Language as Calculus vs. Language as Universal Medium

INTRODUCTION

"An initial reference-point in this area is provided by Leibniz's distinction between two components of his ambitious project in mathematical logic or, rather, project to create a mathematical logic. On the one hand, Leibniz proposed to develop a *characteristica universalis* or *lingua characteristica* which was to be a universal language of human thought whose symbolic structure would reflect directly the structure of the world of our concepts. On the other hand, Leibniz's ambition included the creation of a *calculus ratiocinator* which was conceived of by him as a method of symbolic calculation which would mirror the processes of human reasoning.

When Leibniz's project began to be realized in the nineteenth century, its two components were taken up by different research traditions. The 'algebraic' school represented by Boole, Peirce, and Schröder sought to develop in the spirit of Leibniz's *calculus ratiocinator* mathematical techniques by means of which different kinds of human reasoning could be mastered. In contrast, Frege himself noted, his *Begriffsschrift* was to be primarily a *characteristica universalis* in Leibniz's sense, a *Formelsprache des reinen Denkens* (cf. here Sluga, "Frege against the Boolean", *Notre Dame Journal of Formal Logic*, 28, 1987, pp. 80-98). Admittedly, Frege made claims for it also as a *calculus ratiocinator*, but those claims were not met with enthusiasm. Husserl contradicted them, apparently thinking (as Tarski did later) that a *lingua universalis* cannot be purely formal. In any case, as Jourdain snidely noted, Frege's formalism was singularly clumsy as a means of actual reasoning: "... using Frege's symbolism as a calculus would be rather like using a three-legged stand-camera for what is called 'snap-shot' photography" (Jourdain, "Preface" to Louis Couturat, *The algebra of Logic*, 1914 pp. III-X). Subsequent attempts to find specific help for the purpose of concrete work in logic or in the foundations of mathematics have tended to confirm rather than to disconfirm Jourdain's judgment. The theoretical interest of Frege's ambitious project is due to its being an attempted *characteristica universalis* or at least *lingua characteristica mathematicae*, not to its being a viable *calculus ratiocinator*." (pp. IX-X)


"Answering Schröder's criticisms of *Begriffsschrift*, Frege states that, unlike Boole's, his logic is not a *calculus ratiocinator*, or not merely a *calculus ratiocinato*, but a *lingua characterica*. (1) If we come to understand what Frege means by this opposition, we shall gain a useful insight into the history of logic. The opposition between calculus ratiocinato and lingua characterica has several connected but distinct aspects. These various aspects, most of the time not stated by Frege, have to be brought out by a study of his work. From Frege's writings a certain picture of logic emerges, a conception that is perhaps not discussed explicitly but nevertheless constantly guides Frege. In referring to this conception I shall speak of the universality of logic.

This universality of Frege's *lingua characterica* is, first, the universality that quantification theory has in its vocabulary and that the propositional
calculus lacks. Frege frequently calls Boole's logic an 'abstract logic' (2), and what he means by that is that in this logic the proposition remains
unanalyzed. The proposition is reduced to a mere truth value. With the introduction of predicate letters, variables, and quantifiers, the proposition
becomes articulated and can express a meaning. The new notation allows the symbolic rewriting of whole tracts of scientific knowledge, perhaps of
all of it, a task that is altogether beyond the reach of the propositional calculus. We now have a lingua, not simply a calculus. Boole's logic, which
cannot claim to be such a lingua, remains the study, in ordinary language, of algebraic relations between propositions. This study is carried out in
ordinary language and is comparable to many branches of mathematics, say group theory. In Frege's system the propositional calculus subsists
embedded in quantification theory; the opposition between lingua and calculus is, in this respect, not exclusive, and that is why Frege writes that his
own logic is not merely a calculus ratiocinator. (3) However, the opposition between calculus ratiocinator and lingua characterica goes much
beyond the distinction between the propositional calculus and quantification theory. The universality of logic expresses itself in an important feature
of Frege's system. In that system the quantifiers binding individual variables range over all objects. As is well known, according to Frege, the
ontological furniture of the universe divides into objects and functions. Boole has his universe class, and De Morgan his universe of discourse,
denoted by '1'. But these have hardly any ontological import. They can be changed at will. The universe of discourse comprehends only what we
agree to consider at a certain time, in a certain context. For Frege it cannot be a question of changing universes. One could not even say that he
restricts himself to one universe. His universe is the universe. Not necessarily the physical universe, of course, because for Frege some objects are
not physical. Frege's universe consists of all that there is, and it is fixed." (pp. 324-325)

Notes

(1) Schröder's criticisms are contained in his review of Begriffsschrift, published in Zeitschrift für Mathematik und Physik 25 (1880), Historisch-
literarische Abteilung, 81-94. Frege's reply was an address to a learned society, delivered on 27 January 1882 and published in its proceedings,
'Über den Zweck der Begriffsschrift', Sitzungs-berichte der Jenaischen Gesellschaft für Medicin und Naturwissenschaft für das Jahr 1882 (Jena
the expression 'lingua characterica' see Günther Patzig's footnote 8, on p. 10 of Gottlob Frege, Logische Untersuchungen, Göttingen 1966.
(2) See, for instance, Frege's comments on Boole in 'Über den Zweck der Begriffsschrift' (mentioned in footnote 1), pp. 1-2.
(3) In 'Über die Begriffsschrift des Herr Peano and meine eigene', Berichte über die Verhandlungen der Königlichen Sächischen Gesellschaft der
Wissenschaften zu Leipzig, Mathematisch-physische Classe 48 (1897), 361-378, [English translation in: Gottlob Frege, Collected Papers on
logic is a calculus ratiocinator, but no lingua characterica; Peano's mathematical logic is in the main a lingua characterica and, subsidiarily, also a
calculus ratiocinator, while my Begriffsschrift intends to be both with equal stress." Here the terms are used with approximately the meanings given
in the present paragraph: Boole has a propositional calculus but no quantification theory; Peano has a notation for quantification theory but only a
very deficient technique of derivation; Frege has a notation for quantification theory and a technique of derivation.


"Sir Isaiah Berlin has shown how to understand Tolstoi on the basis of the insight that Tolstoi was a fox who believed that he was a hedgehog (1). It
is time we realize similarly what Frege was: a semanticist who did not believe in semantics. This insight we owe largely to van Heijenoort, who describes it by speaking of two conceptions of language and logic (2). He called them conceptions of logic as language and logic as calculus. More generally, and perhaps a shade more aptly, we might label them conceptions of language as the inescapable medium of communication (in brief, "language as medium") and language as calculus.

The most general form of the former I can think of is that we cannot according to this view get "outside" our language, as it were look on it from outside. The reason is that the results of all such "viewing" must be expressible in our language. Now this language presupposes in all its uses certain semantical relations (relations of representation) between language and reality. (Otherwise we could not use language in our transactions with reality.) But since these semantical relations are presupposed in each and every use of language, they cannot be expressed in language. Any attempt to do so involves a circularity and hence results in nonsense or tautology.

I am not putting forward these views as being unchallengeable. Indeed, they are challenged by the view of language and its logic as calculus. According to this view we can do all or most of the things the contrary opinion deemed impossible. Among other things, we can think of the representative relationships between language and the world as being varied radically and in a large scale. The point of using the term "calculus" is hence not to compare language to an uninterpreted calculus, a mere game with characters, but to emphasize that language, including our very own home language, is in principle freely reinterpretable like a calculus, at least for the purposes of a semanticist.

As van Heijenoort already pointed out, the development of all systematic logical semantics (model theory) thus presupposes some variant of the view of language as calculus. For one of the leading ideas of all model theory is to vary the interpretation of some part of the language in question in a way the view of language as medium does not countenance. As we saw, the stronger forms of this view even forbid saying anything significant and nonvacuous about the basic semantical relationships (relationships of naming, reference, or otherwise named representation)." (pp. 716-717)

Notes


On the website "Theory and History of Ontology" (www.ontology.co)

Selected Bibliography on Language as Calculus vs. Language as Universal Medium

*Mathesis Universalis: the Search for a Universal Science*