics, effectivness, justification, rationality, and form. Second, most of the articulations of the pre-theoretic notions(s) of logical consequence make essential use of a boundary between logical and non-logical terminology. This suggests a sort of relativism/pluralism explicitly noted by Bernard Bolzano and Alfred Tarski: logical consequence is relative to the logical/non-logical boundary. An argument may be valid on one collection of logical terms, invalid on another. Third, it is possible that at least some aspects of the notion of logical consequence are vague: there may be borderline cases of valid arguments. If so, we have to turn to what the correct account of vagueness is. On some theories of vagueness, consequence ends up as relative to something, such as a sharpening or a conversational context, and on others, we end up with a kind of pluralism. Finally, there are a number of interesting and important mathematical theories that employ a non-classical logic, and are rendered inconsistent if classical logic is imposed. This suggests a fourth kind of relativism/pluralism: relativity to structure."


"In Bolzano's view, a proposition is necessarily true iff it is derivable from true propositions that include no intuition (Anschauung). This analysis is historically important because it displays close similarities to Quine's and Kripke's ideas. Its systematic significnce, however, is reduced by the fact that derivability is defined with recourse to the method of variation, which we are allowed to apply even to propositions containing none of the respective variables. This liberaly leads to the result that, according to Bolzano's analysis, every truth is necessarily true. Even by introducing his condition of relevance (shared variables), Bolzano cannot avoid that some propositions come out as necessarily true which are merely contingently true."


"In the second volume of his Wissenschaftslehre (2) from 1837, the Bohemian philosopher, theologian, and mathematician Bernard Bolzano (1781-1848) introduced his concept of consequence, named derivability (Ableitbarkeit), together with a variety of theorems and further considerations. Derivability is an implication relation between sentences in themselves (Sätze an sich), which are not meant to be linguistic symbols but the contents of declarative sentences as well as of certain mental episodes. When Schmidt utters the sentence 'Schnee ist weiss', and Jones judges that snow is white, the sentence in itself expressed by Schmidt is the same as the one to which Jones agrees in thought. This sentence in itself is an abstract entity: in some sense, it exists; but it is unreal insofar as it lacks a position in space and time, does not stand in causal relationships, and is independent of the existence of thinking beings and languages. (3)"

(*) On the whole, this contribution is a summary of my book Der Begriff der Ableitbarkeit bei Bolzano (Siebel 1996).

(2) I refer to it by 'WL' plus number of volume, section, and page. It is partly translated by Rolf George: Theory of Science, Oxford 1972; but here translations are mine.

"Kant's famous definition of analyticity states that a judgement is analytic if its subject contains its predicate. Bolzano objects that (i) Kant's definiens permits an interpretation too wide, (ii) the definiens is too narrow, (iii) the definiendum is too limited, and (iv) the definition does not capture the proper essence of analyticity. Objections (i), (iii) and (iv) can be countered. Objection (ii) remains because, among other things, the Kantian definition has an eye only for an analysis of the subject within a judgement."

"Bolzano's theory of Collections (Inbegriffe) has usually been taken as a rudimentary set theory. More recently, Frank Krickel has claimed it is a mereology. I find both interpretations wanting. Bolzano's theory is, as I show, extremely broad in scope; it is in fact a general theory of collective entities, including the concrete wholes of mereology, classes-as-many, and many empirical collections. By extending Bolzano's ideas to embrace the three factors of kind, components and mode of combination, one may develop a coherent general account of collections. But it is most difficult to take Bolzano's view to fit modern set theory. So while Krickel's positive thesis is rejected, his negative thesis is confirmed."

"Bolzano's work will in due course be wholly accessible in print and should present relatively few problems of interpretation. I foresee a steadily growing reputation, but whether he comes to his just recognition will depend on attracting sufficiently many interested and talented commentators. The most promising centre of Bolzano studies is currently Hamburg, where a number of young enthusiasts have gathered around Wolfgang Künne.

Of the three philosophers I have mentioned, Bolzano is without doubt the most considerable. Meinong's theories are in the end unacceptably extreme and Brentano's work is often unclear in its implications, though both say things which are of much value to present-day discussions. On the other hand, whether one agrees with his semantic Platonism or not, Bolzano's views are up to the highest standards of contemporary discussion and in their clarity above much of it. His correspondence with Ferdinand Exner has been called the first text of modern analytical philosophy. Most work has to date concentrated on his logic and semantics, but his ethics, political philosophy, philosophy of religion and philosophy of mathematics all deserve greater exposure. The Complete Edition will serve as a definitive textual basis, but it is very expensive, and we badly need cheap study texts in English and German to complement it, and a good introduction to Bolzano in English. We also need to revise our histories of nineteenth-century philosophy to take adequate account of its greatest representative."


Translated from the Polish and German by Magda Stroinka and David Hitchcock. "We provide for the first time an exact translation into English of the Polish version of Alfred Tarski’s classic 1936 paper, whose title we translate as ‘On the concept of following logically’. We also provide in footnotes an exact translation of all respects in which the German version, used as the basis of the previously published and rather inexact English translation, differs from the Polish. Although the two versions are basically identical, to an extent that is even uncanny, we note more than 400 differences. Several dozen of these are substantive differences due to revisions by Tarski to the Polish version which he did not incorporate in the German version. With respect to these revisions the Polish version should be regarded as more authoritative than the German. Hence scholars limited to an English translation should use ours."

"After the original of this paper had appeared in print, H. Scholz in his article ‘Die Wissenschaftslehre Bolzanos, Eine Jahrhundert-Betrachtung’, Abhandlungen der Fries'schen Schule, new series, vol. 6, pp. 399-472 (see in particular p. 472, footnote 58) pointed out a far-reaching analogy between this definition of consequence and the one suggested by B. Bolzano about a hundred years earlier." [Note added by Tarski in English in Tarski (1956, 1983).] (p. 67).


"The aim of the paper is to present and evaluate Bolzano's theory of grounding, that is, his theory of the concept expressed and the relation brought into play by 'because'. In the first part of the paper (Sections 1-4) the concept of grounding is distinguished from and related to three other concepts: the concept of an epistemic reason}, the concept of causality, and the concept of deducibility (i.e., logical consequence). In its second part (Sections 5-7) Bolzano's positive account of grounding is reconstructed in axiomatic form and critically discussed."


"Bolzano holds that every sentence can be paraphrased into a sentence of the form "A has b". Bolzano's arguments for this claim are reconstructed and discussed. Since they crucially rely on Bolzano's notion of paraphrase, this notion is investigated in detail. Bolzano has usually been taken to require that in a correct paraphrase the sentence to be paraphrased and the paraphrasing sentence express the same proposition. In view of Bolzano's texts and systematical considerations this interpretation is rejected: Bolzano only holds that the sentence to be paraphrased and the paraphrasing sentence must be equipollent ("gleichgeltend"). It is shown that even this modest view of paraphrase does not help Bolzano in sustaining his claim that all sentences have the form "A has b"."


"How can we truly say that a is tired in the morning, and not tired at noon? Bolzano holds that every proposition about a contingent thing contains an idea representing a time in its subject-part. In this paper I reconstruct and assess Bolzano's arguments for his view of propositions about contingent things, comparing them to those of his main opponent, the view according to which every proposition about a contingent thing contains a copula combined with an idea that represents a time at which the object represented by the subject-part of the proposition has the property represented by the predicate-part (copula-modification view)."


"According to Bolzano, an object has necessary being if, and only if, there is a conceptual truth that ascribes being to it. I (Textor, 1996, chapter 5) proposed that the notion of conceptual truth bears the explanatory weight in Bolzano's theory of necessity because, ultimately, the truth of such a proposition depends only on the nature of the concepts it contains. Rusnock (2012) argues against this interpretation and proposes, in turn, that for Bolzano necessity and contingency are tied to free choice. In this article I will provide conceptual and historical background for Bolzano's view of necessity and use it to motivate my interpretation as well as to rebut Rusnock's criticism."


"Bernard Bolzano was born in 1781, the year of the publication of the first edition of Kant’s *Critique of Pure Reason*; he died in 1848, the year of Gottlob Frege's birth. These dates are symbolic. Bolzano’s work is a link between Kant’s philosophy and early analytic philosophy of which Frege is a key exponent. In this chapter I will discuss how Bolzano’s criticism of Kant shapes Bolzano's theory of propositions. In connection with this I will outline how Bolzano discovered the method of variation and give an overview of his results in employing this method."


"Bolzano incorporated Kant's distinction between intuitions and concepts into the doctrine of propositions by distinguishing between conceptual (*Begriffssätze an sich*) and intuitive propositions (*Anschauungssätze an sich*). An intuitive proposition contains at least one objective intuition, that is, a simple idea that represents exactly one object; a conceptual proposition contains no objective intuition. After Bolzano, philosophers dispensed with the distinction between conceptual and intuitive propositions. So why did Bolzano attach philosophical importance to it? I will argue that, ultimately, the value of the distinction lies in the fact that conceptual and intuitive truths have different objective grounds: if a conceptual truth is grounded at all, its ground is a conceptual truth. The difference in grounds between conceptual and intuitive truths motivates Bolzano's criticism of Kant's view that intuition plays the fundamental role in mathematics, a conceptual science by Bolzano's lights."


"Here I revisit Bolzano's criticisms of Kant on the nature of logic. I argue that while Bolzano is correct in taking Kant to conceive of the traditional logic as a science of the activity of thinking rather than the content of thought, he is wrong to charge Kant with a failure to identify and examine this content itself within logic as such. This neglects Kant's own insistence that traditional logic does not exhaust logic as such, since it must be supplemented by a transcendental logic that will in fact study nothing other than
thought's content. Once this feature of Kant's views is brought to light, a much deeper accord emerges between the two thinkers than has hitherto been appreciated, on both the nature of the content that is at issue in logic and the sense of logic's generality and formality.


"Bernard Bolzano (1781-1848) was a contemporary of the founders of non-Euclidean geometry and of the renovation of projective geometry. However, he did not participate in the movement transforming concepts and methods which crystallized in a new order of geometry at the beginning of the nineteenth century. On the contrary, throughout his life Bolzano tried to demonstrate Euclid's postulate of parallel lines.

Two ontological convictions played the role of epistemological obstacle for Bolzano and prevented him even from imagining the possibility that non-Euclidean geometries might exist. In the first place, Bolzano thought that Euclidean geometry had an intrinsic structure and thus geometrical space must be intrinsically Euclidean. Secondly, the description of this structure contained the existence of an "objective" connection between geometrical truths; a basic truth was, by its nature, "simple and general". This article forms part of the body of work aimed at identifying obstacles in the history of mathematics in order to confront them with obstacles to learning and to establish their epistemological character.


"Like Peirce, whom he preceded by roughly half a century, Bernard Bolzano (1781–1848), the brilliant mathematician, logician and semiotician who taught and wrote in Prague, was little recognized in his lifetime. Like Peirce, he endured persecution for his uncompromising attitudes, in his case both in science and political-religious life: also Bolzano’s teaching career, like Peirce’s, was cut short, in Bolzano’s case because of official displeasure of the Vatican and the Vienna court over his resolute and unwavering liberalism in religious, social and political matters and towards the relation of Czechs and Germans in the Bohemian crownlands of the Austro-Hungarian monarchy. Bolzano’s principal scientific contribution was, like Peirce’s, in the area of mathematics and logic; and Bolzano’s logic, like Peirce’s, contained major contributions to semiotics, which Bolzano called the theory of signs (Zeichenlehre) and Semiotik, though Bolzano’s Zeichenlehre was certainly not as comprehensive and systematic as Peirce’s semeiotic. Unlike Peirce, Bolzano is known primarily to logicians and to specialists in Catholic theology, while his semiotics has received relatively little attention."
On the website "Theory and History of Ontology"

The Philosophy of Bernard Bolzano: Logic and Ontology

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